



Using *Fathom* to Investigate Scatter Plots


Focus Identify trends in scatter plots.

This table lists the Academy Award winners for Best Actress from 1973–2002, and the age of each actress in that year.




Academy Award Winners for Best Actress					
Actress	Year	Age	Actress	Year	Age
Glenda Jackson	1973	37	Jodie Foster	1988	26
Ellen Burstyn	1974	42	Jessica Tandy	1989	80
Louise Fletcher	1975	41	Kathy Bates	1990	42
Faye Dunaway	1976	35	Jodie Foster	1991	29
Diane Keaton	1977	31	Emma Thompson	1992	33
Jane Fonda	1978	41	Holly Hunter	1993	35
Sally Field	1979	33	Jessica Lange	1994	45
Sissy Spacek	1980	30	Susan Sarandon	1995	44
Katharine Hepburn	1981	74	Frances McDormand	1996	39
Meryl Streep	1982	33	Helen Hunt	1997	34
Shirley MacLaine	1983	49	Gwyneth Paltrow	1998	26
Sally Field	1984	38	Hilary Swank	1999	25
Geraldine Page	1985	61	Julia Roberts	2000	33
Marlee Matlin	1986	21	Halle Berry	2001	33
Cher	1987	41	Nicole Kidman	2002	35

To use *Fathom* to draw a scatter plot for these data, follow these steps:


1. Open *Fathom*. From the File menu, select New.
2. To enter the title:
Click on the New Collection icon , then click on the screen. Double click Collection 1. Type **Academy Award Winners for Best Actress** and click OK, as shown at the right.

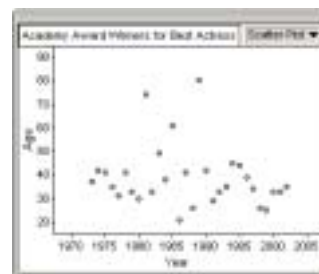


3. To enter the data:
Click on the New Case Table icon , then click on the screen.

Click on <new>; type **Year**, then press Enter. A new column appears to the right with the heading <new>. Click on the word <new>, type **Age**, then press Enter. Under the headings *Year* and *Age*, input the data from the table on page 185.

4. To graph the data:

Click on the New Graph icon , then click on the screen. Two axes appear. Click on the column heading, *Year*, and drag it to the horizontal axis. Click on the column heading, *Age*, and drag it to the vertical axis. *Fathom* creates a scatter plot, as shown at the right.



The range of the data is: $80 - 21$, or 59 years.

However, most of the data are located between 31 and 42 years.

The data form a **cluster** between these ages.

This means that, from 1973 to 2002, most winners were between 31 and 42 years old.

There are two large sections without points — between 49 and 61 years, and between 61 and 74 years. These represent gaps in the data. From 1973 to 2002, none of the winners were between 49 and 61 years old, or between 61 and 74 years old.

From the scatter plot, we can see that there is no pattern or **trend** in the data.

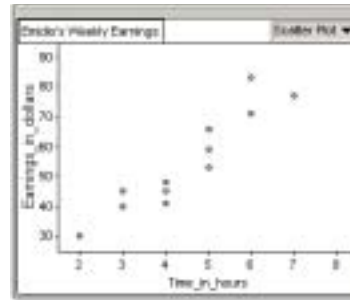
Here is another example of data in a scatter plot:

Emidio works part-time as a waiter in a restaurant. The table and scatter plot on page 187 show the hours he worked each week for 12 weeks, and his earnings, in dollars, for each week.

From the scatter plot, the points show an **upward trend** from left to right. Emidio's earnings increase as time increases. That is, the more hours per week Emidio works, the more he earns.

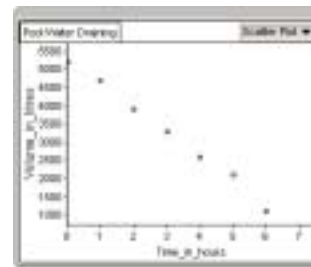


Emilio's Weekly Earnings	
Time, in hours	Earnings, in dollars
1	35
2	40
3	45
4	48
5	49
6	45
7	55
8	53
9	58
10	60
11	71
12	77



Here is a third example of data in a scatter plot:
 Water is draining out of a swimming pool. The table and scatter plot show the volume of water, in litres, remaining in the pool at one-hour intervals.

Pool Water Draining	
Time, in hours	Volume, in litres
1	5200
2	4700
3	3900
4	3000
5	2600
6	2100
7	1100



From the scatter plot, the points show a **downward trend** from left to right.
 The volume of water in the pool decreases as time increases.
 That is, the more hours the pool drains, the less water there is in it.

Check

Time	Speed (km/h)
12 noon	0
12:30	70
1:00	85
1:30	100
2:00	100
2:30	100
3:00	0
3:30	0
4:00	100
4:30	65
5:00	0

- Emilio drove from Toronto to Cornwall. She left Toronto at 12 noon and arrived in Cornwall at 5 p.m. The table at the left shows the speed of the car recorded at different times during her trip. Use *Fathom* to draw a scatter plot to represent the data. Describe any trends. What do you know from looking at the graph?

Reflect

Describe 3 different types of scatter plots.
 Provide an example for each type.

Explore

Work with a partner.

Madhu measured the mass of her pet guinea pig every 5 months, until it was 25 months old.

The data are shown at the left.

Age (months)	Mass (g)
5	200
10	350
15	480
20	510
25	520

- One person draws a bar graph. The other person draws a line graph.
- During which period did the guinea pig gain the most mass?
- What happened to the mass of the guinea pig after 15 months? Does this make sense?
- Predict the mass of the guinea pig at 30 months.



Reflect & Share

Discuss with your partner which graph is better for displaying the data. Justify your answer.

Estimate the mass of the guinea pig at 18 months.

Which graph is better to make this estimate? Explain.

Connect



A **line graph** displays data that change over time.

The line graph on page 189 shows how Leah's height changes as she gets older.

From 2 to 13 years, each line segment goes up to the right.

This shows that Leah's height increases.

From 13 to 16 years, the line segments still go up to the right, but they are not as steep.

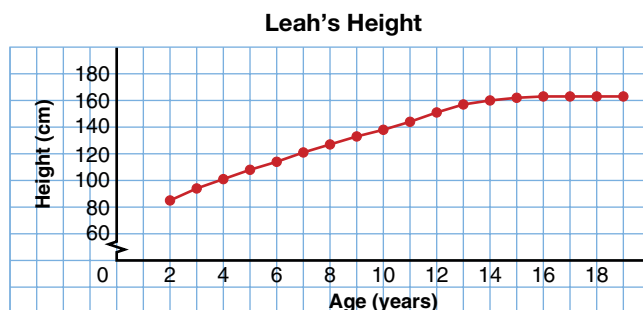
This shows that Leah's height increases, but at a slower rate than before.

From 16 years on, the line segments are horizontal.

This shows that Leah's height has stopped increasing.

She has stopped growing taller.

This symbol ↘ on the vertical axis means that the numbers from 0 to 60 are not shown.



We can use this line graph to find values between data points. At $11\frac{1}{2}$ years, Leah was about 148 cm tall.

Example

- Draw a line graph to display these data.
- Use the graph. Describe any trends in Canada's population.
- Use the graph. During which period did Canada's population increase the most? The least? How can you tell?
- Estimate Canada's population in 2011. Explain how you used the graph to do this. What assumption did you make?

Canada's Population (to the nearest million)	
Years	Number (millions)
1901	5
1911	7
1921	9
1931	10
1941	12
1951	14
1961	18
1971	22
1981	25
1991	28
2001	31

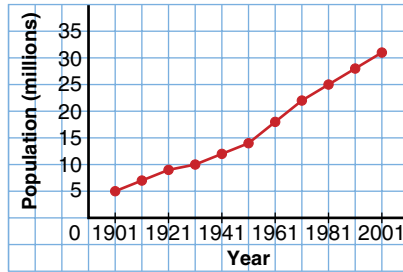
Solution

- Step 1.** Draw axes on grid paper. The horizontal axis represents time, in years. Use 1 square for each 10-year interval, starting in 1901. The vertical axis represents population, in millions. The greatest population to be graphed is 31 million. Make 35 the greatest number on the vertical scale. Use 1 square for every 5 million, starting at 0.

Step 2. Plot a point on the grid for each pair of entries in the table. Use a ruler to join adjacent points.

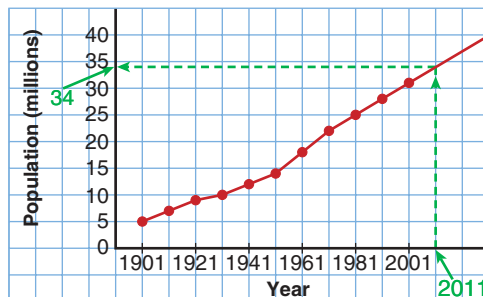
Step 3. Label the axes. Give the graph a title.

Canada's Population



- b) The graph goes up to the right.
This means that Canada's population increased from 1901 to 2001. In general, the line segments from 1951 to 2001 are steeper than the segments from 1901 to 1951.
This means that the population was increasing at a faster rate in the 2nd half of the century.
- c) The period in which the line segment is the steepest represents the greatest increase in population. This happened from 1951 to 1971. From 1921 to 1931, the line segment is the least steep. This is when Canada's population had the least increase.
- d) To predict Canada's population in 2011, extend the last line segment to 2011. From 2011 on the horizontal axis, draw a vertical line to the graph. From the graph, draw a horizontal line to the vertical axis. This line meets the axis at about 34 million. The population of Canada in 2011 will be approximately 34 million.
We assume that Canada's population will continue to grow at the same rate as it did from 1991 to 2001.

Canada's Population



Practice

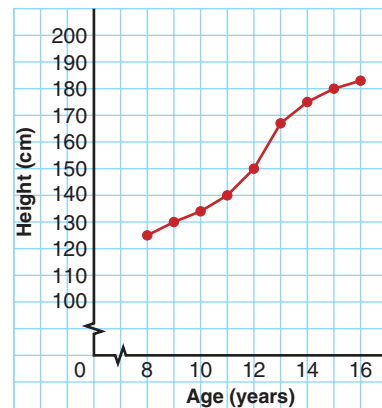
Calculator Skills

Three consecutive 2-digit numbers have a product of 551 286.

What is the sum of the three numbers?

1. a) What does this line graph show?
 b) About how tall was Nathan at each age?
 i) 8 years
 ii) 12 years
 iii) 15 years
 c) During which year did Nathan grow the most?
 The least?
 How does the graph show this?
 d) Predict Nathan's height at 18 years.
 Explain your reasoning.
 e) Predict Nathan's height at 50 years. Explain.

Nathan's Growth



Average Monthly Rainfall (cm)

Month	Vancouver	Ottawa
January	14.0	1.5
February	12.0	1.6
March	9.6	3.2
April	5.8	5.8
May	4.9	7.5
June	4.7	7.7
July	2.6	8.8
August	3.5	9.2
September	5.4	8.3
October	11.7	7.0
November	13.8	6.3
December	16.4	3.3

2. a) What does this table show?
 b) Draw a line graph for each city on the same grid.
 c) Describe the trends in rainfall for the two cities.
 How do the line graphs illustrate these trends?
 d) In which month(s) do the line graphs cross?
 What does this mean?
 e) What is the average annual rainfall for each city?
3. a) Research your region on one of these topics:
 i) the average precipitation for each month
 ii) the maximum temperature for the first day of each month
 iii) the average temperature for each month
 b) Organize the data in a table. Draw a line graph.
 c) Repeat part a for a city or region in a different part of Canada.
 d) Draw a line graph to display these data on the same grid as in part b.
 e) How are the line graphs alike? How are they different?
 f) Write all that you know from looking at the graphs.

Stopping Distance for a Car		
Speed (km/h)	Dry Pavement (m)	Wet Pavement (m)
0	0	0
10	3	6
20	7	11
30	12	17
40	16	26
50	23	38
60	30	50
70	38	64
80	49	78
90	60	97
100	72	120

4. a) What does stopping distance depend on?
 b) On the same grid, draw a line graph for dry pavement and for wet pavement.
 c) Why are line graphs suitable for these data?
 d) Describe the trends in the graphs.
 e) A car travels at 75 km/h on dry pavement. What is its stopping distance?
 f) A car takes 30 m to stop on wet pavement. How fast was it travelling?
 g) Write a question you could answer using the graph but not the table. Explain why you need the graph instead of the table.

5. **Assessment Focus** Nina owns a shoe store. These tables show data about the shoe store.

i)

Yearly Sales	
Year	Sales (\$)
1997	579 000
1998	621 000
1999	598 000
2000	634 000
2001	657 000
2002	642 000
2003	675 000

ii)

Sizes of Shoes Sold in May	
Size	Number of Pairs Sold
6	60
7	239
8	217
9	156
10	61
11	43
12	36



- a) Which data would be best represented with a line graph? Explain.
 b) Draw a line graph for the table you chose in part a.
 c) Describe the trends in the data.
 d) Which type of graph would be suitable for the other table? Explain. Graph the data from the other table.
 e) What do you know from looking at each graph?

Reflect

Why is a line graph the best type of graph to use to make predictions?



Using Spreadsheets to Record and Graph Data

Focus Display data on graphs using spreadsheets.

Spreadsheet software, such as *AppleWorks*, can be used to record, then graph, data.


This table shows the favourite sports of Grade 7 students in Mona's school.

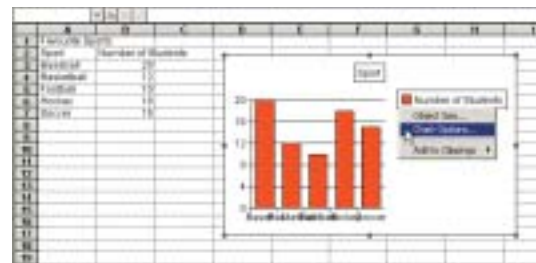
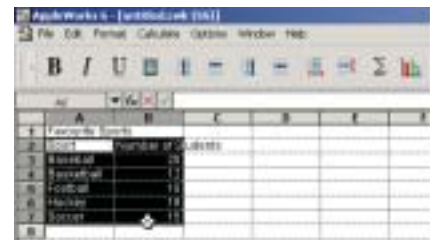
To graph these data using *AppleWorks*, follow these steps:

Open *AppleWorks*. Choose Spreadsheet. Enter the data into rows and columns in the spreadsheet.

Favourite Sports	
Sport	Number of Students
Baseball	20
Soccer	15
Hockey	18
Football	10
Basketball	12

To create a bar graph

1. Highlight the data. Include the column heads, but do not include the table title.
2. Click the graph icon  on the tool bar. A Chart Options dialogue box appears. Choose Bar, then click OK, as shown below left.

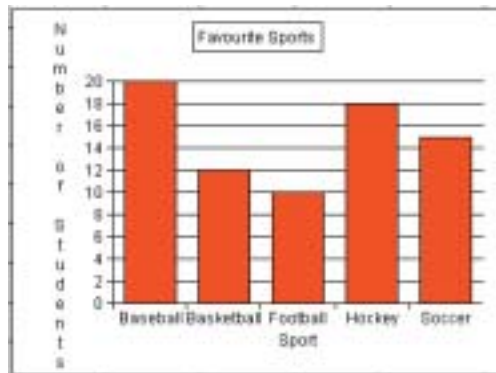


3. The software creates a legend, which is not needed for a bar graph. To remove the legend, right-click the graph. Choose Chart Options, as shown above right. Select the Labels tab. Click the box next to Show Legend to remove the check mark.

- In the Title box, type **Favourite Sports**, as shown below left. To insert labels, click the Axes tab. Select X axis. Type **Sport**. Select Y axis. Type **Number of Students**, as shown below right. Then click OK.



Your graph should look like the one below.




To create a circle graph

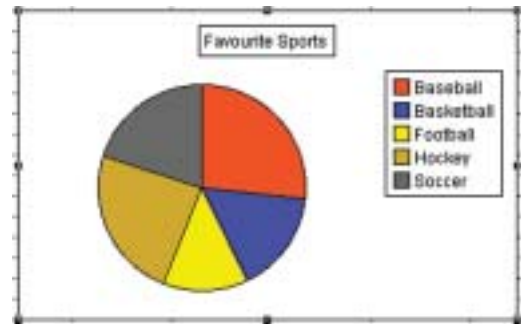
- Use the data on page 193. Highlight the data as shown below left. Do not include the column heads or title.

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		A3	fx Baseball
	A	B	C
1	Favourite Sports		
2	Sport	Number of Students	
3	Baseball	20	
4	Basketball	12	
5	Football	10	
6	Hockey	18	
7	Soccer	15	
8			



A spreadsheet program uses the term “pie” for a circle graph.

- Click the graph icon  on the tool bar. A Chart Options dialogue box appears. Choose Pie, then click OK, as shown bottom right, page 194. The circle graph shows a legend at the right. The legend shows what sport each sector represents.
- To add a title, right-click the graph. Choose Chart Options. Select the Labels tab. In the Title box, type **Favourite Sports**, as shown below left. Then click OK. Your graph should look like the one below right.




To create a line graph

Average Annual Income of Canadian Women

Year	Income (\$)
1989	28 219
1990	29 050
1991	29 654
1992	30 903
1993	30 466
1994	30 274
1995	30 959
1996	30 606
1997	30 484
1998	32 553
1999	32 026

These data came from the Statistics Canada website.

- Enter the data into rows and columns in the spreadsheet.
- Highlight the data. Include the column heads, but do not include the table title.
- Click the graph icon  on the tool bar. A Chart Options dialogue box appears. Choose X-Y Line, then click OK, as shown below.



4. The software creates a legend, which is not needed for a line graph. To remove the legend, right-click the graph. Choose Chart Options. Select the Labels tab. Click the box next to Show Legend to remove the check mark. In the Title box, type **Average Annual Income of Canadian Women**. Then click OK.

Your graph should look like the one below.

- What trend does the graph show? Explain.
- How often did the average income increase? Decrease?

How can you tell this from the graph?

- When did the average annual income have the greatest increase? Greatest decrease?
- Write what else you know from the graph.



1. Think about where you met your best friend. The table at the right lists the places where some Canadians met their best friends.
- Use a spreadsheet to draw a bar graph and a circle graph.
 - Which graph represents the data better? Explain.

Place	Number
School	5700
Work	4100
Club or organization	1400
Religious organization	700
Home/neighbourhood	4300
Through family	1200
Through friend	1100
Other	600

Average Value of \$1U.S.	
Year	Value in Canadian Dollars
1997	1.385
1998	1.484
1999	1.486
2000	1.485

2. Use a spreadsheet to draw a line graph for the data in the table at the left.
- What trend does the graph show?
 - Predict the average value, in Canadian dollars, of \$1U.S. in 2002 and in 2003. What assumptions did you make?
 - Research to find the average value, in Canadian dollars, of \$1U.S. in 2002 and 2003. Where could you get this information? How accurate was your prediction?

3. The data below are from the Statistics Canada website.
- Use a spreadsheet.
Create a graph to display the data.
 - Which type of graph best represents the data?
Explain.
 - Which type of graph could not be used to represent the data? Explain.

Math Link

Your World

Newspapers often publish survey results. The headlines may be misleading.

Try to find headlines like these. Cut them out. Explain why they are misleading.

Why would newspapers do this?

Average Hours of Television Viewing per Week, Fall 2001

Province	Adolescents 12–17 years
Newfoundland/Labrador	15.8
Prince Edward Island	12.4
Nova Scotia	14.1
New Brunswick	14.1
Quebec	14.5
Ontario	12.8
Manitoba	12.8
Saskatchewan	12.5
Alberta	13.3
British Columbia	10.9



Focus Understand which measure of central tendency best describes a set of data.

Explore

Work on your own.

Record on the board how many siblings you have.

Use the class data.

Find the mean, the median, and the mode.

Find the range.

The mean, median, and mode are **measures of central tendency**.

Each measure is an average.

Reflect & Share

With a classmate, discuss which measure of central tendency best describes the average number of siblings.

Connect



A clothing store sold jeans in these sizes in one day:

28 30 28 26 30 32 28 32 26 28 34 38 36 30 34 32 30

To calculate the mean jeans size sold, add the sizes, then divide by the number of jeans sold.

$$\begin{aligned}\text{Mean} &= \frac{28 + 30 + 28 + 26 + 30 + 32 + 28 + 32 + 26 + 28 + 34 + 38 + 36 + 30 + 34 + 32 + 30}{17} \\ &= \frac{522}{17} \\ &\approx 30.7\end{aligned}$$

The mean size is approximately 30.7.

To calculate the median, order the jeans sold from least size to greatest size. The middle number is the median.

There are 17 numbers. The middle number is 9th.

26, 26, 28, 28, 28, 28, 30, 30, **30**, 30, 32, 32, 32, 34, 34, 36, 38

The median size is 30.

The mode is the number that occurs most often.

There are two numbers that occur most often.

They are 28 and 30. So, the mode sizes are 28 and 30.

In this situation, the mean, 30.7, is of little use.

The mean does not represent a size.

When there is an odd number of data, to find the middle number: Add 1 to the number of data, then divide by 2. This gives the position of the middle number.

For example:

$$\frac{17 + 1}{2} = \frac{18}{2} = 9;$$

the middle number is 9th.

The median, 30, shows about one-half of the customers bought jeans of size 30 or smaller, and about one-half of the customers bought jeans of size 30 or larger.

The modes, 28 and 30, tell which sizes are purchased more often. The mode is most useful to the storeowner.

He may use the mode to order extra stock of the most popular sizes.

Example



A bookstore has 15 books in its young adult section. There are 5 different prices.

This table shows the number of books at each price.

Young Adult Books	
Price (\$)	Number of Books
8.99	3
9.99	5
13.99	5
32.99	1
37.99	1

- Find the mean, median, and mode prices.
- Which measure best represents the average price of a young adult book?
- What is the range of the prices?

Solution

a) Mean price:

- Multiply each price by the total number of books at that price, then add the prices.

$$(8.99 \times 3) + (9.99 \times 5) + (13.99 \times 5) + 32.99 + 37.99 = 217.85$$

- Divide the total price by the total number of books: 15

$$\frac{217.85}{15} \doteq 14.52$$

The mean price per book is approximately \$14.52.

Median price:

There are 15 books.

The table shows the books in order from least price to greatest price.

The median price is the 8th price. The 8th price is \$9.99.

The median price is \$9.99.

Mode price:

There are two mode prices. They are \$9.99 and \$13.99.

- The mean price is not charged for any of the books. Only two books cost more than the mean of \$14.52.

There are two mode prices.

One mode, \$9.99, is the same as the median price.

One-half the books cost less than the median price.

One-half cost more.

So, the median price, \$9.99, best represents the average cost of a young adult book at the store.

- c) For the range, subtract the lowest price from the highest:

$$37.99 - 8.99 = 29.00$$

The range of prices is \$29.00.

Practice

1. Here are Ira's practice times, in seconds, for the 100-m backstroke: 122, 118, 123, 119, 118, 120, 118, 121, 119
- Find the mean, median, and mode of these data.
 - Of the mean, median, and mode, which do you think best describes Ira's race time? Explain.
 - What is the range of these data?

2. Caitlin received these test marks in each subject.

- a) Find the mean, median, and mode for each subject.

- b) Explain what information each measure of central tendency gives.

- c) Which subject is Caitlin best at? Worst at? Explain your reasoning.

Caitlin's Marks							
Math	85	69	92	55	68	75	78
Spelling	72	81	50	69	81	96	92
History	68	74	82	80	76	67	74

Weekly Tips Earned (\$)		
Waiter	Week 1	Week 2
James	1150	600
Kyrra	700	725
Tamara	800	775
Jacob	875	860
George	600	1165

3. The table shows the tips earned by five waiters and waitresses during two weeks in December.

- Calculate the mean, median, and mode tips for each week.
- Calculate the mean, median, and mode tips for the two-week period.
- Compare your answers in parts a and b. Which are the same? Which are different? Explain why.
- Which measure of central tendency best represents the tips earned during the two weeks? Explain.



4. Jamal was training for a 400-m race. His times, in seconds, for the first five races were: 120, 118, 138, 124, 118
- Find the median and mode times.
 - Jamal wants his median time after 6 races to be 121 s. What time must he get in his 6th race? Explain.
5. Find 5 numbers that have a mean of 24 and a median of 25. Justify your answer. How many different sets of 5 numbers can you find? Show that each set has the mean and median stated.
6. A quality control inspector randomly selects boxes of crackers from the production line. She measures their masses. On one day she selects 30 boxes. The inspector records these data:
- 12 boxes: 405 g each
 - 4 boxes: 395 g each
 - 8 boxes: 390 g each
 - 4 boxes: 385 g each
 - 2 boxes: 380 g each
- Which expression can be used to calculate the mean mass? Use it to find the mean mass.
 - $\frac{(405 \times 12 + 395 \times 4 + 390 \times 8 + 385 \times 4 + 380 \times 2)}{30}$
 - $405 \times 12 + 395 \times 4 + 390 \times 8 + 385 \times 4 + 380 \times 2 \div 30$
 - For the shipment of crackers to be acceptable, the mean mass must be at least 395 g. Is this shipment acceptable? Explain.
7. **Assessment Focus**
- Use these data: 28, 30, 30, 31, 32, 33, 34, 35, 37, 38, 39, 41
- Find the mean, median, and mode.
 - What happens to the mean, median, and mode in each case?
 - Each number is increased by 10.
 - Each number is doubled.
 Explain the results.

Number Strategies

Use only the digits 0, 2, 3, and a decimal point. Use each digit only once each time.

Make as many numbers with 2 digits as possible.

Find the sum of the numbers.

Reflect

Use your answers from *Practice*. Describe a situation for each case.

- The mean is the best measure of central tendency.
- The median is the best measure of central tendency.
- The mode is the best measure of central tendency.

Explore

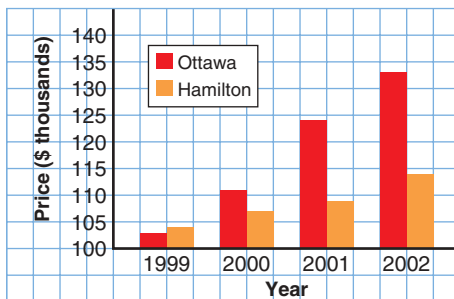
Work on your own.

What do the two graphs below show?

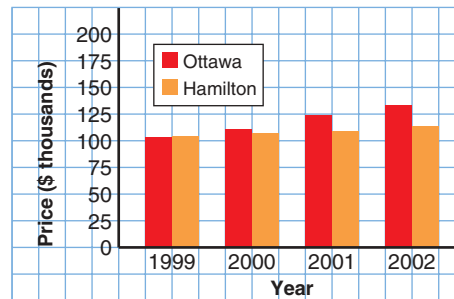
How are the graphs similar? How are they different?

Explain.

Average New House Prices



Average New House Prices



Reflect & Share

Discuss with a classmate:

What impression does each graph give? Explain.

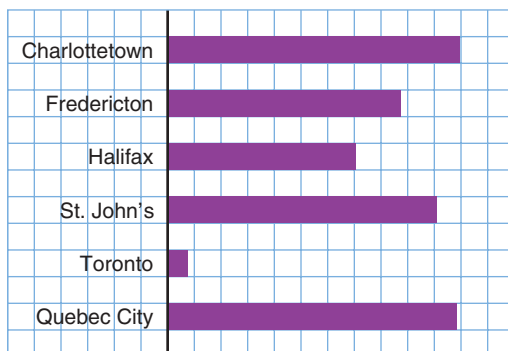
Who may want to use each type of graph?

Connect

Sometimes, graphs are used to distort information and to mislead.

Look at these graphs.

Average Annual Snowfall



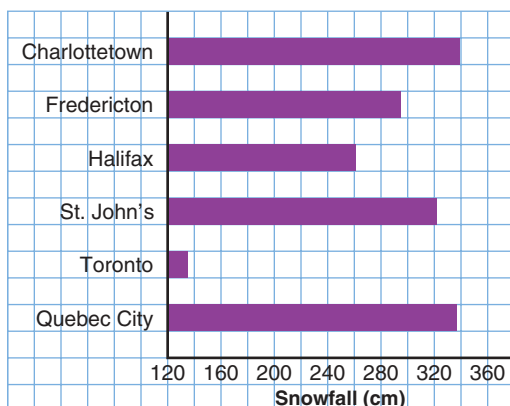
This bar graph is misleading.

It suggests that Quebec City has more than 10 times as much snow as Toronto.

This graph has no measurements of the depth of snow.



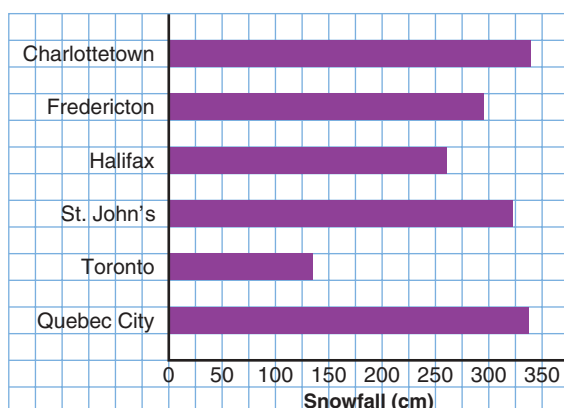
Average Annual Snowfall



Visually, this graph suggests the same information as the first graph. However, the horizontal scale is labelled with the snowfall.

The scale shows that Toronto has about 135 cm of snow and Quebec City has about 335 cm.

Average Annual Snowfall



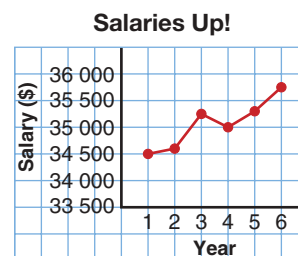
This graph accurately shows the data. The horizontal scale starts at 0.

The lengths of the bars are shown in the correct ratio. Quebec City has between 2 and 3 times as much snow as Toronto.

Example

This line graph is used to suggest that salaries have doubled in 6 years.

- Why is this graph misleading?
- Redraw the graph to show accurately how salaries have changed in 6 years.



Solution

- Use the vertical scale.

The salary in year 1 is about \$34 500.

The salary in year 6 is about \$35 750.

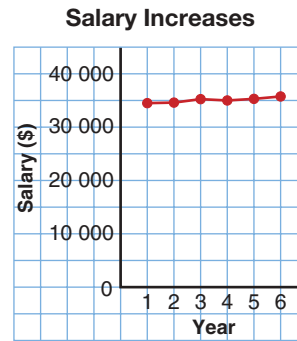
The increase in salary is: $\$35\,750 - \$34\,500 = \$1\,250$

The salaries have increased by only \$1250 in 6 years.

- Make a table from the line graph. Estimate each salary.

Year	Salary (\$)
1	34 500
2	34 600
3	35 250
4	35 000
5	35 300
6	35 750

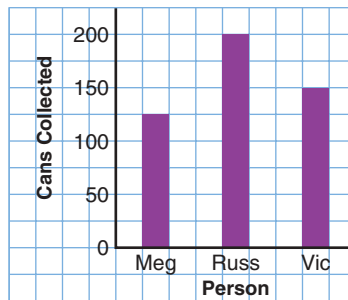
Draw a graph.
Start the vertical scale at \$0.
Use a scale of 1 square to represent \$5000.



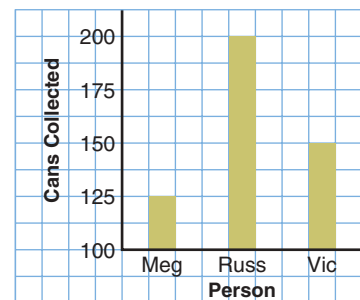
Practice

1. Which graph is misleading? Explain why it is misleading.

Food Drive Results



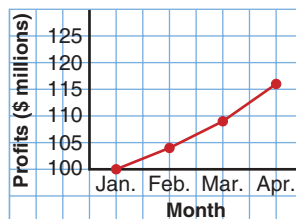
Food Drive Results



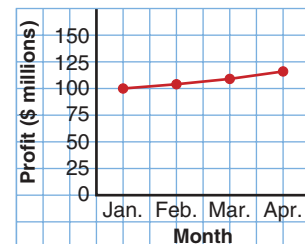
2. The two graphs display the Read Books Company's profits for a four-month period.



Read Books Co. Profits



Read Books Co. Profits



- Which graph might the president of the company choose to report profits to the shareholders? Why?
- Which graph might the rival company use to compare profits? Why?
- Predict the company's profit for May. What assumptions do you make?

Mental Math

Estimate each product.

- 3.9×4.1
- 0.5×10.2
- 1.1×11.1
- 20.5×0.9

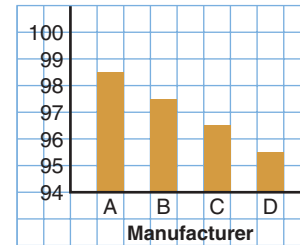
3. Why do some graphs display data in a misleading way? Describe how a graph might be drawn to misrepresent data.

4. A graph similar to this is part of an advertisement for a truck manufacturer.

With the graph,

Manufacturer A says that more than 98 out of 100 of its trucks sold in the last 10 years are still on the road.

Number of Trucks Still on the Road



- What impression does this graph give?
- How many trucks, out of 100, are still on the road for Manufacturer B? C? D?
- Do you think Manufacturer A's trucks are more dependable than the other manufacturers' trucks? Explain.
- Redraw the graph to give an accurate representation of the data.

Board of Directors' Expenses

Quarter	Amount (\$)
1st	85 000
2nd	104 000
3rd	125 000
4th	155 000

5. Draw a graph to display the data at the left in each way:
- The directors want the expenses to look low.
 - The shareholders want to show the expenses are too high.
 - Draw an accurate representation of the data.
- For each graph you draw, explain how it shows what you intended.

6. Assessment Focus

Use newspapers, magazines, or the Internet. Find a graph that creates a false impression.

- Describe how the graph creates a false impression.
- Why might the misleading graph be used?
- How could the graph be changed to present the data more accurately?
- Use your suggestions in part c to draw an accurate graph.

Reflect

Describe two ways in which a graph may be misleading.

Making a Study Sheet



1 Be sure that you know exactly what will be covered on the test or quiz. Ask your teacher if you are not sure. Highlight these areas in your notebook.

2 List the math vocabulary you will need. Write definitions and provide examples if you need to.

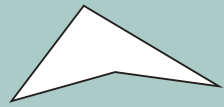

Congruent figures have exactly the same size and shape. Corresponding angles have the same measure. Corresponding sides have the same length.

3 Colour code any formulas you will need.

Area of rectangle = Length \times Width
Area of triangle = $\frac{1}{2} \times$ Base \times Height

4 Use pictures to help you remember.

Polygon: A closed figure formed by three or more line segments

YES:  NO: 

5

Copy out a sample problem. Include all necessary examples and discussion.

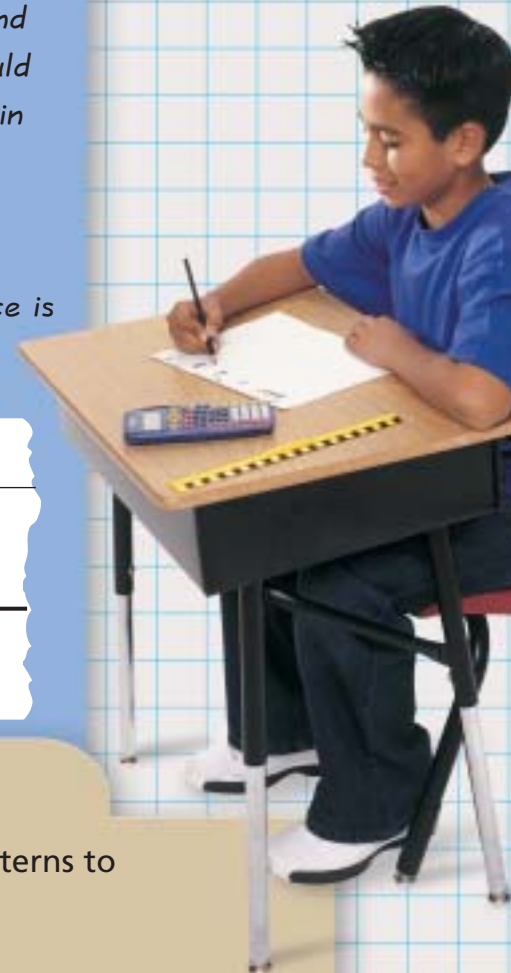
Sample Problem: You have a choice of recess times:

- You may have 30 min a day for the next two weeks,
OR
- You may have 1 min of recess the 1st day, 2 min the 2nd day, 4 min the 3rd day, 8 min the 4th day, and so on, for the next two weeks. Which choice would give you more recess time in the 2 weeks? Explain your answer.

Solution

This problem is about finding out which recess choice is better. The strategy I will use is to make a table.

Day	1	2	3
Choice 1 (min)	30	30	30
Total time (min)	30	60	90
Choice 2 (min)	1	2	4
Total time (min)	1	3	7



6

Use word association and patterns to help you remember.

A hexagon has six sides. (Both have an X.)

Octagon -- Octopus (Both have 8 sides/arms.)

What Do I Need to Know?

- ✓ **Primary data** are data you collect.
- ✓ **Secondary data** are found from databases on the Internet, or a library, or other sources.
- ✓ In a set of data:

The **mean** is the sum of numbers divided by the number of numbers in the set.

The **median** is the middle number, when there is an odd number of data in the set. When there is an even number of data, the median is the mean of the two middle numbers.

The **mode** is the number that occurs most often. A set of data can have no mode, one mode, or more than one mode.

What Should I Be Able to Do?

For extra practice, go to page 442.

LESSON

- 5.1** 1. A recreation group wants to find out the favourite summer activity of teenagers in the town.
- a) Write an unbiased survey question.
 - b) Write a biased survey question. Explain how it is biased.



- 5.2** 2. The data in this table are from Statistics Canada.
- 5.5**

Average Weekly Earnings in 2001

Job	Earnings (\$)
Forestry	831
Mining	1153
Utilities	1000
Construction	800
Real Estate	612
Education	696
Health Care	584
Transportation	742

- a) What do the data show?
- b) Draw a graph to display the data.
- c) Explain your choice of graph. Could another type of graph have been used? Why?
- d) Which job has the highest weekly earnings?
The lowest weekly earnings?
How is this shown in your graph?
- e) How can the data be used to find the average annual earnings? The average monthly earnings?
How might the graph be used to find these?
- f) Which average: mean, median, or mode, do you think is shown in the table?
Explain your answer.

5.5 3. Jacob collected the data in the table below. The table lists 5 TV shows that air on Sunday evenings. The survey question was “Which of these shows is your favourite?”

Everyone Loves Jordan___, Girl Meets World___, Lost in Time___, Metro PD___, Reality Shock___”

TV Show	Tally	Frequency
Everyone Loves Jordan	### ### ## //	
Girl Meets World	### ###	
Lost in Time	### ////	
Metro PD	////	
Reality Shock	### ### ## ##	

- a) Write the frequency of each response.
- b) How many people did Jacob survey?
- c) Suppose you were a TV producer and had to take one show off the air. Which show would it be? Explain.
- d) Suppose you were an advertiser. Which show would you advertise with? Why?
- e) Display the data using:
 - i) a pictograph
 - ii) a bar graph

5.3 4. A quality control inspector measures the masses of boxes of raisins. He wants to know if the average mass of a box of raisins is 100 g. He randomly chooses boxes from the production department. The masses, in grams, are recorded below.

Masses of Boxes of Raisins in Grams

99.1, 101.7, 99.8, 98.9, 100.8, 100.3, 98.3, 100.0, 97.8, 97.6, 98.5, 101.7, 100.2, 100.2, 99.4, 100.3, 98.8, 102.0, 100.3, 98.0, 99.4, 99.0, 98.1, 101.8, 99.8, 101.3, 100.5, 100.7, 98.7, 100.3, 99.3, 102.5

- a) Draw a stem-and-leaf plot.
- b) What can you tell from the plot that you could not easily see from the data?
- c) Will this shipment be approved? Explain.
- d) What is the median?
- e) What is the mode?

- f) Would the shipment be approved if the mode was used? If the median was used? Explain.

- 5.4** **5.** The mean monthly rainfalls for Calgary and Charlottetown are given.

Mean Monthly Rainfall (cm)		
Month	Calgary	Charlottetown
January	1.2	9.8
February	1.0	8.2
March	1.5	7.6
April	2.5	7.5
May	5.3	8.0
June	7.7	7.9
July	7.0	7.4
August	4.9	9.0
September	4.8	9.2
October	1.6	9.9
November	1.2	11.5
December	1.3	10.0

- a) Draw a line graph for each city on the same grid.
 b) Describe each graph. Are there any trends? Explain.
 c) Do the line graphs cross? What does this mean?
 d) Determine the mean annual rainfall for each city. Explain your steps.
 e) Determine the median monthly rainfall for each city.
 f) What else do you know from the table or the graph?

- 5.5** **6.** In each case, which is most useful: the mean, median, or mode? Justify your answer.

- a) A storeowner wants to know which sweater sizes he should order. Last week he sold 5 small, 15 medium, 6 large, and 2 X-large sweaters.
 b) Five of Robbie's friends said their weekly allowances are: \$5, \$8, \$10, \$6, and \$5. Robbie wants to convince his parents to increase his allowance.
 c) Tina wants to know if her math mark was in the top half or bottom half of the class.

- 5.5** **7.** A small engineering company has an owner and 5 employees.
5.6 This table shows their salaries.

Company Salaries	
Position	Annual Salary (\$)
Owner	130 000
Manager	90 000
2 Engineers	50 000
Receptionist	28 000
Secretary	28 000

Which measure of central tendency would you use to describe the average annual salary in each case?

- a) You want to attract a new employee.
 b) You want to suggest the company does not pay its employees well.

Practice Test

800-m Race Times

3:25, 3:07, 1:55, 3:11
 2:41, 2:47, 3:04, 1:58
 2:35, 3:25, 3:08, 2:53
 2:29, 3:15, 2:55, 2:28
 3:14, 2:47, 3:07, 2:39
 2:39, 2:43, 3:19, 2:54
 3:11, 2:59, 2:42, 2:57

1. a) Explain the difference between primary and secondary data.
 b) State if primary data or secondary data are used in each case. Justify your answers.
 - i) Anna searched the Internet to find the top ten movies of the year.
 - ii) Rory phoned his friends to ask if they were coming to his party.

2. Here are the times, in minutes and seconds, of 28 people who competed in an 800-m race.
 - a) Display the data in a stem-and-leaf plot.
 - b) What is the range of times for the race?
 - c) Find the median time.
 - d) Is there a mode time? If so, what is it? If not, explain how you know.

3. Parham received these marks on 7 math tests:
 91, 75, 95, 80, 83, 86, 68
 What mark will he need on his next test in each case?
 Justify each answer.
 - a) The mean of his 8 tests is 84.
 - b) The mode of his 8 tests is 86.
 - c) The median of his 8 tests is 84.

Country of Origin for Visitors to Canada

Country	Number of Visitors
Japan	500 000
France	400 000
Germany	400 000
Australia	175 000
United Kingdom	850 000
Mexico	150 000
Hong Kong	125 000

4. a) Graph the data at the left. Explain your choice of graph.
 b) In the year 2000, approximately 15 000 000 people visited Canada from the U.S. Suppose you want to add these data to your graph in part a. How would your graph change?
 c) Redraw the graph in part a. Include the data from part b.
 d) Compare the two graphs. Which graph gives you more information? Justify your answer.

Your help is needed to organize a school Winter Carnival Day.

Think about:

- the classes or grades that should participate
- which activities students would enjoy
- which snacks and drinks should be offered

You will collect, display, and analyse data.

Part 1

Brainstorm with your classmates. Keep a record of the ideas.

Here are some topics you may want to discuss:

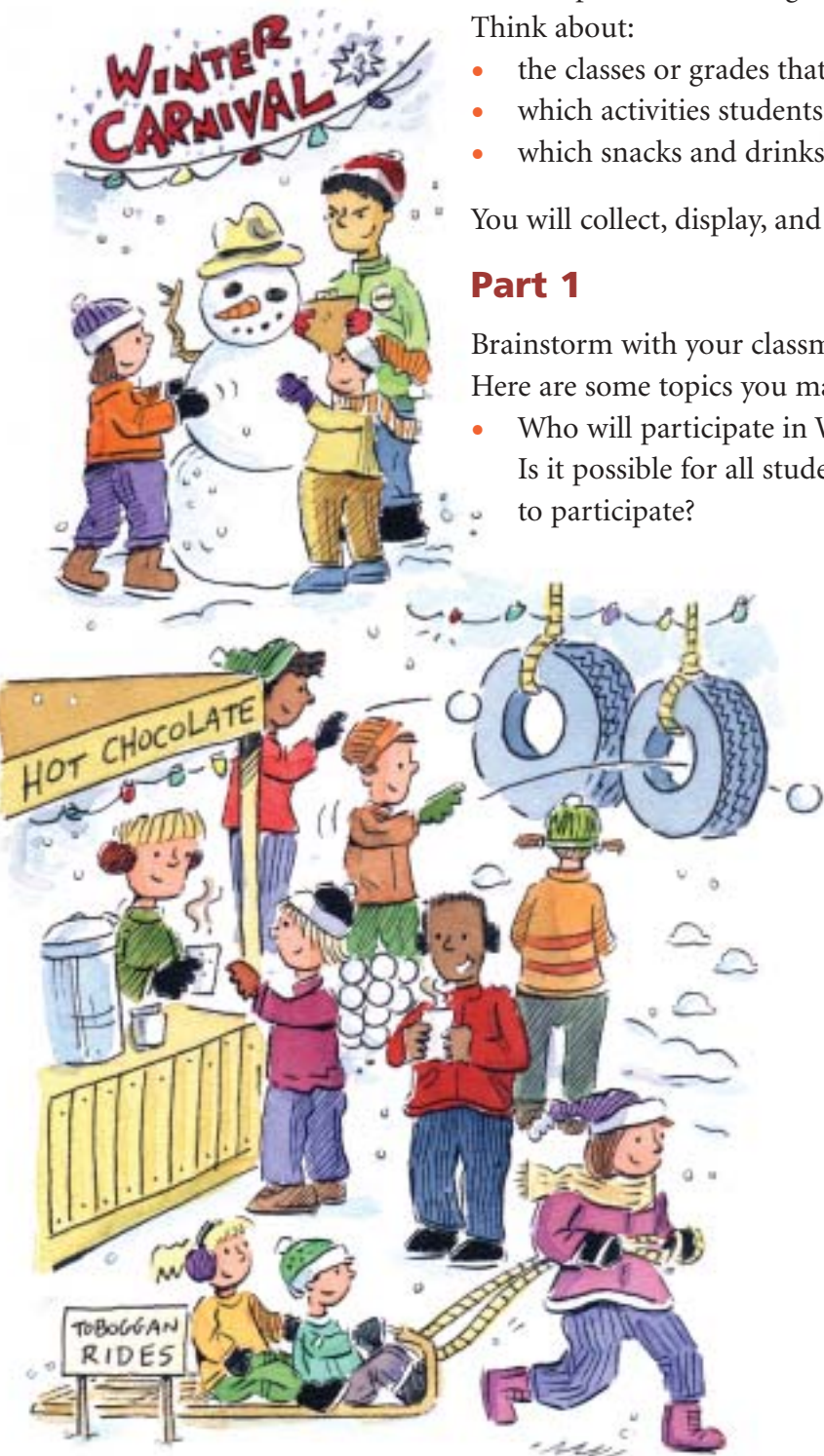
- Who will participate in Winter Carnival Day?
Is it possible for all students in every grade in your school to participate?

Are there special considerations for different age groups?

- What type of indoor and outdoor sports, games, and crafts do you think students in your school would enjoy?
- What types of snacks and drinks should be offered?
How much will this cost?

After brainstorming, break into five groups. Each group will be responsible for collecting data and reporting on one of the following topics:

- Indoor sports
- Outdoor sports
- Games
- Crafts
- Snacks and drinks



Part 2

Your group will collect, display, and analyse data related to your task.

- Write a survey question or questions related to your task. Explain how bias can be avoided when writing the question.
- Conduct the survey.
- Display the collected data in different ways. Justify your displays.

- Analyse the data.

What decisions can be made about Winter Carnival Day from your results?

- Prepare a report of your findings.

Check List

Your work should include:

- ✓ your survey question and data collection plan
- ✓ at least two displays of your data
- ✓ justification for the procedures and displays you chose
- ✓ your analysis and recommendations

Part 3

Present the results of your group work to the rest of the class.



Reflect on the Unit

Describe the different types of graphs you drew.
What can you tell about the data from each type of graph?
Use examples to explain.

UNIT

6

Measuring Perimeter and Area

You see geometric figures all around you.

Look at these pictures.
Identify a figure.

What would you need to know to find the area of that figure?

What would you need to know to find the perimeter of the figure?

What You'll Learn

- Use formulas to find the areas of a parallelogram, a triangle, and a trapezoid.
- Find the area and perimeter of an irregular figure.

Why It's Important

- The ability to measure is a life skill. You measure to find how much paint you need for a wall; how much fencing you need for a garden; how much material you need for drapes; and so on.





Key Words

- perimeter
- area
- parallelogram
- triangle
- trapezoid



Skills You'll Need

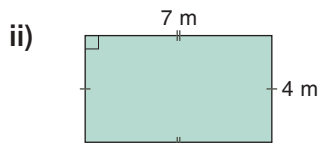
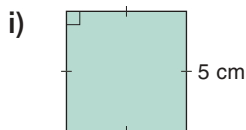
Perimeter and Area of a Rectangle

Perimeter is the distance around a figure.

Area is the amount of surface a figure covers.

Example 1

a) Find the perimeter of each figure.



b) Find the area of each figure in part a.

Solution

a) i) The figure is a square.

The perimeter of a square is: $P = 4s$

Substitute $s = 5$.

$$P = 4 \times 5$$

$$= 20$$

The perimeter is 20 cm.

ii) The figure is a rectangle.

The perimeter of a rectangle is: $P = 2(b + h)$

Substitute $b = 7$ and $h = 4$.

$$P = 2(7 + 4)$$

$$= 2 \times 11$$

$$= 22$$

The perimeter is 22 m.

b) i) The area of a square is: $A = s^2$

Substitute $s = 5$.

$$A = 5^2$$

$$= 5 \times 5$$

$$= 25$$

The area is 25 cm².

ii) The area of a rectangle is: $A = bh$

Substitute $b = 7$ and $h = 4$.

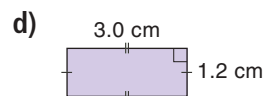
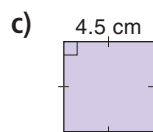
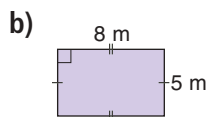
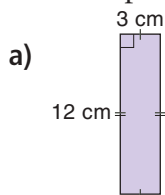
$$A = 7 \times 4$$

$$= 28$$

The area is 28 m².

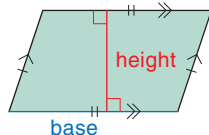
✓ Check

1. Find the perimeter and area of each figure.

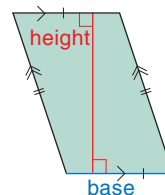


Focus Use a formula to find the area of a parallelogram.

This is a **parallelogram**.
How would you describe it?



Here is the same parallelogram.
Any side of the parallelogram is a base.
The height is perpendicular to the base.



Explore

Work with a partner.

You will need a tangram and grid paper.

- One tan is a parallelogram. Find its area.
- Make another parallelogram by combining tans. Find the area of the parallelogram.
- Continue to combine tans to make different parallelograms. Find the area of each parallelogram you make.
- Record your work. Draw each parallelogram on grid paper.
- Use variables. Write a formula to find the area of a parallelogram.



Reflect & Share

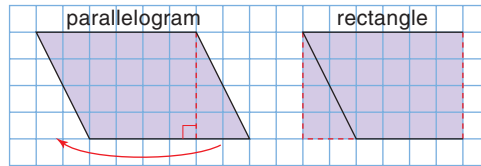
How did you find the area of each parallelogram?

Which different strategies did you use?

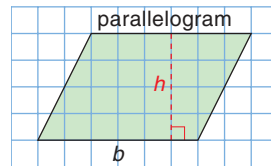
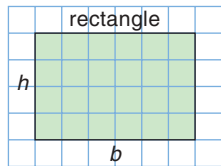
Which strategy helped you write the formula for the area?

Recall that both a rectangle and a square are parallelograms.

Any parallelogram that is not a rectangle can be “cut” and rearranged to form a rectangle.



The parallelogram and the rectangle have the same area.
 The area of a parallelogram is equal to the area of a rectangle with the same height and base.
 To find the area of a parallelogram, multiply the base by the height.



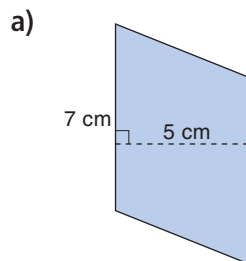
b represents the base.
 h represents the height.

Area of rectangle:
 $A = bh$

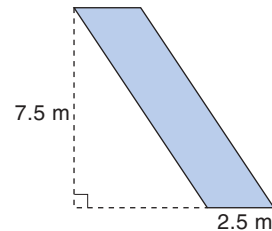
Area of parallelogram:
 $A = bh$

Example

Calculate the area of each parallelogram.



b) The height can be drawn outside the parallelogram.



Solution

a) $A = bh$
 Substitute $b = 7$ and $h = 5$.
 $A = 7 \times 5$
 $= 35$
 The area of the parallelogram is 35 cm^2 .

b) $A = bh$
 Substitute $b = 2.5$ and $h = 7.5$.
 $A = 2.5 \times 7.5$
 $= 18.75$
 The area of the parallelogram is 18.75 m^2 .

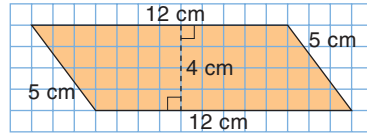
Practice

Recall that you can use a protractor to draw the height perpendicular to the base.

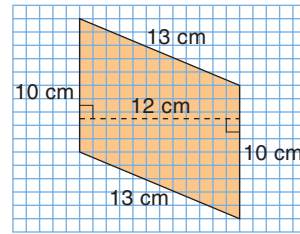
The base of a parallelogram is not always horizontal.

1. Identify one base and height of each parallelogram.

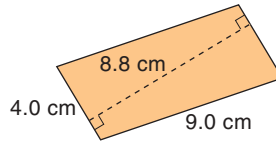
a)



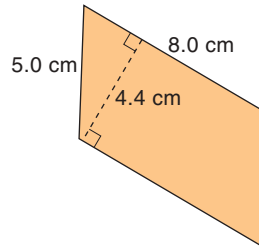
b)



c)



d)



2. Find the area of each parallelogram in question 1.

3. a) On 1-cm grid paper, draw 3 different parallelograms with each base and height.

i) base: 3 cm; height: 5 cm ii) base: 3.5 cm; height: 7.0 cm

- b) Find the area of each parallelogram you drew in part a. What do you notice?

4. On 1-cm grid paper, draw as many different parallelograms as you can with each area.

a) 10 cm^2

b) 18 cm^2

c) 28 cm^2

5. **Assessment Focus** Use 1-cm grid paper.

Draw a parallelogram, which is not a rectangle, with base 6 cm and height 4 cm.

a) What is the area of the parallelogram?

b) Change the base to draw a parallelogram with twice the area. What is the base?

c) Change the height to draw a parallelogram with twice the area. What is the height?

d) Change the base and height to draw a parallelogram with twice the area.

How many different pairs of base and height can you find? Show your work.



Mental Math

It is Wednesday, January 14.
Kim and Sun-Yi are working together.

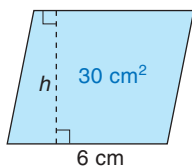
Kim works every Wednesday.
Sun-Yi works every 5th day.

When will Kim and Sun-Yi next work together?

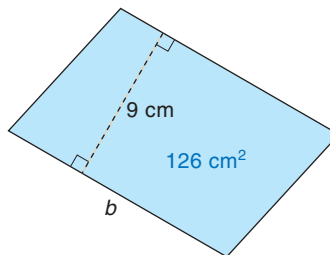


6. The area of each parallelogram is given. Find each unknown measure.

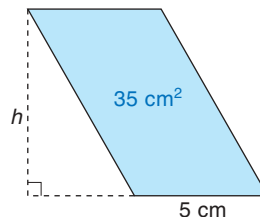
a) the height



b) the base



c) the height

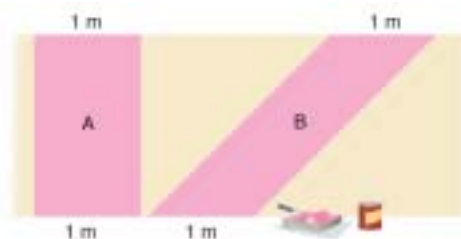


7. Use 1-cm grid paper.

Draw a rectangle with the same area as each parallelogram in question 6.

How many different ways can you do this?

8. Sasha is buying paint for a design on a wall. Here is part of the design. Sasha says figure B will need more paint than figure A. Do you agree? Explain.



9. You will need 1-cm grid paper, ruler, and tracing paper. Draw a parallelogram with base 10 cm and height 6 cm. Draw a diagonal to make two triangles.

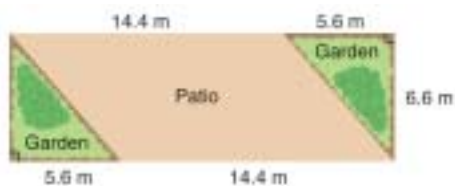
a) What do you notice about the two triangles?

How can you check your observation?

b) What is the area of the parallelogram?

c) What is the area of each triangle? How do you know?

Take It Further



10. A restaurant owner built a patio in front of his store to attract more customers.

a) What is the area of the patio?

b) What is the total area of the patio and gardens?

c) How can you find the area of the gardens?

Show your work.

Reflect

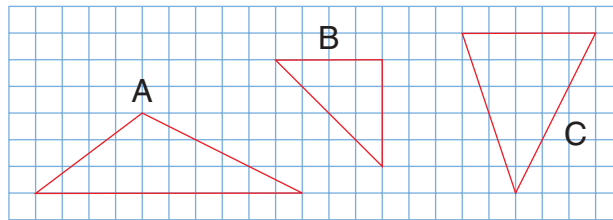
What do you need to know to find the area of a parallelogram?
Use an example to explain.

Explore

Work with a partner.

You will need a ruler and 1-cm grid paper.

Draw each triangle below on 1-cm grid paper.



- How many different ways can you find the area of each triangle? What strategies did you use?
- Use what you know about parallelograms. Find the area of each triangle.
- Use variables. Write a formula to find the area of a triangle.

Reflect & Share

How did you use a parallelogram to find the area of a triangle?

Compare your formula with that of another pair of classmates.

If the formulas are different, can both of them be used to find the area of a triangle? Explain.

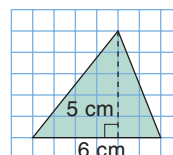
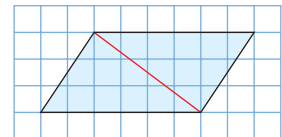
Connect

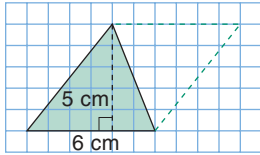
When we draw a diagonal in a parallelogram, we make 2 congruent triangles.

Congruent triangles have the same area.

So, the area of one triangle is $\frac{1}{2}$ the area of the parallelogram.

To find the area of this triangle:





Complete a parallelogram on one side of the triangle.

The area of the parallelogram is:

$$A = \text{base} \times \text{height, or } A = bh$$

$$\text{So, } A = 6 \times 5$$

$$= 30$$

The area of the parallelogram is 30 cm^2 .

So, the area of the triangle is: $\frac{1}{2}$ of $30 \text{ cm}^2 = 15 \text{ cm}^2$

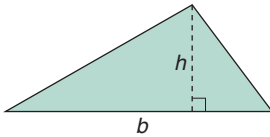
We can write a formula for the area of a triangle.

$$A = \frac{1}{2} \text{ base} \times \text{height}$$

$$A = \frac{1}{2} bh$$

$$\text{or } A = bh \div 2$$

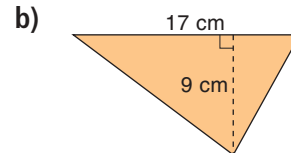
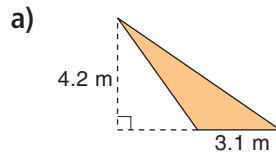
$$\text{or } A = \frac{bh}{2}$$



Example

Find the area of each triangle.

For an obtuse triangle, the height might be drawn outside the triangle.



Solution

a) $A = \frac{bh}{2}$

Substitute $b = 3.1$

and $h = 4.2$.

$$A = \frac{3.1 \times 4.2}{2}$$

$$A = 6.51$$

The area is 6.51 m^2 .

b) $A = \frac{bh}{2}$

Substitute $b = 17$ and $h = 9$.

$$A = \frac{17 \times 9}{2}$$

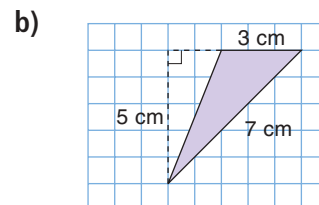
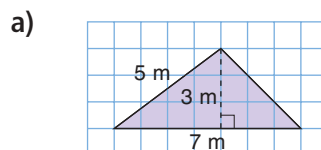
$$A = \frac{153}{2}$$

$$= 76.5$$

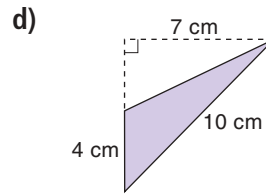
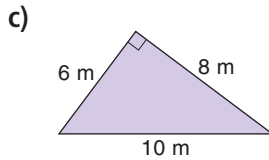
The area is 76.5 cm^2 .

Practice

1. Identify one base and height of each triangle.

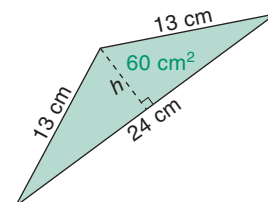
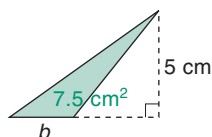
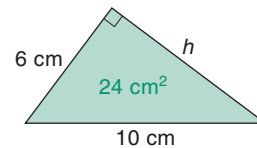
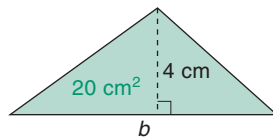


In a right triangle, one base and height are two sides of the triangle.



2. Find the area of each triangle in question 1.
3. a) On 1-cm grid paper, draw 3 different triangles with each base and height.
 - i) base: 4 cm; height: 3 cm
 - ii) base: 7.5 cm; height: 6.5 cm
 b) Find the area of each triangle you drew in part a. What do you notice?
4. On 1-cm grid paper, draw two different triangles with each area.
 - a) 16 cm^2
 - b) 8 cm^2
 - c) 10 cm^2
5. Use 1-cm grid paper.
 - a) Draw a triangle with area 12 cm^2 .
 - b) Investigate the different ways you can draw a triangle that has:
 - i) double the area
 - ii) one-half the area
 Write a report of your findings.
6. Use 1-cm grid paper.
 - a) Draw different triangles with base 4 cm and height 6 cm.
 - b) Find the area of each triangle you draw.
 - c) Measure the side lengths of each triangle you draw. How do you know all the triangles are different?

7. The area of each triangle is given. Find each unknown measure.
 - a) the base
 - b) the height



Calculator Skills

Which is the best deal?
How do you know?

250 g cheese for \$2.99

400 g cheese for \$4.99

600 g cheese for \$6.79



Math Link

Your World

You use rulers and protractors to measure in the classroom.

Which measuring instruments do you have at home?

What do these instruments measure?

List all the instruments you can find.

Give an example of what each one measures.

8. When you know the area of a triangle, and its base, how can you find its height? Use an example to explain.

9. **Assessment Focus** The owner of a house paints this attic wall. There is a small rectangular window in the wall.

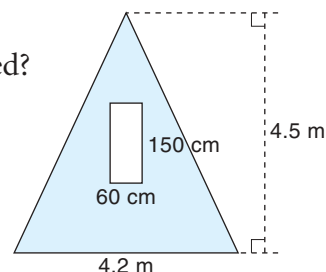
One litre of paint covers 6.5 m^2 .

a) What is the area that is to be painted?

b) The paint comes in 1-L cans.

How many cans does the owner need?

Explain your answer.



Take It Further

10. A local park has a pavilion to provide shelter. The pavilion has a roof the shape of a rectangular pyramid.

a) What is the total area of all four parts of the roof?

b) One sheet of plywood is

240 cm by 120 cm.

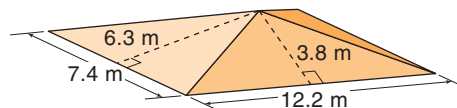
It costs \$24.95.

What is the least number of sheets of plywood

needed to cover the roof?

What is the cost?

Explain how you got your answer.



Reflect

A triangle and a parallelogram have the same base and height.

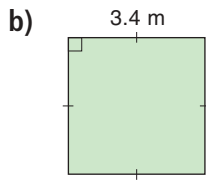
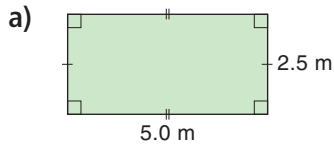
How are the areas of the triangle and parallelogram related?

Use an example to explain.

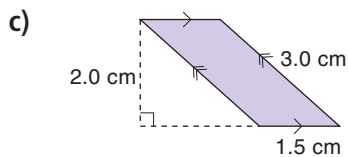
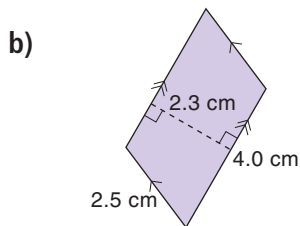
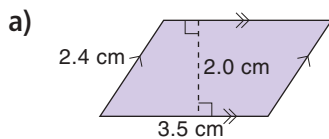
Mid-Unit Review

LESSON

- 6.1 1.** Find the perimeter and area of each figure.



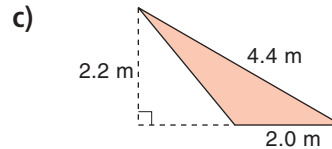
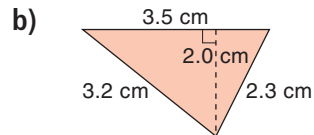
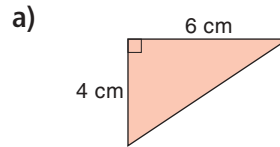
- 2.** Find the area of each parallelogram.



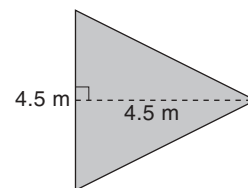
- 3.** A parallelogram has height 45 cm and base 60 cm.

- Find its area.
- What is the base and height of a parallelogram with twice the area?
- What is the base and height of a parallelogram with one-half the area?

- 6.2 4.** Find the area of each triangle.



- 5.** Po Ling is planning to pour a concrete patio beside her house. It has the shape of a triangle. The contractor charges \$125.00 for each square metre of concrete poured.



What will the contractor charge for the concrete?

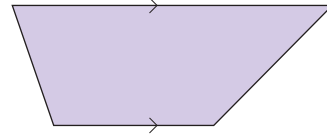


Focus Use a formula to find the area of a trapezoid.

This is a **trapezoid**.

How would you describe it?

Recall that a rectangle, a square, and a parallelogram are trapezoids, too.

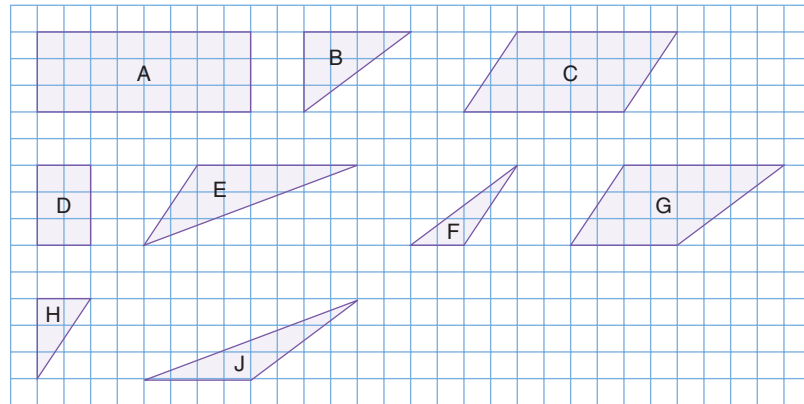


Explore

Work with a partner.

You will need scissors.

Your teacher will give you a copy of the figures below.

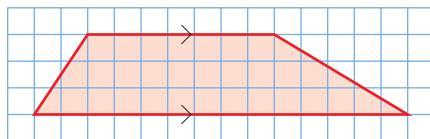


- Find the area of each figure.
- Cut out the figures.
- Identify the trapezoid that is not a parallelogram.
- How many different ways can you use the figures to find the area of the trapezoid?
- For each way you find, write a formula in words for the area of a trapezoid.
- Find the perimeter of the trapezoid.

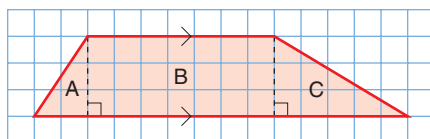
Reflect & Share

How did you use what you know about the areas of a triangle, a rectangle, and a parallelogram to find the area of a trapezoid?

We can find the area of a trapezoid by dividing it into other figures. Here are 3 ways to find the area of this trapezoid.

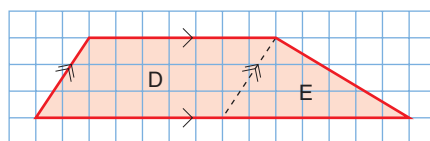


- Make 2 triangles and a rectangle.



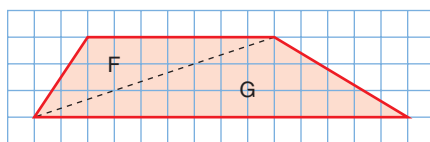
$$\text{Area of trapezoid} = \text{area of triangle A} + \text{area of rectangle B} + \text{area of triangle C}$$

- Make 1 triangle and a parallelogram.



$$\text{Area of trapezoid} = \text{area of parallelogram D} + \text{area of triangle E}$$

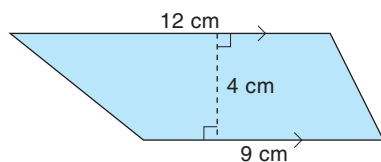
- Make 2 triangles.



$$\text{Area of trapezoid} = \text{area of triangle F} + \text{area of triangle G}$$

Example

- Estimate the area of this trapezoid.
- Calculate the area to check your estimate.

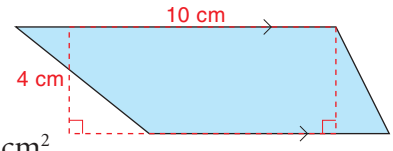


Solution

- a) Sketch a rectangle with width 4 cm and length between 9 cm and 12 cm, maybe 10 cm. The area of the rectangle is an estimate of the area of the trapezoid.

$$\begin{aligned} \text{Area of rectangle} &= 10 \times 4 \\ &= 40 \end{aligned}$$

The area of the trapezoid is about 40 cm².



- b) Divide the trapezoid into 2 triangles.

$$\text{Area of triangle A} = \frac{bh}{2}$$

Substitute $b = 12$ and $h = 4$.

$$\begin{aligned} \text{So, area} &= \frac{12 \times 4}{2} \\ &= 24 \end{aligned}$$

$$\text{Area of triangle B} = \frac{bh}{2}$$

Substitute $b = 9$ and $h = 4$.

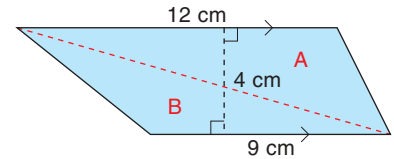
$$\begin{aligned} \text{So, area} &= \frac{9 \times 4}{2} \\ &= 18 \end{aligned}$$

Area of trapezoid = area of triangle A + area of triangle B

$$= 24 + 18$$

$$= 42$$

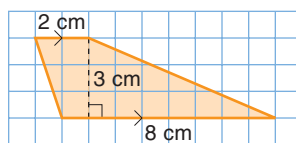
The area of the trapezoid is 42 cm².



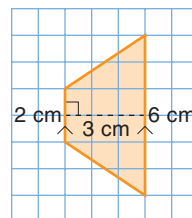
Practice

1. Find the area of each trapezoid by dividing it into 2 triangles.

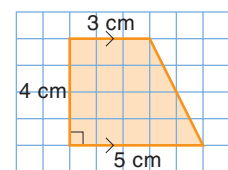
a)



b)

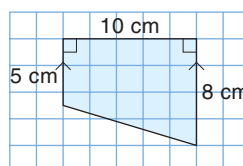


c)

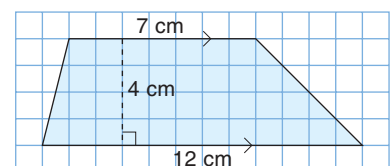


2. Find the area of each trapezoid by dividing it into 1 or 2 triangles and a rectangle.

a)



b)



Calculator Skills

Predict each product.

Check your prediction.

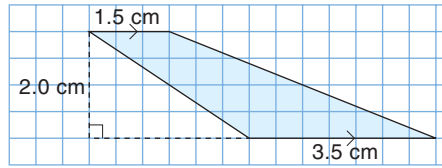
What patterns do you see in the answers?

- 9×9
- 99×99
- 999×999
- 9999×9999

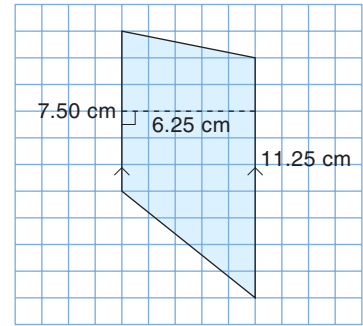
Use the pattern to predict the product of $99\ 999 \times 99\ 999$.

3. Find the area of each trapezoid.

a)

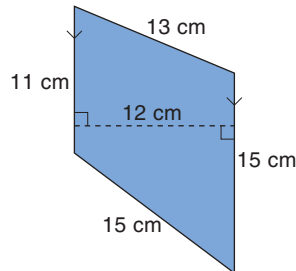


b)

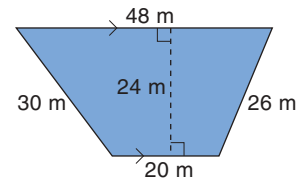


4. Find the area and perimeter of each trapezoid.

a)



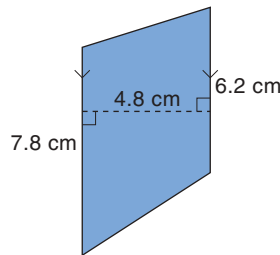
b)



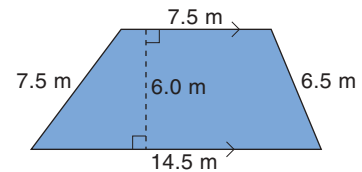
5. a) Estimate the area of each trapezoid.

Check your answer by calculating the area.

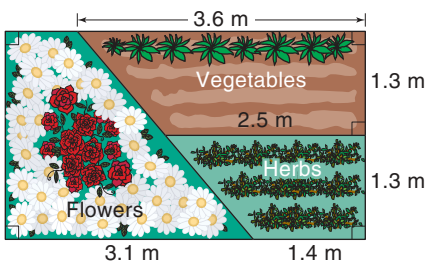
i)



ii)



b) Can you find the perimeter of each trapezoid in part a? Explain.



6. a) What is the area of each part of this garden?

b) Find the area of the whole garden two different ways.

7. Suppose you have a piece of string, 4 pushpins, a ruler, and grid paper.

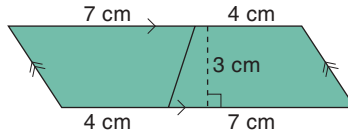
a) Describe how to make a trapezoid with perimeter 20 cm.

Use your strategy to make the trapezoid.

b) Draw the trapezoid on grid paper.

c) Find the approximate area of the trapezoid.

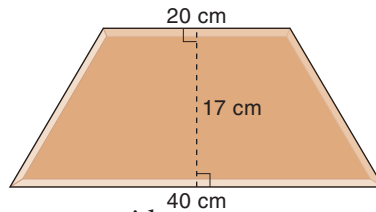
- 8. Assessment Focus** Two congruent trapezoids join to form a parallelogram.



- How can you use the area of the parallelogram to find the area of each trapezoid?
- Use grid paper. Draw a trapezoid. Use the area of a parallelogram to find the area of your trapezoid. Show your work.

Take It Further

- 9.** A patio is made with congruent brick tiles.



Each tile is a trapezoid.

- What is the area of the top face of each tile?
- Use red Pattern Blocks on triangular grid paper. Sketch a patio that uses these trapezoid tiles. How many tiles are in your patio?
- What is the area of your patio?
- When a patio is built, there is a 3-mm space between tiles for the grout. Would your completed patio be larger or smaller than the area you calculated in part c? Explain. How much larger or smaller would it be?



- 10.** Use any of the methods you know to find the area of a trapezoid. Use variables. Write a formula for the area of a trapezoid.

Reflect

How can you use the strategies for finding the area of a trapezoid to find the areas of a square, rectangle, and parallelogram? Use examples to explain.



Measuring for Construction

Before construction begins on a shopping mall, the site is precisely measured in different ways for different reasons. Measuring does not stop once construction begins. Initial measurements are checked and rechecked because estimates and plans sometimes change as the project continues.

The first “measurers” on the site are members of a survey team. The first thing they do is to verify the perimeter and area with an older, existing plan. In some cases, the last survey for the site might have been carried out 200 years before.

Most surveyors today have new, technology-based surveying tools. However, the team might use a transit (an angle-measuring device based on a telescope) and stadia (a graduated measuring rod). These measuring devices have been in use since the early 19th century. The survey team may have aerial or satellite photographs, EDM (Electronic Distance Measuring) equipment based on microwaves or lasers, or GPS (Global Positioning System) equipment.



A construction project requires the services of many different suppliers and contractors. One company provides security fencing around the site. Another company lays asphalt for the roads and parking lots. A third company installs the flooring and carpets inside the mall. Some people work from the architect’s blueprints to calculate how much to charge for their materials and labour. Other people will send an estimator to do her own measuring. Estimators use a variety of measuring tools: tape measure, trundle wheel, hand-held EDM, and so on. The estimators have to know how to use the measurements they collect.

Recently, a company introduced a digital measuring device. This device is wheeled around the perimeter of the region to calculate the area of the region. Why might this seem to be an impossible calculation? Can you explain how it might work?

Interpreting a Problem

Problem

How many different trapezoids can you draw with area 24 cm^2 ?

Interpret the problem

A trapezoid has at least 1 pair of parallel sides.

A trapezoid could have 2 pairs of parallel sides.

It would then be a rectangle or a parallelogram.

Solve the problem

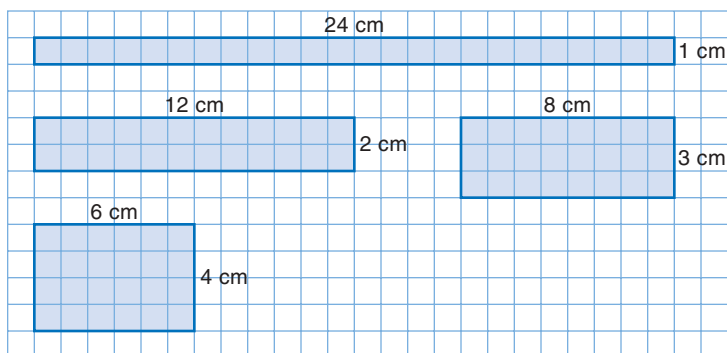
Solution 1

To draw a rectangle with area 24 cm^2 , find two factors of 24.

The factors in each pair are the base and the height of the rectangle:

1×24 , 2×12 , 3×8 , 4×6

Each of these rectangles has area 24 cm^2 .



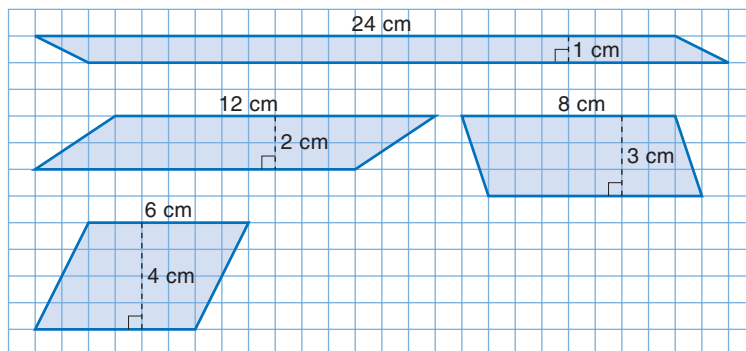
Solution 2

To draw a parallelogram with area 24 cm^2 ,

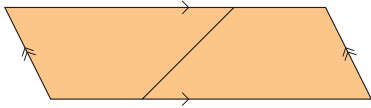
use the factors in *Solution 1*.

The factors in each pair are the base and the height of the parallelogram.

Each of these parallelograms has area 24 cm^2 .



Solution 3



To draw a trapezoid (that is not a parallelogram) with area 24 cm^2 :

Two congruent trapezoids join to form a parallelogram. If each trapezoid has area 24 cm^2 , then the parallelogram has area 48 cm^2 .

Work backward.

Draw a parallelogram with area 48 cm^2 .

Divide it into 2 congruent trapezoids.

A parallelogram with area 48 cm^2 can have base 12 cm and height 4 cm .

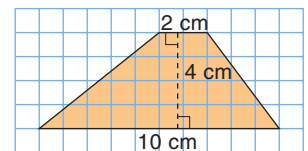
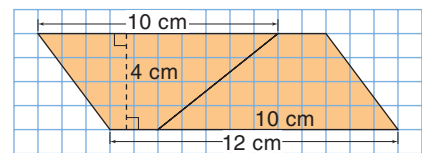
Choose a length for one base of the trapezoid, less than 12 cm .

Choose 10 cm . Mark a point on the top side of the parallelogram 10 cm from the left vertex.

Mark a point on the bottom side of the parallelogram 10 cm from the right vertex.

Join these points to form two congruent trapezoids.

So, one trapezoid with area 24 cm^2 looks like this:



Look back

- What if we had chosen a parallelogram with base 8 cm and height 4 cm ?
What could the trapezoid look like?
- What if we had chosen 5 cm for one base of the trapezoid?
What would the trapezoid look like?



Problems

1. Draw 3 different trapezoids with area 30 cm^2 .
2. Think of other methods to find the area of a trapezoid.
Use a different method to draw a trapezoid with area 20 cm^2 .

Reflect

Choose an area for a trapezoid. Explain how to draw a trapezoid with that area. Include a diagram.

6.4

Measuring Irregular Figures

Focus Find the area and perimeter of an irregular figure.

In *Section 6.3*, you calculated the area of a trapezoid by dividing it into other figures.

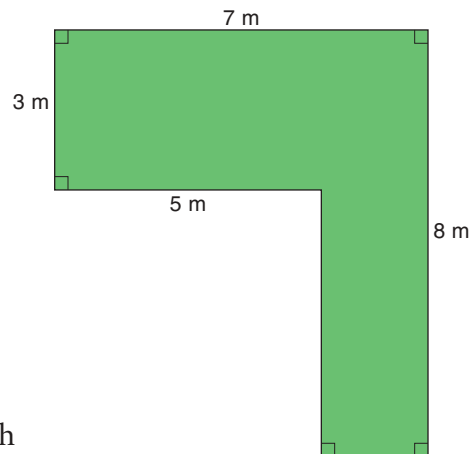
You can use a similar strategy to find the areas of other irregular figures.



Explore

Work with a partner.
A garden in a backyard has this plan.

How many different ways can you find the area of the garden?
What is its perimeter?

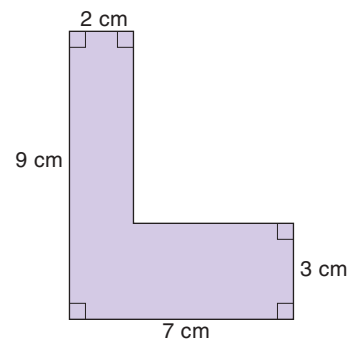


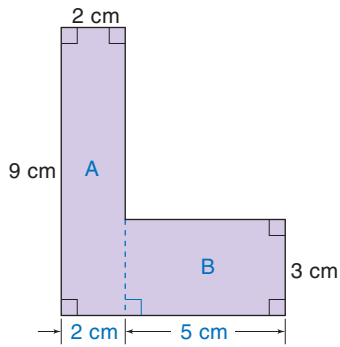
Reflect & Share

Compare your strategies with those of another pair of students.
What other strategies could you have used?

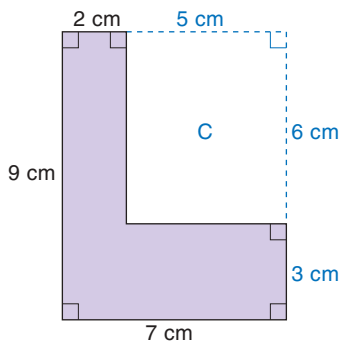
Connect

Here are two ways to find the area of this irregular figure.

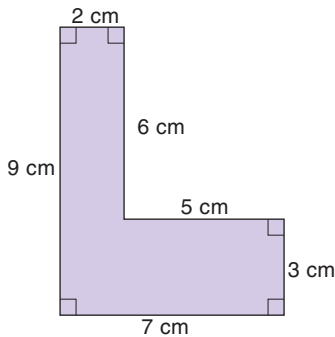




- Divide the figure into smaller figures whose areas you can find.
 Rectangle A has length 9 cm and width 2 cm.
 Area of rectangle A: $9 \text{ cm} \times 2 \text{ cm} = 18 \text{ cm}^2$
 Since the width of rectangle A is 2 cm,
 the length of rectangle B is: $7 \text{ cm} - 2 \text{ cm} = 5 \text{ cm}$
 Area of rectangle B: $3 \text{ cm} \times 5 \text{ cm} = 15 \text{ cm}^2$
 So, the area of the figure is: $18 \text{ cm}^2 + 15 \text{ cm}^2 = 33 \text{ cm}^2$



- Draw a rectangle around the figure.
 The area of the large rectangle is: $9 \text{ cm} \times 7 \text{ cm} = 63 \text{ cm}^2$
 Figure C is a rectangle.
 Its width is: $7 \text{ cm} - 2 \text{ cm} = 5 \text{ cm}$
 Its length is: $9 \text{ cm} - 3 \text{ cm} = 6 \text{ cm}$
 Area of rectangle C: $5 \text{ cm} \times 6 \text{ cm} = 30 \text{ cm}^2$
 So, the area of the figure is: $63 \text{ cm}^2 - 30 \text{ cm}^2 = 33 \text{ cm}^2$

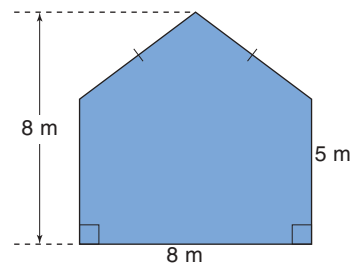


- To find the perimeter of the figure, add the side lengths.
 Perimeter
 $= 2 \text{ cm} + 6 \text{ cm} + 5 \text{ cm} + 3 \text{ cm} + 7 \text{ cm} + 9 \text{ cm}$
 $= 32 \text{ cm}$
 The perimeter is 32 cm.

Example

Here is a plan of the back wall of a barn.

- a) What is the area of the wall?
- b) One can of paint covers 40 m^2 . How many cans are needed to paint this wall?



Solution

The wall is a rectangle with an isosceles triangle above it.

a) For the triangle:

The base, b , is 8 m.

The height, h , is $8 \text{ m} - 5 \text{ m} = 3 \text{ m}$.

$$\text{Area} = \frac{bh}{2}$$

Substitute $b = 8$ and $h = 3$.

$$\begin{aligned} \text{Area} &= \frac{8 \times 3}{2} \\ &= 12 \end{aligned}$$

For the rectangle:

$$\text{Area} = bh$$

Substitute $b = 8$ and $h = 5$.

$$\begin{aligned} \text{Area} &= 8 \times 5 \\ &= 40 \end{aligned}$$

$$\begin{aligned} \text{Area of wall} &= 40 \text{ m}^2 + 12 \text{ m}^2 \\ &= 52 \text{ m}^2 \end{aligned}$$

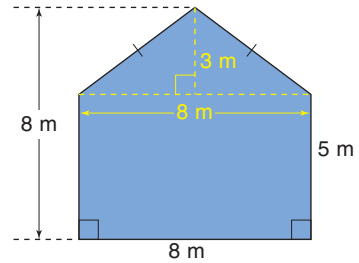
The area of the wall is 52 m^2 .

b) The area of the wall is 52 m^2 .

One can of paint covers 40 m^2 .

One can is not enough.

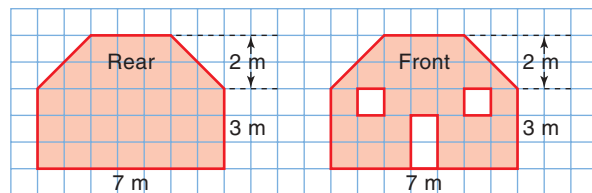
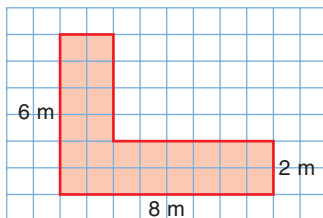
Two cans of paint are needed.



Practice

1. A living room in a home has the shape, below left.

What is the area of the living room?



2. The rear and front walls of a shed are shown, above middle and right.

a) Find the area of the rear wall.

b) Find the area of the front wall, excluding the windows and door.

Number Strategies

Which is the next number in each pattern?

Write each pattern rule.

3, 9, 27, 81, ___

12, 14, 18, 26, ___

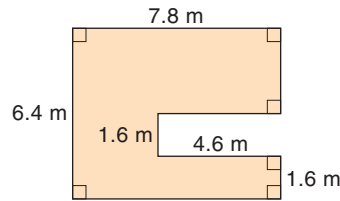
52, 28, 16, 10, ___

5, 14, 41, 122, ___

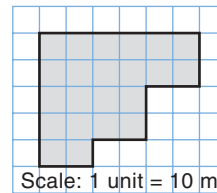


Reflect

3. The diagram shows the basement floor of a home.
- Estimate the area and perimeter of the floor.



- Calculate the area and perimeter of the floor.
 - Compare your estimates with your calculations. Was your estimate reasonable? Explain.
4. This diagram shows a plan of a parking lot.
- Estimate the area and perimeter of the lot.
 - Calculate the area and perimeter of the lot.
 - How could you use the grid to verify your answers?



5. A backyard is a rectangle 15 m long by 10 m wide. In one corner, there is a rectangular garden 5 m by 3 m.
- Use grid paper. Draw a diagram of the backyard.
 - Calculate the area of the backyard, excluding the garden.
 - What if the garden was in a different place in the yard? Would the answer to part b be different? Explain.
6. **Assessment Focus** An L-shaped swimming pool has area 30 m^2 . Each rectangular arm has width 3 m.
- Use grid paper. Draw 3 different pools.
 - Find the perimeter of each pool you drew. What do you notice about the perimeters?
 - What if the width of each arm was 5 m? What effect does this have? Explain.

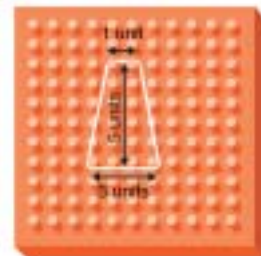
There are different ways to find the area of an irregular figure. Which way do you prefer to use? Use an example to explain your reasoning.

Game

Trapezoid Challenge

HOW TO PLAY THE GAME:

1. Roll 3 number cubes to get the height and lengths of the 2 parallel sides of a trapezoid.



2. Use the geoboard to make a trapezoid with those dimensions.
Choose which number represents which dimension.

YOU WILL NEED

11 by 11 geoboard;
geobands; 3 number cubes
labelled 1 to 6; dot paper

NUMBER OF PLAYERS

3 or 4

GOAL OF THE GAME

To get 50 points



3. The area of the trapezoid is your score for the round.
4. Take turns.
The winner is the first person to reach 50 points.

What strategies did you use to try to win?
Does it matter which of the 3 numbers you use for the height? Explain.

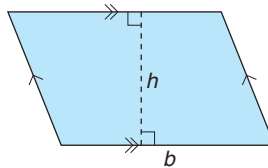
Variation: Record each trapezoid on grid paper. No two trapezoids can be the same. If you cannot create a different trapezoid, you forfeit your turn.

What Do I Need to Know?

✓ The *perimeter* of a figure is found by adding the lengths of its sides.

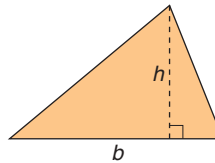
✓ **Area of a Parallelogram**

$$A = bh$$



✓ **Area of a Triangle**

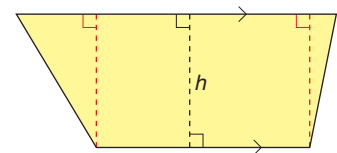
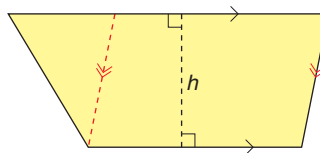
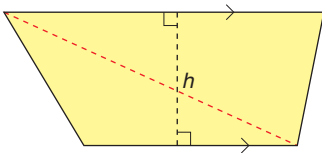
$$A = \frac{bh}{2}$$



✓ **Area of a Trapezoid**

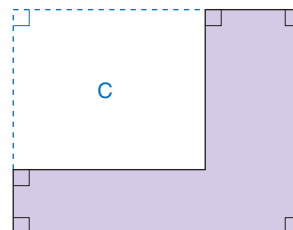
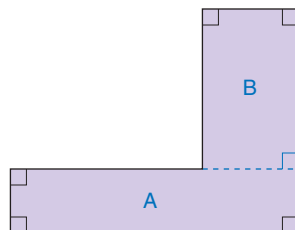
Divide the trapezoid into:

- Two triangles
- A parallelogram and a triangle
- A rectangle and 2 triangles



✓ **Area of an Irregular Figure**

- Divide the figure into figures whose area you can find, then add the areas.
- Or, draw a rectangle around the figure; subtract the areas of the newly formed figures from the area of the rectangle.



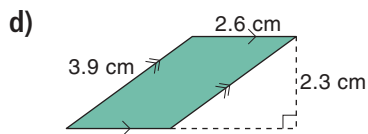
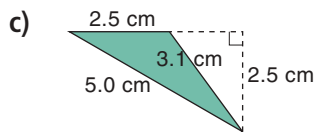
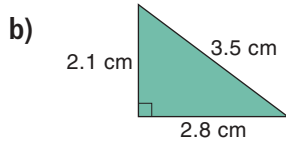
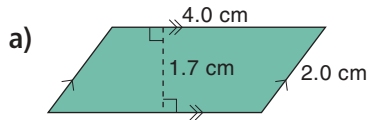
What Should I Be Able to Do?

For extra practice, go to page 443.

LESSON

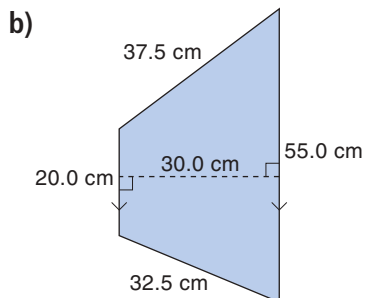
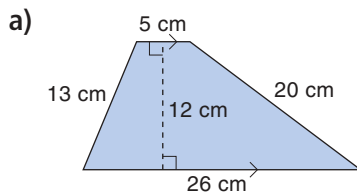
6.1 1. Find the area of each figure.

6.2 Explain your strategy.



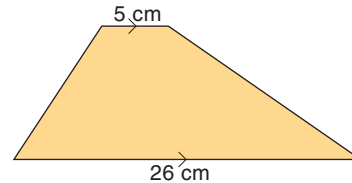
6.3 2. Estimate the area of each trapezoid.

Then, calculate the area to check if your estimate was reasonable.



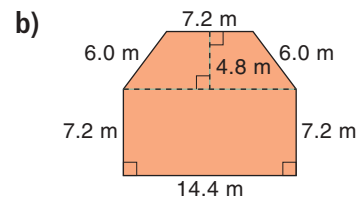
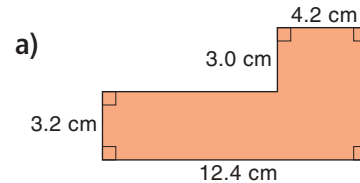
3. Estimate the perimeter of each trapezoid in question 2. Then, calculate the perimeter to check.

4. The area of this trapezoid is approximately 150 cm^2 .

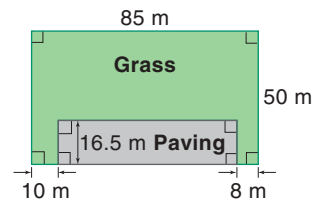


Estimate its height.
Show your work.

6.4 5. Find the perimeter and area of each figure.



6. A school playground has a paved surface and a grass surface.



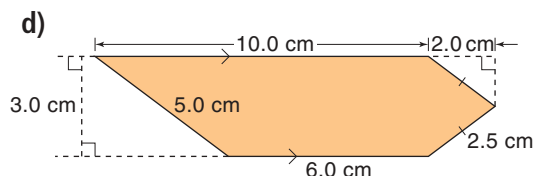
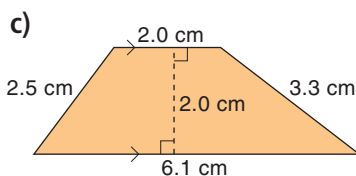
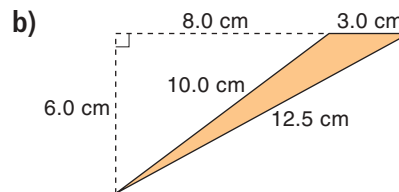
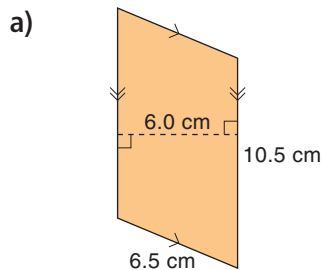
a) What is the area of the paved surface?

b) Fencing costs $\$35.50/\text{m}$. How much would it cost to fence the grass area?

Practice Test

1. Find the area and perimeter of each figure.

Explain your strategies.



2. How does the area of a triangle change in each case?

- Its height is doubled.
- Its base is halved.
- Its height is doubled and its base is halved.

Explain how you know.

3. Use 1-cm grid paper.

Draw an irregular figure with area 64 cm^2 .

Label all the dimensions of the figure.

Find the perimeter of the figure.

4. A design has a series of trapezoids.

For each trapezoid, one parallel

side is always 1.5 m shorter

than the other parallel side.

The height of each trapezoid

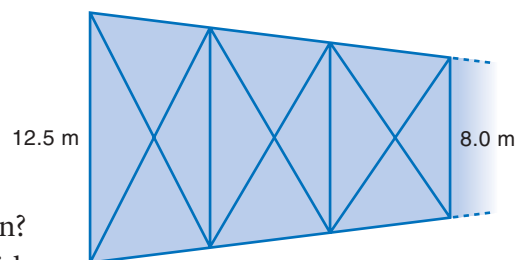
is 6.0 m.

a) What is the area of the

6th trapezoid in the design?

b) How long is the design with

6 trapezoids?



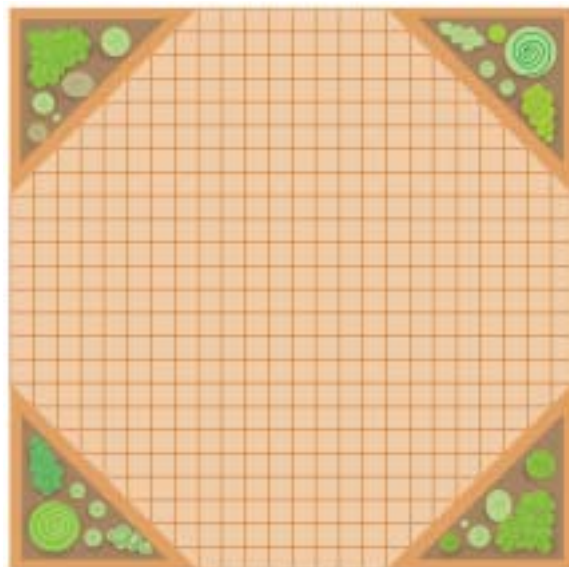


The owners of a large shopping centre want to build a patio in front of the main entrance to attract more customers.

Your task is to design the tiled surface of the patio.

You must use tiles with these shapes: triangle, parallelogram, rectangle, trapezoid

The patio has the shape shown at the right. Each square on this plan has side length 10 cm.



- You must include at least:
 - 3 triangles with different areas
 - 3 parallelograms with different areas
 - 3 rectangles with different areas
 - 3 trapezoids (that are not parallelograms) with different areas
- Use a formula to find the area of each tile.

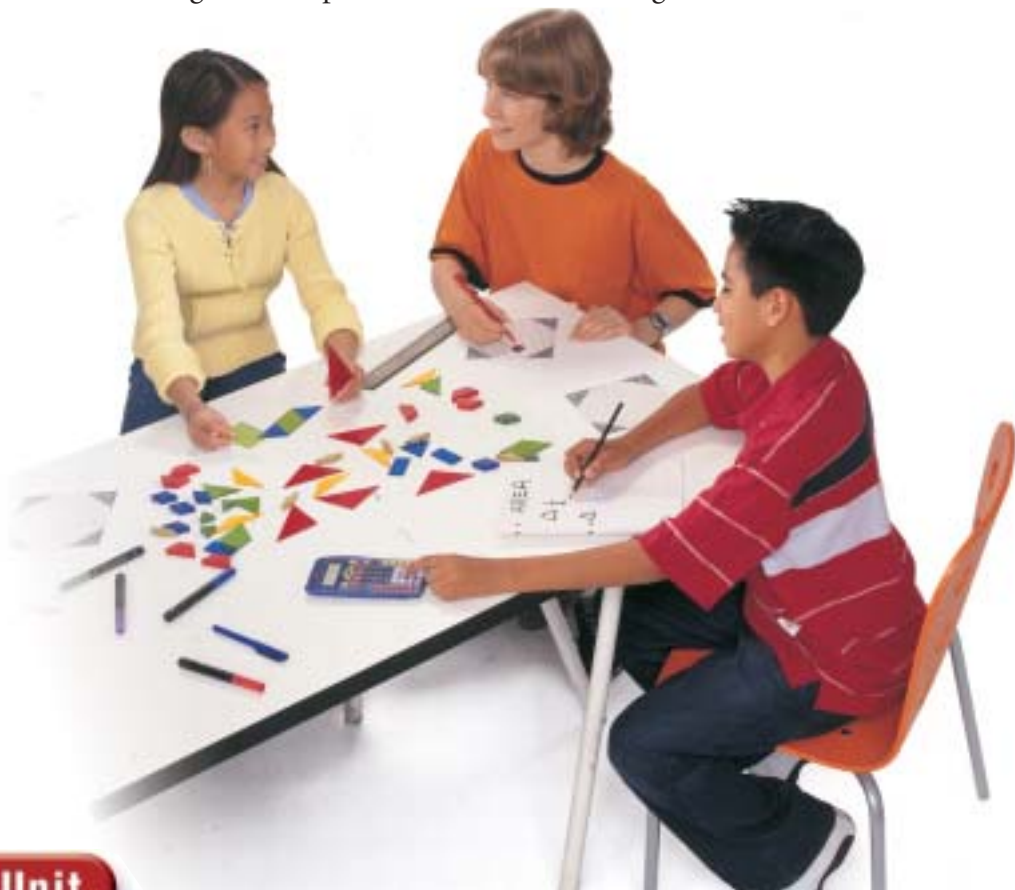
Your teacher will give you a grid to draw your design.
Complete the design.

Colour the design if it helps to show the different figures.

Check List

Your work should show:

- ✓ the area of each different figure you used
- ✓ the formulas you used to calculate the areas
- ✓ a diagram of your design on grid paper
- ✓ an explanation of how you created the design



Reflect on the Unit

What do you need to know to find the area of a parallelogram, a triangle, and a trapezoid? Explain.

Include a diagram and an example to show how you found each area.



UNIT

7

Geometry

Patterns are pleasing to the eye. They are used by designers, architects, and engineers to make their products more attractive. Look at the quilt pattern. Which figures are used as quilt blocks? Which other figures have you seen in quilts?

What You'll Learn

- Identify, describe, compare, and classify figures.
- Identify the conditions that make two figures congruent.
- Construct and analyse tiling patterns.
- Recognize the image of a figure after a transformation.
- Create and analyse designs using transformations.

Why It's Important

- Geometry is used daily by scientists, architects, engineers, and land developers.
- Geometric attributes, such as congruence and symmetry, enable you to see the world around you in a different way.



Key Words

- convex polygon
- concave polygon
- tiling the plane
- tessellations

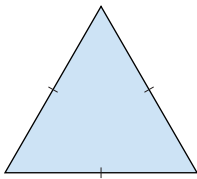
Skills You'll Need

Classifying Triangles

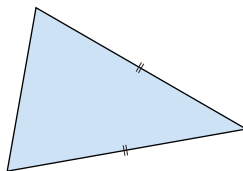
Here are two ways to classify triangles.

- By side length

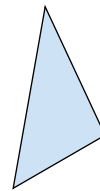
An equilateral triangle has all sides equal.



An isosceles triangle has 2 sides equal.

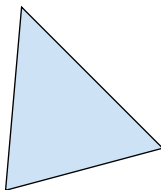


A scalene triangle has all sides different.

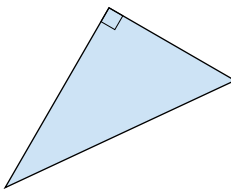


- By angle measure

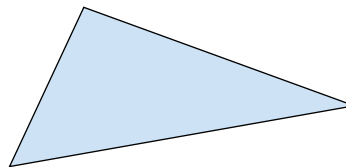
An acute triangle has all angles less than 90° .



A right triangle has one 90° angle.



An obtuse triangle has one angle greater than 90° .



✓ Check

Use square dot paper or isometric dot paper.

1. Draw an isosceles triangle.
Is it acute, obtuse, or right? How do you know?
2. Draw an obtuse triangle.
Is it equilateral, scalene, or isosceles? How do you know?
3. Can you draw an obtuse isosceles triangle?
If you can, draw it.
If you cannot draw the triangle, say why it cannot be drawn.
4. Can you draw a right equilateral triangle?
If you can, draw it.
If you cannot draw the triangle, say why it cannot be drawn.

Constructing a Triangle

Here are two ways to construct a triangle, using a ruler, compass, and protractor.

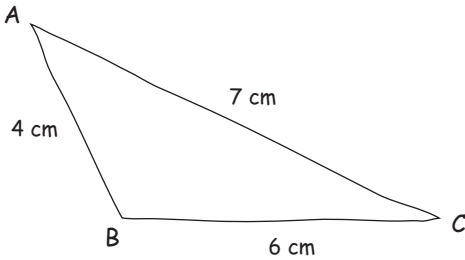
Example 1

Construct $\triangle ABC$ with $AB = 4$ cm, $BC = 6$ cm, and $CA = 7$ cm.

Solution

You will need a ruler and compass.

Step 1 Sketch the triangle.



Step 2 Construct the triangle:

Use a ruler to draw side $BC = 6$ cm.

With the compass point and pencil 7 cm apart, put the compass point on C and draw an arc.

All points on this arc are 7 cm from C.

With the compass point and pencil 4 cm apart, put the compass point on B and draw an arc.

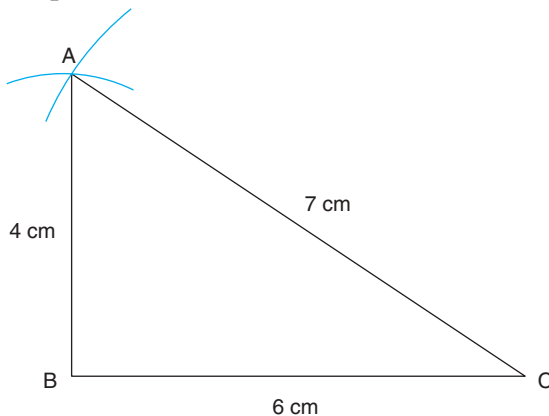
All points on this arc are 4 cm from B.

Make sure the arc intersects the first arc you drew.

Mark a point where the arcs intersect.

This point is 7 cm from C and 4 cm from B.

Label the point A. Join AB and AC. Label each side with its length.



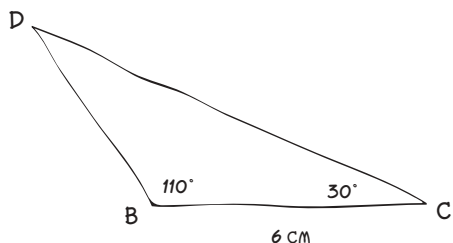
Example 2

Construct $\triangle BCD$ with $BC = 6$ cm, $\angle B = 110^\circ$, and $\angle C = 30^\circ$.

Solution

You will need a ruler and protractor.

Step 1 Sketch the triangle.



Step 2 Construct the triangle:

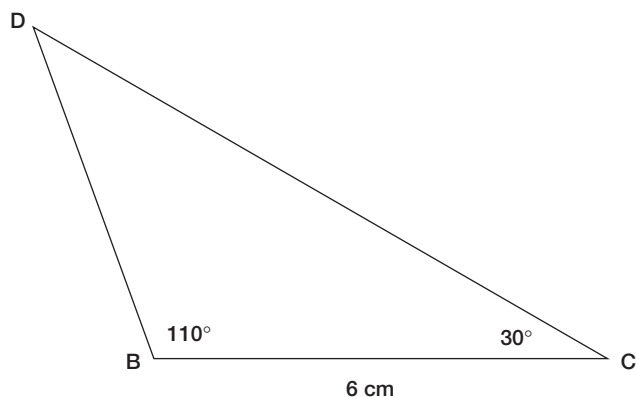
Use a ruler to draw side $BC = 6$ cm.

Use a protractor to make an angle of 110° at B.

Use a protractor to make an angle of 30° at C.

Label point D where the arms of the angles intersect.

Label the known side and angles.



✓ Check

5. Construct each triangle.

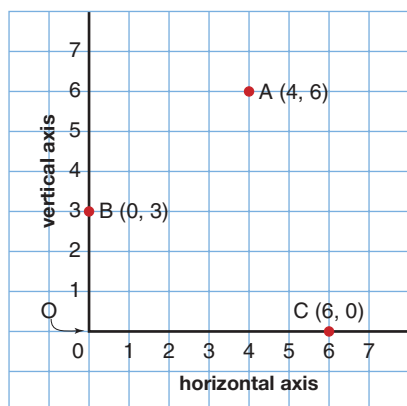
- $\triangle CDE$ with $CD = 4$ cm, $DE = 7$ cm, $CE = 9$ cm
- $\triangle DEF$ with $DE = 7$ cm, $\angle D = 80^\circ$, $\angle E = 30^\circ$

Plotting Points on a Coordinate Grid

When we draw a horizontal axis and a vertical axis on grid paper, we have a coordinate grid.

The axes intersect at the origin, O.

We label each axis with numbers, beginning with 0 at the origin.



A point on a grid is described by its coordinates.

Point A has coordinates (4, 6).

To plot point A, start at 4 on the horizontal axis, then move up 6 spaces.

Mark a point. This is point A.

Point B has coordinates (0, 3).

To plot point B, start at 0, then move up 3 spaces. Point B is on the vertical axis.

Point C has coordinates (6, 0).

To plot point C, mark a point at 6 on the horizontal axis.

✓ Check

6. On grid paper, draw a coordinate grid.

Plot each point on the grid.

A(5, 7), B(3, 8), C(10, 4), D(9, 1), E(0, 8), F(5, 0)

7. a) Where are all the points with horizontal coordinate 0?

b) Where are all the points with vertical coordinate 0?

7.1

Classifying Figures

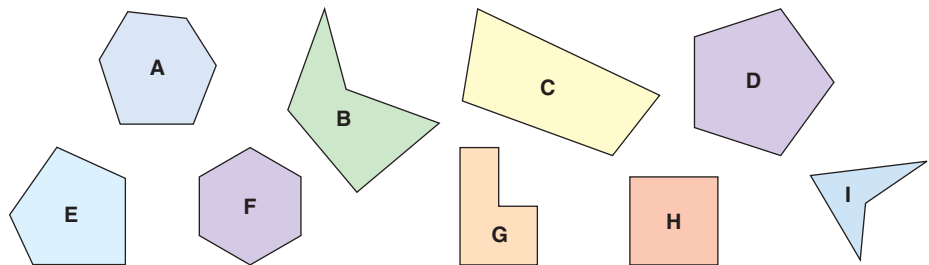
Focus Identify, describe, and classify geometric figures.

Look around the classroom.
Name the different figures you see.
Which figure is most common?



Explore

Work with a group.
You will need a ruler and a protractor.
Your teacher will give you a large copy of these figures.



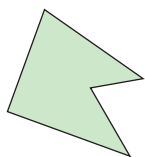
- Identify each figure.
Describe it.
- Choose two figures.
How many different ways can you compare them?
- Choose three figures. How are they the same?
How are they different?

Reflect & Share

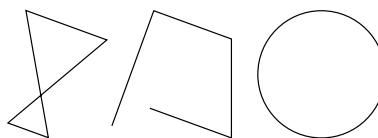
Share your results with another group of classmates.
Work together to classify the figures in different ways.

A polygon is a closed figure with sides that are line segments.
 Exactly 2 sides meet at a vertex.
 The sides intersect only at the vertices.

This figure is a polygon.



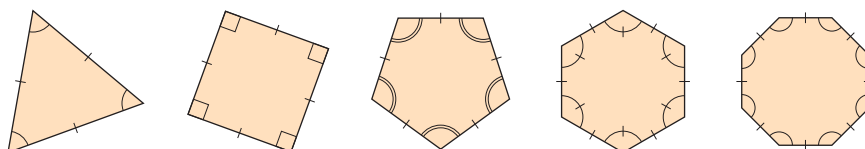
These figures are *not* polygons.



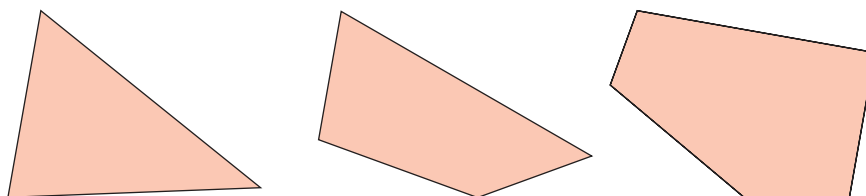
A regular polygon has line symmetry and rotational symmetry.

Recall that matching arcs or symbols in angles show that the angles are equal.

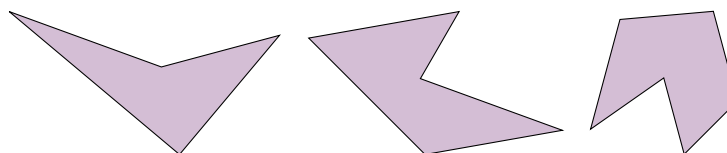
A regular polygon has all sides equal and all angles equal.
 These polygons are regular.



A **convex polygon** has all angles less than 180° .
 These polygons are convex.



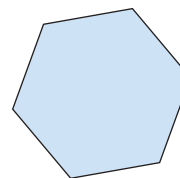
A **concave polygon** has at least one angle greater than 180° .
 These polygons are concave.



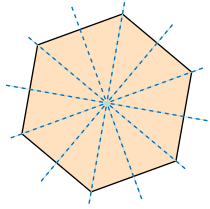
Example

Here is a regular hexagon.

- a) How many lines of symmetry does it have?
- b) What is the rotational symmetry?



Solution

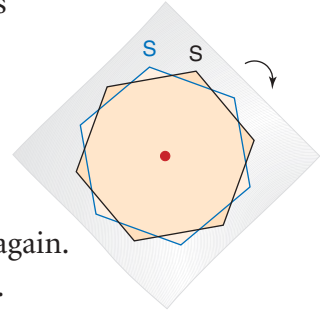


Label two corresponding vertices. Then you know when the tracing is back at the starting position.

- a) Trace the hexagon.
Fold the tracing paper so that one part of the hexagon coincides with the other.
The fold line is a line of symmetry.
Repeat the folding as many times as possible.

A regular hexagon has 6 lines of symmetry:
3 lines join opposite vertices, and 3 lines join the midpoints of opposite sides.

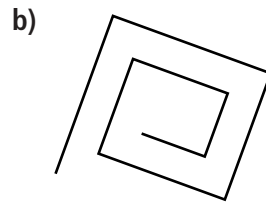
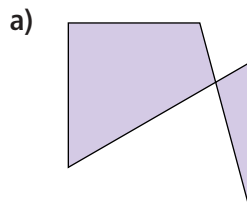
- b) Trace the hexagon. Place the tracing to coincide with the hexagon.
Rotate the tracing about its centre until the tracing coincides with the hexagon again.
Count how many times you can do this.



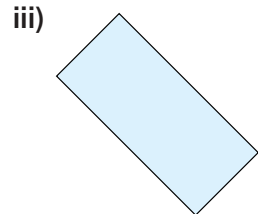
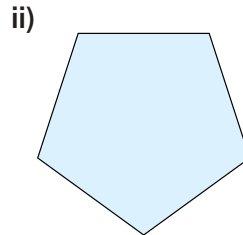
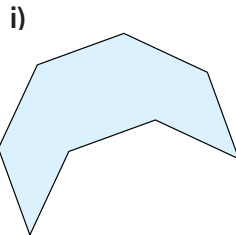
The tracing coincides with the hexagon 6 times.
So, a regular hexagon has rotational symmetry of order 6.

Practice

1. Explain why each figure is not a polygon.



2. a) Is each polygon regular? How do you know?



- b) Which polygons in part a have:
i) line symmetry? ii) rotational symmetry?
How do you know?

3. Identify the figures in each flag.

Describe each figure as many ways as you can.

- a) Congo b) Bosnia-Herzegovina c) Guyana d) Seychelles

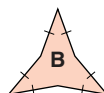


4. Describe each figure. How are the figures the same? Different?

a)



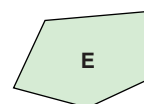
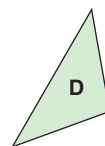
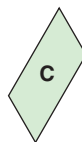
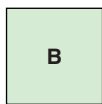
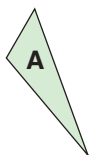
b)



c)



5. Match each polygon with its description below.



- a) an isosceles triangle with an angle of 40°
 b) a rhombus with a right angle
 c) a pentagon with an angle of 120°
 d) a parallelogram with an angle of 60°
 e) an obtuse triangle with an angle of 110°



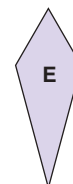
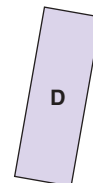
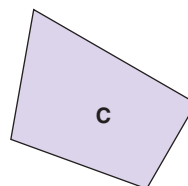
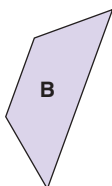
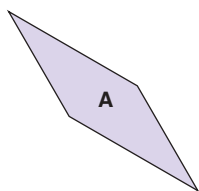
6. Use square dot paper or isometric dot paper.

Draw each polygon.

- a) an isosceles triangle with a height of 4 units
 b) a parallelogram with an angle of 45°
 c) a trapezoid with a 90° angle and a 45° angle
 d) a kite with exactly one right angle
 e) a parallelogram with a 90° angle
 f) a scalene obtuse triangle
 g) an isosceles right triangle
 h) a hexagon with exactly 3 right angles

7. Identify each polygon.

Describe it as many ways as you can.



A quadrilateral is a polygon with 4 sides.

Number Strategies

Find the next 3 numbers in each pattern.

What is each pattern rule?

- 23, 28, 26, 31, 29, ...
- 6, 9, 15, 27, 51, ...
- 1, 3, 9, 27, ...

8. Use dot paper.
- a) Draw a quadrilateral. Label it A.
 - b) Draw another quadrilateral that differs from quadrilateral A in only one way. Label it B.
 - c) Continue to draw quadrilaterals that differ in only one way. Label each one you draw.
How many different quadrilaterals can you draw?

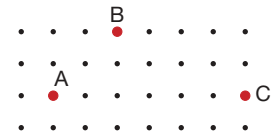
9. Assessment Focus

The 3 points A, B, C are vertices of a polygon.

Copy the points on dot paper.

- a) Find other vertices and sketch each figure.

- i) a trapezoid with line symmetry
- ii) a kite
- iii) a parallelogram
- iv) a pentagon



- b) How many other figures can you

make that have these points as 3 vertices?

Identify each figure. Describe it as many ways as you can.

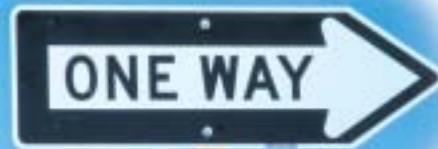
10. The lengths of three sides of a quadrilateral are 5 cm, 5 cm, and 8 cm.

- a) Sketch and name the different quadrilaterals possible.
- b) Suppose one angle is 90° . Which quadrilaterals are possible now? Justify your answer.

Math Link

Your World

The Department of Highways uses different figures for road signs. Which road signs use each of these figures: pentagon, octagon, square, circle, rectangle, triangle?



Reflect

Choose 3 different polygons. Sketch each polygon as many different ways as you can. Describe each polygon.

Explore



Work on your own.

You will need a ruler, protractor, and compass.

For each set of measurements given, how many different triangles can you draw?

- Construct a triangle with sides of length 5 cm, 7 cm, and 9 cm.
- Construct a triangle with two sides of length 9 cm and 5 cm, and one angle of 30° .
- Construct a triangle with one side of length 5 cm and two angles of 40° and 60° .

Reflect & Share

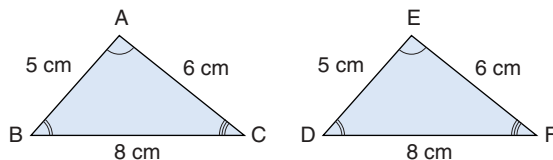
Compare your triangles with those of several classmates.

- How many different triangles can you draw in each case?
 - when you know 3 sides
 - when you know 2 sides and 1 angle
 - when you know 1 side and 2 angles
- What measurements do you need to know to be able to draw exactly one triangle?

Connect

- When 3 sides of a triangle are given, only one triangle can be drawn. So, if we know that two triangles have the same 3 sides, those triangles must be congruent.

Congruent figures have the same size and shape. These triangles are drawn to scale.



We say: Triangle ABC is congruent to triangle EDF.

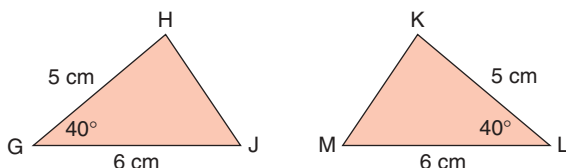
We write: $\triangle ABC \cong \triangle EDF$

We list the corresponding vertices of the triangles in the same order.

$\triangle ABC$ and $\triangle EDF$ have:
 corresponding sides equal and corresponding angles equal
 $AB = ED$ $\angle A = \angle E$
 $BC = DF$ $\angle B = \angle D$
 $AC = EF$ $\angle C = \angle F$

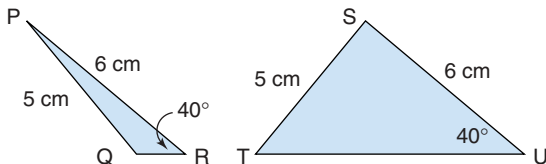
You may have to flip or rotate one triangle so both triangles face the same way.

- When 2 sides and 1 angle of a triangle are given, there are two cases to consider.
- The given angle is between the 2 sides.
Only one triangle can be drawn.



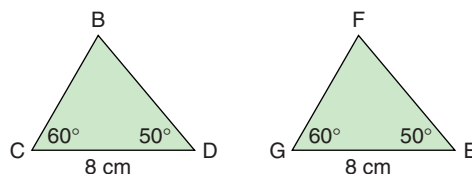
So, triangle GHJ is congruent to triangle LKM or $\triangle GHJ \cong \triangle LKM$

- The given angle is *not* between the 2 sides.
Sometimes more than one triangle can be drawn.



$\triangle PQR$ and $\triangle STU$ are *not* congruent.

- When 2 angles and 1 side are given, only one triangle can be drawn.

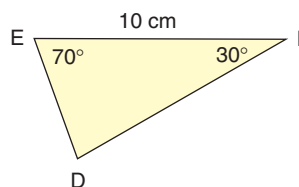
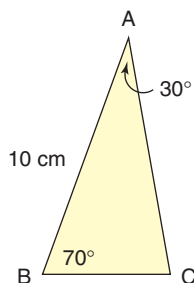


$\triangle BCD \cong \triangle FGE$

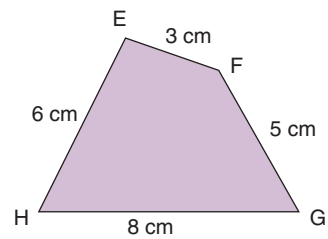
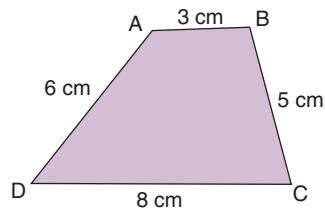
Example

Are the figures in each pair congruent? How do you know?

a)



b)



Solution

a) $\triangle ABC$ and $\triangle FED$ have 2 pairs of corresponding angles equal and 1 pair of corresponding sides equal:

$$\angle A = \angle F = 30^\circ$$

$$\angle B = \angle E = 70^\circ$$

$$AB = FE = 10 \text{ cm}$$

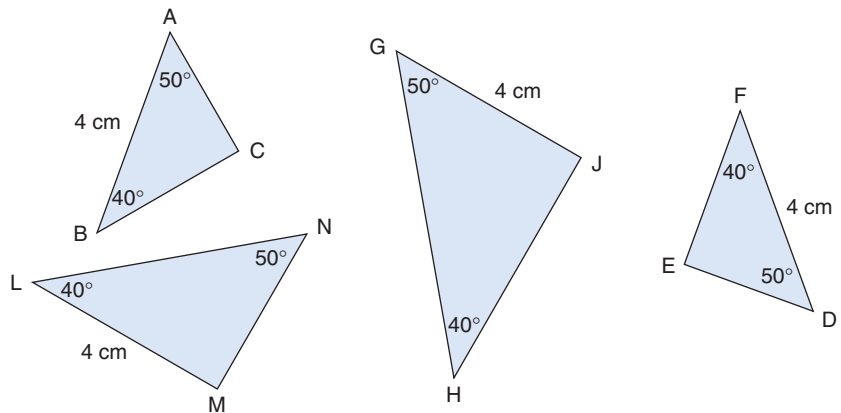
So, $\triangle ABC$ and $\triangle FED$ are congruent: $\triangle ABC \cong \triangle FED$

b) Quadrilateral ABCD and quadrilateral EFGH have 4 pairs of corresponding sides equal. But the quadrilaterals have different shapes. So, the quadrilaterals are not congruent.

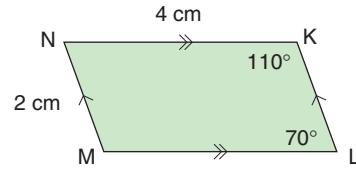
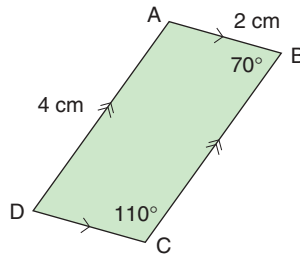
Part b of the *Example* shows that for two quadrilaterals to be congruent, it is not sufficient that 4 pairs of corresponding sides are equal. We need to know that the corresponding angles are equal, too.

Practice

1. Look at the triangles below.
Find pairs of congruent triangles.
Explain why they are congruent.



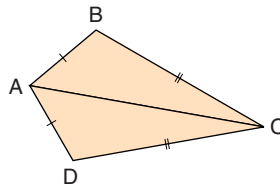
2. Are quadrilaterals ABCD and KLMN congruent?
How do you know?



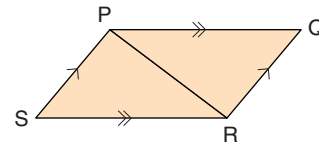
When you name congruent triangles, remember to list corresponding vertices in the same order.

3. In each figure below, name pairs of congruent triangles.
Explain how you know they are congruent.
Try to find more than one way to show the triangles are congruent.

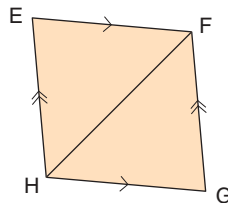
a) ABCD is a kite.



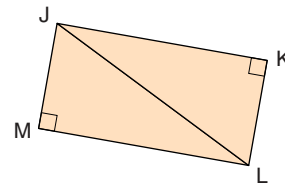
b) PQRS is a parallelogram.



c) EFGH is a rhombus.



d) JKLM is a rectangle.



4. For each figure below:

a) Sketch the figure.

b) What are the fewest measurements you need to know to draw the figure?

c) How does your answer to part b help you identify congruent figures of this type?

i) parallelogram

ii) rectangle

iii) square

5. $\triangle ABC$ and $\triangle DEF$ have $AB = DE = 6$ cm and $BC = EF = 7$ cm.

a) Sketch the triangles.

b) What else do you need to know to tell if the triangles are congruent?

Mental Math

Which three factors of 24 have a sum of 20?

- 6. Assessment Focus** Use dot paper.
- Draw two quadrilaterals with equal sides, but the quadrilaterals are not congruent.
Explain why the quadrilaterals are not congruent.
 - Use the 4 side lengths in part a. Draw two congruent quadrilaterals with these side lengths.
Explain how you know the quadrilaterals are congruent.
 - Explain how the quadrilaterals in parts a and b are different.

- 7.** Alex called a carpet store. He wanted a piece of carpet to repair a damaged rug. Alex asked for a piece measuring 3 m by 4 m by 5 m by 6 m. Explain why the salesperson could not help Alex.



- 8.**
- Are all isosceles triangles with two 50° angles congruent? Explain.
 - Are all isosceles triangles with two 50° angles and exactly one side of length 10 cm congruent? Explain.
- 9.** Construct a right triangle with one side 5 cm and the longest side 8 cm.
- Can you draw two different triangles with those measurements?
 - If your answer to part a is yes, draw the triangles.
 - If your answer to part a is no, explain how you know that only one triangle can be drawn with these measurements.

Take It Further

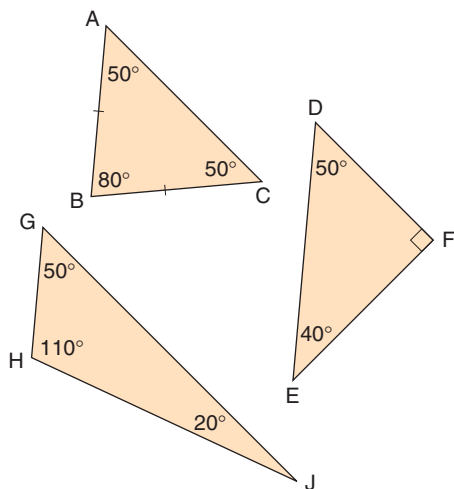
Reflect

Describe the different ways you can tell if two triangles are congruent.

Mid-Unit Review

LESSON

7.1 1. Identify each figure.



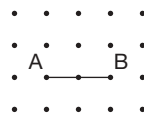
- a scalene triangle with an angle of 50°
- an isosceles triangle with an angle of 50°
- a right triangle with an angle of 50°
- an obtuse triangle with an angle of 50°
- an acute triangle with an angle of 50°

7.1 2. Use dot paper.

- Draw 2 congruent concave hexagons. How do you know the hexagons are congruent? How do you know they are concave?
- Draw 2 congruent convex hexagons. How do you know the hexagons are convex? How do you know they are congruent?

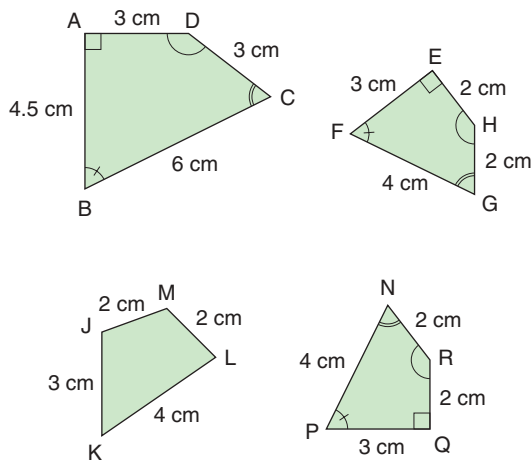
7.2 3. Segment AB is one side of $\triangle ABC$. Use dot paper.

a) Draw $\triangle ABC$.



- Draw a triangle congruent to $\triangle ABC$. How do you know the triangles are congruent?
- Draw a triangle that is *not* congruent to $\triangle ABC$. How do you know the triangles are *not* congruent?

4. Use these figures.

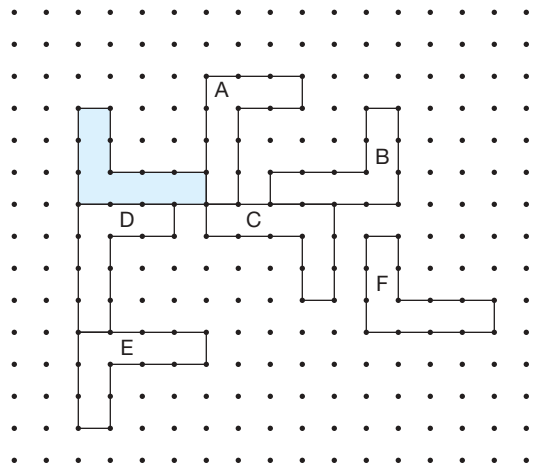


- Identify 2 figures that are *not* congruent. How do you know they are not congruent?
- Identify 2 congruent figures. How do you know they are congruent?

Explore

Work with a partner.

Your teacher will give you a large copy of these figures.



Use tracing paper and a Mira if they help.

The shaded figure has been translated, rotated, and reflected.

Each labelled figure is the image after a transformation.

Identify the transformation that produced each image.

Explain how you know.

Reflect & Share

Discuss your strategies for identifying each transformation.

What is special about a reflection image? A translation image?

A rotation image?

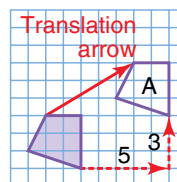


We can show transformations on a grid.

Translation

The translation image and the shaded figure are congruent.

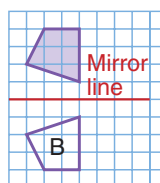
The shaded figure is translated 5 units right and 3 units up. Its translation image is figure A. The translation arrow shows the movement in a straight line.



Reflection

The reflection image and the shaded figure are congruent.

The shaded figure is reflected in a horizontal line 1 unit below the figure. Its reflection image is figure B.

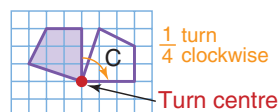


The figures have different orientations. That is, you flip one figure to make it coincide with the other figure.

Rotation

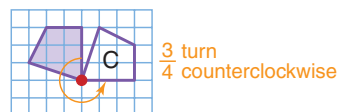
The rotation image and the shaded figure are congruent.

The shaded figure is rotated a $\frac{1}{4}$ turn clockwise. The turn centre is the vertex indicated.

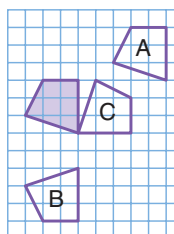


The rotation image is figure C.

We get the same image if the shaded figure is rotated a $\frac{3}{4}$ turn counterclockwise about the turn centre.



Here are the three images and the shaded figure on the same grid.

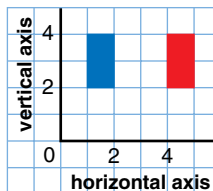


We can show transformations on a coordinate grid.

Example

Look at these rectangles.

Is one rectangle a transformation image of the other? Explain.



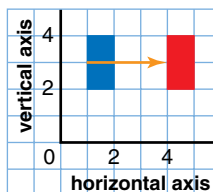
Solution

Let the blue rectangle be the original figure.

And let the red rectangle be the image.

Solution 1

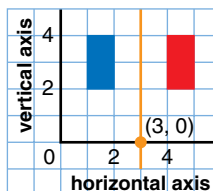
The red rectangle is the image after a translation of 3 units right. The translation arrow shows the movement.



Solution 2

Use a Mira to verify the image.

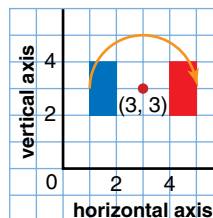
The red rectangle is the image after a reflection in a vertical line through $(3, 0)$ on the horizontal axis.



Solution 3

Use tracing paper to verify the image.

The red rectangle is the image after a rotation of $\frac{1}{2}$ turn about the point with coordinates $(3, 3)$.



The *Example* shows that an image may be the result of any one of the 3 transformations.

It also shows a rotation about a turn centre that is not on the figure.

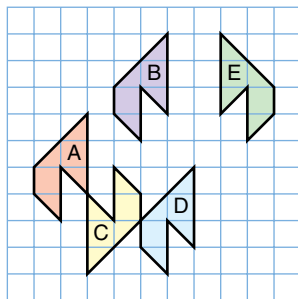
Practice

Number Strategies

Add or subtract, as indicated.

- $\frac{5}{8} + \frac{7}{6}$
- $\frac{17}{10} - \frac{3}{4}$
- $\frac{11}{12} - \frac{2}{3}$
- $\frac{4}{5} + \frac{5}{6}$

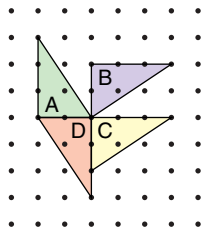
1. Use the figures below.



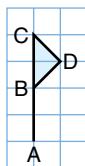
Identify the transformation for which:

- a) Figure B is the image of Figure A.
- b) Figure C is the image of Figure A.
- c) Figure E is the image of Figure B.
- d) Figure A is the image of Figure D.
- e) Figure C is the image of Figure D.

2. Identify each transformation.



- a) Figure A is the image of Figure B.
- b) Figure B is the image of Figure C.
- c) Figure C is the image of Figure D.
- d) Figure D is the image of Figure A.

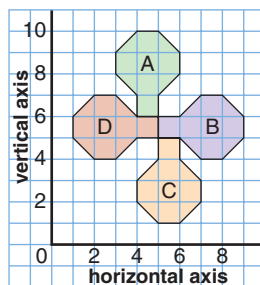


3. Draw this flag on a coordinate grid. The coordinates are A(11, 11), B(11, 13), C(11, 15), and D(12, 14).

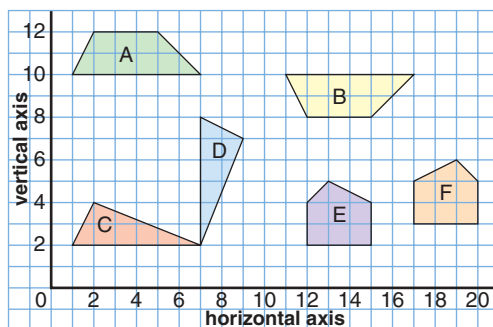
Draw the image of the flag after each transformation.

- a) a translation 3 units right
- b) a translation 5 units down
- c) a reflection in a vertical line through (9, 0)
- d) a reflection in a horizontal line through (0, 8)
- e) a rotation of a $\frac{1}{2}$ turn about point A
- f) a rotation of a $\frac{1}{4}$ turn clockwise about point C

4. How many different ways can each figure be described as a transformation of another figure? Explain.



5. a) Which pairs of congruent figures do *not* represent a figure and its transformation image? How do you know?

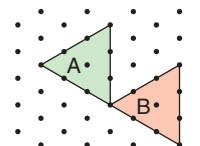


- b) For each pair of congruent figures that do show a transformation, identify the transformation.

6. **Assessment Focus** Use grid paper. In each case, describe the figure you drew.
- Draw a figure for which a translation image is also a reflection image and a rotation image. Draw the translation image.
 - Draw a figure for which a translation image is also a reflection image, but *not* a rotation image. Draw the translation image.
 - Draw a figure for which a translation image is *not* a reflection image *nor* a rotation image. Draw the translation image.

Take It Further

7. Describe Figure A as a transformation image of Figure B as many different ways as possible.



Reflect

When you see a figure and its transformation image on a grid, how do you identify the transformation? Use diagrams in your explanation.

Explore

You can rotate or flip the figure to try to make it fit.

Work on your own.

You will need index cards, a ruler, and scissors.

- Draw a triangle on a card. Cut it out.
Use tracings of the triangle to cover a piece of paper.
- Draw a quadrilateral on a card. Cut it out.
Use tracings of the quadrilateral to cover a piece of paper.
- Draw a pentagon on a card. Cut it out.
Use tracings of the pentagon to cover a piece of paper.

Reflect & Share

Share your results with the class.

- Will congruent triangles cover a page and leave no gaps? Explain.
- Will congruent quadrilaterals cover a page and leave no gaps? Explain.
- Will congruent pentagons cover a page and leave no gaps? Explain.

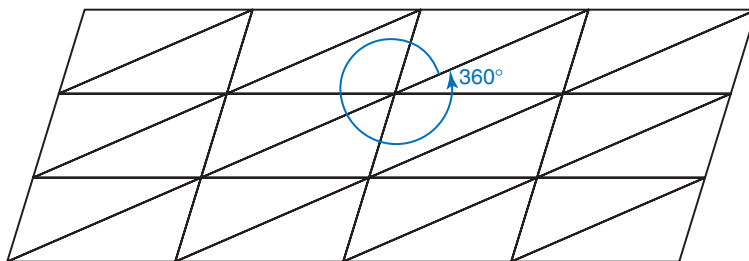
How can you tell if congruent figures will cover a page and leave no gaps?

Connect

When congruent copies of a figure cover a page and leave no gaps, we say the figure **tiles the plane**.

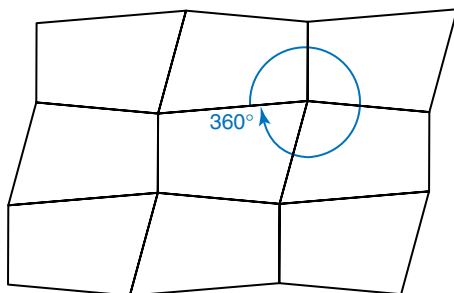
- A triangle always tiles the plane.

At any point where vertices meet, the angles add to 360° .



➤ A quadrilateral always tiles the plane.

At any point where vertices meet, the angles add to 360° .



Example

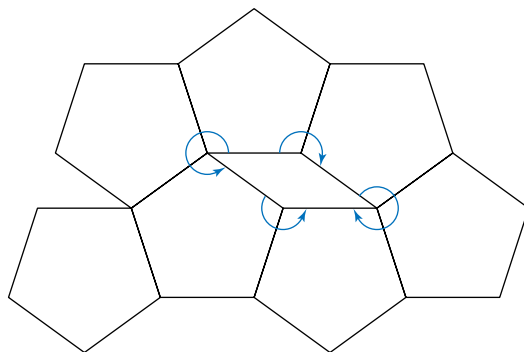
Will a pentagon always tile the plane? Explain.

Solution

If we can find a pentagon that does *not* tile the plane, we can say that a pentagon does not always tile the plane.

Draw a regular pentagon.

Use tracing paper to repeat the pentagon to try to cover the page.



This pentagon does not cover the page.

It leaves gaps that are rhombuses.

Five vertices do not meet.

There are points where 3 vertices meet and the sum of the angles is less than 360° .

There are points where 2 vertices meet and the sum of the angles is less than 360° .

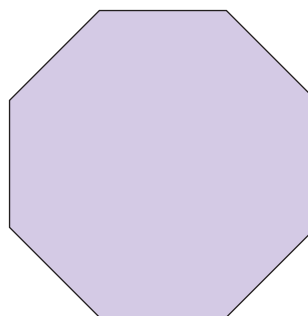
So, a pentagon does not always tile the plane.

In the *Practice* questions, you will investigate to find which other figures *do* tile the plane.

Practice



1. Use dot paper.
Draw a convex hexagon that is not regular.
Try to cover the dot paper with copies of this hexagon.
Does the hexagon tile the plane? Explain.
2. Use dot paper.
Draw a concave hexagon.
Try to cover the dot paper with copies of this hexagon.
Does the hexagon tile the plane? Explain.
3. Here is a regular octagon.



Trace this octagon.
Try to tile the plane.
What do you notice?

4. Look at the picture called *Reptiles*, drawn by M.C. Escher.



Which figure do you think Escher started with?
Explain how Escher's reptiles tile the plane.

Calculator Skills

Evaluate.

- $23.56 + 27.39 \times 4.35$
- $(23.56 + 27.39) \times 4.35$

Why are the answers different?

5. A floor tile is a regular hexagon.
What happens when you try to tile a rectangular floor with a regular hexagon?
Use isometric dot paper to find out.
6. Why do most tiling patterns in floors and patios use squares or rectangles?



7. Assessment Focus

Not all pentagons tile the plane. Use grid paper.

- a) Find a pentagon that will tile the plane.
Describe the pentagon.
Explain how it tiles the plane.
- b) How many different pentagons can you find that will tile the plane? Draw each pentagon and show how it tiles the plane.
- c) Explain why some pentagons tile the plane, while others do not.

8. In question 3, you discovered that a regular octagon will not tile the plane.
Use grid paper. Find an octagon that will tile the plane.
Explain how it tiles the plane.

Take It Further

9. Think about “tiling” in nature. Which figures are used?

Reflect

How can you tell if a polygon will tile the plane?
Use examples in your explanation.

Focus Create and analyse designs using transformations.

Explore

Work on your own.
You will need isometric dot paper.
Choose two or more of these Pattern Blocks.



Make a design to cover a page.
Copy your design on dot paper.
Label each figure in your design.
Explain your design in terms of transformation images.
That is, how do you rotate, translate, or reflect each Pattern Block to generate the design? Write your instructions carefully.

Reflect & Share

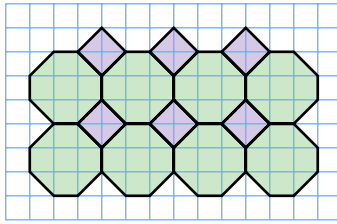
Trade instructions with a classmate.
Generate your classmate's pattern.
Check your version of the pattern with your classmate's.
How do they compare?

Connect

In *Section 7.4*, you investigated tiling patterns.
You used congruent copies of one figure.
You discovered that not all octagons tile the plane.
But an octagon and a square can tile the plane,
as shown in the *Example* that follows.

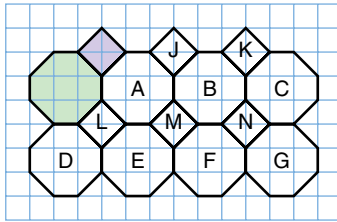
Example

Use transformations to describe how to construct this design.



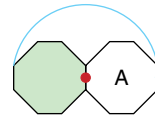
Solution

Label the figures in the design, as shown.



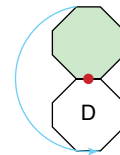
Start with the shaded octagon.

- Step 1* To get octagon A, rotate the shaded octagon a $\frac{1}{2}$ turn about a turn centre that is at the midpoint of the right side.



Repeat a similar rotation to get figure B from figure A.

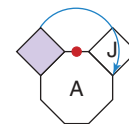
- Step 2* To get octagon D, rotate the shaded octagon a $\frac{1}{2}$ turn about a turn centre that is at the midpoint of the bottom side.



Repeat a similar rotation to get octagon E from octagon A.

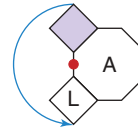
Look at the shaded square.

- Step 3* To get square J, rotate the shaded square a $\frac{1}{2}$ turn about the midpoint of the top side of octagon A.



Repeat a similar rotation to get square K from square J.

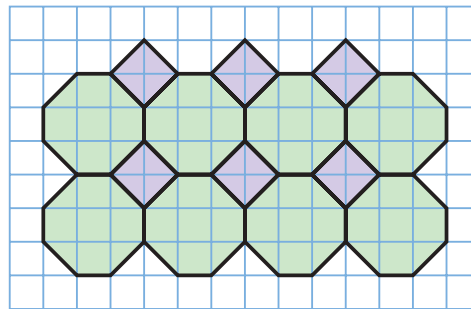
Step 4 To get square L, rotate the shaded square a $\frac{1}{2}$ turn about the midpoint of the left side of octagon A.



Repeat a similar rotation to get square M from square J.

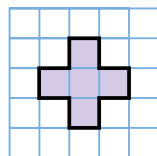
Practice

1. Here is the design from the *Example*.



- Use translations to describe how to construct this design.
- Use reflections to describe how to construct this design.

2. Use this figure and transformations to create a design on grid paper.



Describe the design in terms of transformations.

- Use isometric paper. Use a parallelogram and an equilateral triangle to make a design. Use transformations to describe the design.
- Draw a figure. Use transformations of the figure to make a border design for a photo frame. Draw the design. Describe how you made it.

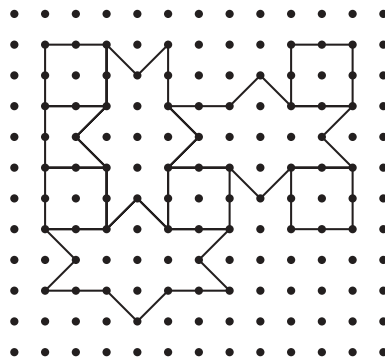
Number Strategies

Which number is the least?

- the sum of all the factors of 30
- the sum of all the factors of 46
- the product of all the factors of 67



5. The Alhambra is a walled city and fortress in Granada, Spain. It was built in the 14th century. Here is part of one of its many tiling patterns.

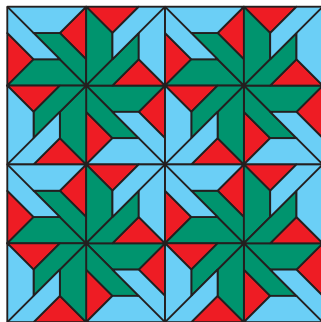


Copy this pattern on dot paper.
Continue the pattern to cover the page.
Use transformations to describe the pattern.

6. Assessment Focus

Use dot paper or grid paper.
Create a design that uses 2 or more figures that together tile the plane.
Colour your design.
Use transformations to describe your design.
Try to describe your design as many ways as you can.

7. Here is a flooring pattern.



Use a copy of this pattern.
Use transformations to describe the patterns in one square.

Reflect

When you use transformations to describe a design, how do you decide which transformation to use? Include a design in your explanation.



Using a Computer to Transform Figures

Focus Use technology to create and analyse designs.

Software, such as *The Geometer's Sketchpad*, can be used to transform figures.


Follow these steps:

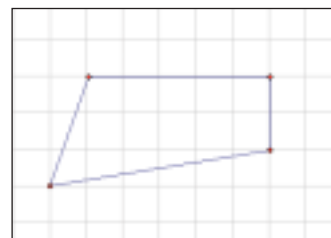
1. Open *The Geometer's Sketchpad*.
From the File menu, choose **New Sketch**.


To make a “grid paper” screen:

2. From the **Edit** menu, click on **Preferences**.
Select the **Units** tab.
Check that the Distance Units are cm.
Click **OK**.
3. From the **Graph** menu, choose **Define Coordinate System**.
The screen has grid lines and two numbered axes.
4. Click on each axis and the two red dots.
The axes and the dots are highlighted.
From the **Display** menu, choose **Hide Objects**.
The axes and dots disappear.
The screen appears like a piece of grid paper.
5. From the **Graph** menu, choose **Snap Points**.

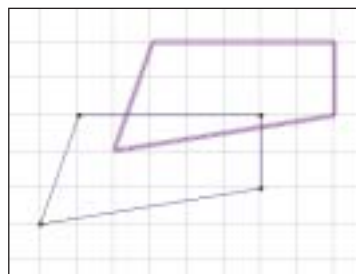
Translating a Figure

6. To create a quadrilateral:
From the **Toolbox**, choose .
Click and drag to construct a quadrilateral.





7. To translate the quadrilateral:
From the **Toolbox**, choose .
Click each side of the quadrilateral to select it.
The quadrilateral is highlighted.

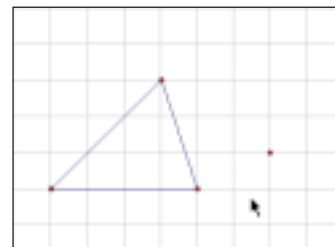
8. From the **Transform** menu, choose **Translate**.
Under **Translation Vector**:, choose **Rectangular**.
Under **Horizontal**:, choose **Fixed Distance**.
Enter 2.0 cm for the Horizontal distance.
Under **Vertical**:, choose **Fixed Distance**.
Enter 2.0 cm for the Vertical distance (below left).
Click **Translate** to get the quadrilateral and its image after a translation 2 right, 2 up (below right).




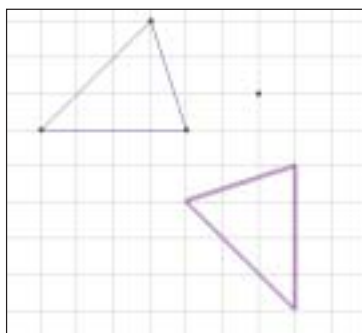
9. Drag any vertex or side of the original figure.
See what happens to the translation image.
10. From the **Edit** menu, choose **Undo Translate Point**.
The screen shows the original quadrilateral and translation image.
To print the quadrilateral and its translation image, from the **File** menu, choose **Print**.
11. Repeat *Steps 7 to 10* using different horizontal and vertical distances.

Rotating a Figure

12. From the **File** menu, choose **New Sketch**.
Follow *Steps 2 to 5* to make a “grid paper” screen.
13. To create a triangle:
From the **Toolbox**, choose .
Click and drag to construct a triangle.
14. To rotate the triangle:
From the **Toolbox**, choose .
Click to place a point near the triangle.




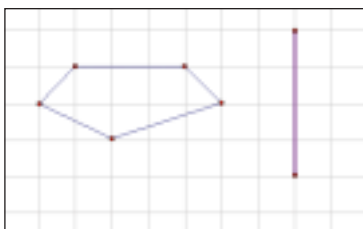
15. Click to select the point.
From the **Transform** menu, choose **Mark Center**.
This is the turn centre for your rotation.
16. From the **Toolbox**, choose .
Click to select each side of the triangle.
The triangle is highlighted.
17. From the **Transform** menu, choose **Rotate**.
Under **Rotate By:**, choose **Fixed Angle**.
Enter 90 degrees. Click **Rotate** to show the triangle and its image after a rotation of 90° counterclockwise.




18. Drag any vertex or side of the original figure.
See what happens to the rotation image.
19. From the **Edit** menu, choose **Undo Translate Point**.
The screen shows the original triangle and rotation image.
20. Repeat *Steps 16 to 19* using a different number of degrees of rotation. Print the figure and its rotation image.

Reflecting a Figure

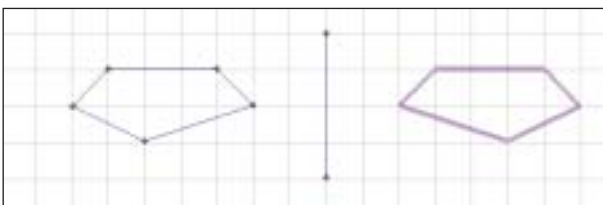
21. Repeat *Step 12*.
22. To create a polygon:
From the **Toolbox**, choose .
Click and drag to construct a polygon.
23. To reflect the polygon:
With the Straightedge Tool still selected, draw a vertical line near your polygon. The line is highlighted.



24. From the **Transform** menu, choose **Mark Mirror**.
The line is a mirror line.

25. From the **Toolbox**, choose .
Click to select each side of the polygon.
The polygon is highlighted.

26. From the **Transform** menu, choose **Reflect**.
The polygon and its reflection image are shown.



27. Drag any vertex or side of the original figure.
See what happens to the reflection image.

28. Drag either end point from the mirror line.
See what happens.

29. From the **Edit** menu, choose **Undo Translate Point**.
Do this two times. The screen shows the original polygon
and its reflection image.

30. Repeat *Steps 23 to 28* using a horizontal mirror line.
Print the figure and its reflection image.



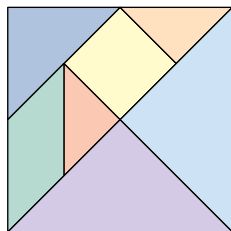
Use any or all of the transformations above to make a design that covers the screen.
Print your design.

Choosing a Strategy

Remember to count squares of different sizes.

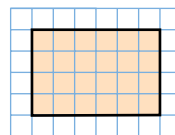
Strategies

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.



1. A train is travelling at a rate of 1 km every 45 s. At this rate, how far will the train travel in 1 h?

2. How many squares are in this rectangle?



3. Use grid paper. Try to draw a quadrilateral with each number of lines of symmetry.

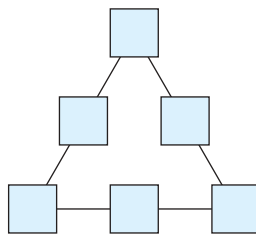
- | | | |
|------|------|------|
| a) 0 | b) 1 | c) 2 |
| d) 3 | e) 4 | f) 5 |

Which quadrilaterals could you not draw? Explain.

4. Use grid paper. Draw this figure:
It has at least 1 line of symmetry.
It has perimeter 24 units.
It has area 23 square units.

5. Copy the diagram below.
Write these numbers in the boxes: $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, 1;
so the sum of the fractions along each side is :

- | | |
|-------------------|------|
| a) $1\frac{1}{2}$ | b) 2 |
|-------------------|------|



6. What fraction of the area of a tangram is triangles?

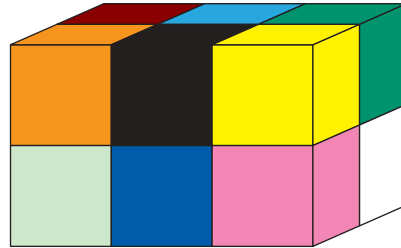
7. Write the next five terms in each pattern.
Describe each pattern rule.

a) $\frac{A}{B \ C \ D} \quad \frac{E \ F}{}$

b) 1, 5, 10, 25, ...

c) O, T, T, F, F, S, S, ...

8. This rectangular prism is made with 12 different coloured cubes. The colours are black, white, red, orange, light green, dark green, light blue, dark blue, brown, yellow, pink, and purple.



Using linking cubes to build the prism.

- a) The colours on the left face are light green, orange, brown, and red.

Draw and colour the back face of the prism.

Another congruent prism is made from the coloured cubes. The colours on the front face are: red, orange, dark green, light blue, black, and white.

The colours on the top face are: red, black, white, pink, purple, and yellow.

The colours on the right face side are: light blue, white, yellow, and brown.

The colours on the left face side are: red, dark green, pink, and dark blue.

- b) Build the prism. Sketch the prism.

- c) What colours are on the bottom face of the prism?

How do you know?

9. In her fitness program Jessie runs on Mondays, Wednesdays, and Saturdays, and swims on Tuesdays and Fridays. Malcolm runs every third day and swims on the day after each run.

Jessie and Malcolm run together on Saturday July 6.

On what days and dates in July will they:

- a) run together again?
 b) swim together?





Office Space Planner

An office space planner plans the best use of the office space. He ensures that employees have a workplace that is functional and attractive. Systems furniture is a series of connected partitions, work surfaces, and cabinets. It is frequently used in offices. The planner uses a computer to design multiple 'standard' work areas. Each work area is designed to meet the needs of a group of employees with similar jobs. The work areas are also designed to 'fit' with other work areas of the same size and shape, and with work areas that have different sizes and shapes. It's all a bit of a puzzle! The space planner must solve the puzzle using geometry and an understanding of how people work and interact (ergonomics).

A workplace, where every work area or 'cubicle' is identical, is often seen as 'cell-like' or dehumanizing. Suppose you are an office planner. What might you do to make groups of work areas more appealing, and still make the best use of the available floor area?



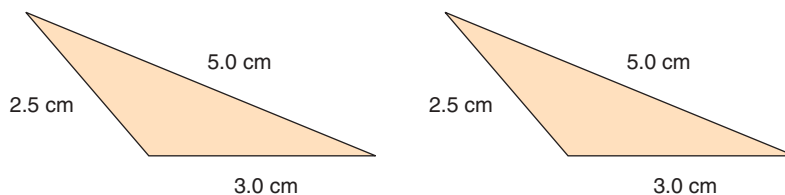
What Do I Need to Know?



Conditions for Congruent Triangles

Two triangles are congruent if:

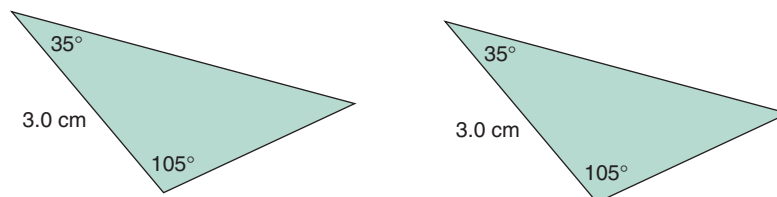
- three pairs of corresponding sides are equal



- two pairs of corresponding sides are equal and the corresponding angles between these sides are equal



- two pairs of corresponding angles are equal and one pair of corresponding sides are equal



Conditions for Congruent Figures

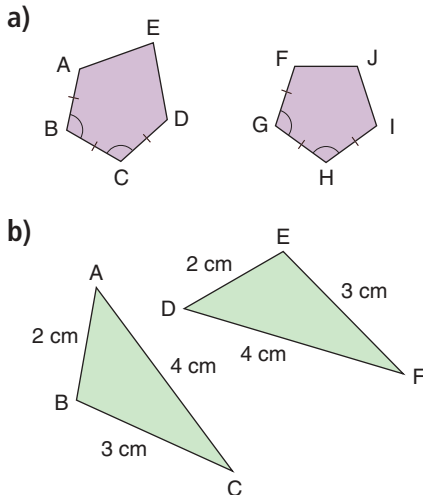
For figures that are not triangles, two figures are congruent if corresponding sides are equal *and* corresponding angles are equal.

LESSON

- 7.1** 1. Use grid paper or dot paper. Draw each figure.
- a concave hexagon
 - a convex pentagon
 - a concave quadrilateral
 - a figure that is not a polygon
 - a regular triangle
- Describe the attributes of each figure. Include angle measures.

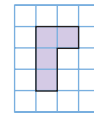
2. Use grid paper or dot paper. Draw each figure.
- a hexagon with exactly 2 lines of symmetry
 - a triangle with rotational symmetry of order 3
 - a pentagon with exactly 3 acute angles
 - a pentagon with exactly 3 obtuse angles
- Describe the attributes of each figure. Include angle measures.

- 7.2** 3. Are the figures in each pair congruent? How do you know?



- 7.3** 4. Plot these points on a coordinate grid: $A(4, 6)$, $B(4, 7)$, $C(7, 9)$, and $D(6, 6)$. Join the points to form a quadrilateral. The coordinates of the vertices of 3 images are given. Identify the transformation that produced each image.
- $C(7, 9)$, $E(5, 12)$, $F(4, 12)$, $G(4, 10)$
 - $I(4, 16)$, $J(6, 16)$, $K(7, 13)$, $L(4, 15)$
 - $M(9, 4)$, $N(8, 1)$, $P(6, 1)$, $Q(6, 2)$

- 7.4** 5. Copy this figure on grid paper.

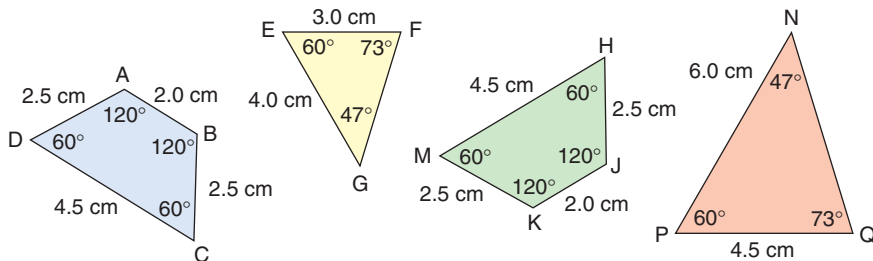


How many different ways can you use the figure to tile the plane? Show each way you find.

- 7.5** 6. Draw 2 different figures on grid paper that will together tile the plane. Use the figures to make a design. Colour one of each figure. Use transformations to explain how to create the design beginning with each coloured figure.

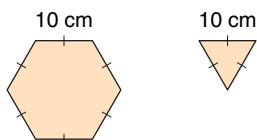
Practice Test

1. Use the figures below.



Identify:

- two congruent figures, and explain how you know they are congruent
 - two figures that are *not* congruent, and explain how you know they are not congruent
 - a scalene triangle with a 60° angle
 - a quadrilateral with a 60° angle
2. Two triangles are congruent if they have 3 matching sides. Suppose two triangles have 3 matching angles. Are the triangles congruent? Justify your answer.
3. Three students looked at a figure and its transformation image. Igal said the picture showed a translation. Shaian said the picture showed a rotation. Cherie said the picture showed a reflection. All three students were correct. What might the picture be? Draw a diagram to show your thinking.
4. Julie will use both of these tiles to cover her floor.



Use isometric paper.

Draw 2 different designs Julie could use.

For each design, use transformations to explain how to create the design.

When we tile the plane with congruent copies of one figure, we make a **tessellation**.

M.C. Escher was a famous Dutch artist.
He designed many different tessellations.



You will create two designs in the Escher style.
The first design is in the style of *Reptiles*, on page 268.

Part 1

Use square dot paper or grid paper.

Tile the plane with a figure of your choice.

Sketch a design on one figure.

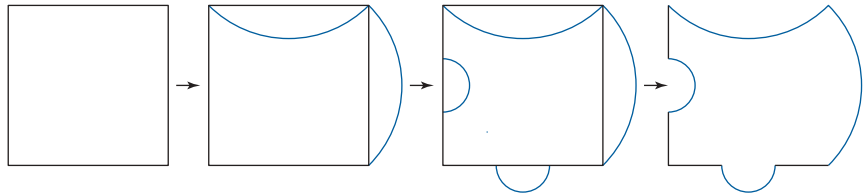
Repeat the sketch until every figure in the plane has the design.

Use transformations to describe how to generate the design beginning with one tile.

You could start with a rectangle, parallelogram, or regular hexagon, instead.

Part 2

Start with a square. Draw congruent curves on 2 sides. A curve that goes “in” on one side must go “out” on the other side. Draw different congruent curves on the other 2 sides.



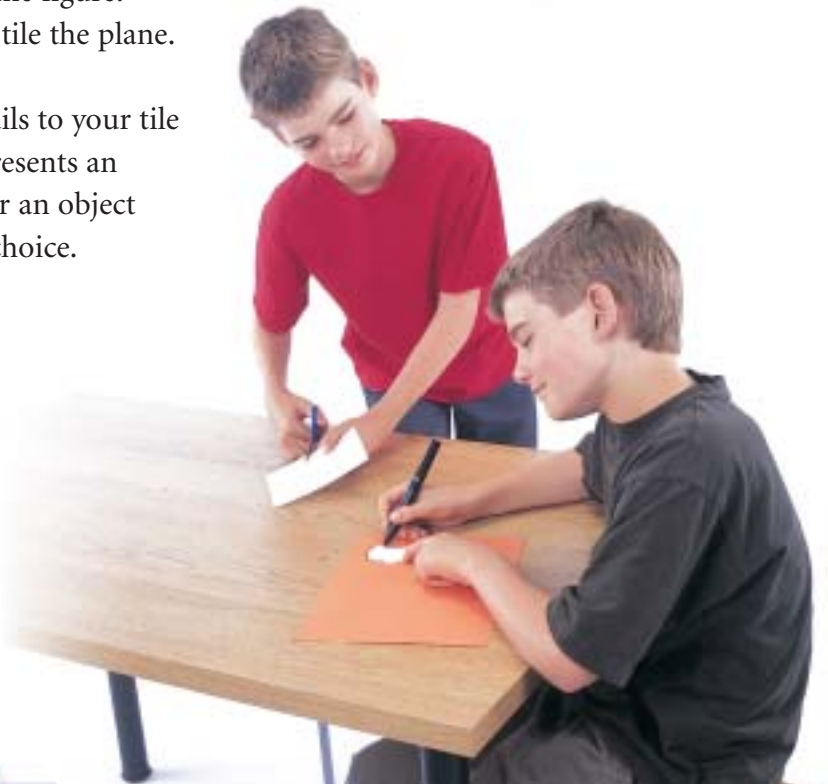
Check List

Your work should show:

- ✓ the initial tile you created for each design
- ✓ the designs you created
- ✓ how you used transformations to create the designs
- ✓ the correct use of mathematical language

Trace the new figure on cardboard.
Cut out the figure.
Use it to tile the plane.

Add details to your tile so it represents an animal or an object of your choice.



Reflect on the Unit

How are transformations related to congruent figures?
Include diagrams in your explanation.

Cross Strand Investigation

Pick's Theorem

Materials:

- 11 pin by 11 pin geoboard
- geobands
- square dot paper
- 0.5-cm grid paper

Work with a partner.

When a polygon is drawn on square dot paper, there are dots on the perimeter and dots inside.

Is there a relationship between the numbers of these dots?

As you complete this *Investigation*, include all your work in a report that you will hand in.

- Make a convex polygon on the geoboard.

Draw the polygon on square dot paper.

Count:

- the number of dots on the perimeter of the polygon
- the number of dots inside the polygon

Find the area of the polygon in square units.

Record your results in a table.

Polygon	Number of dots on the perimeter	Number of dots inside	Area in square units

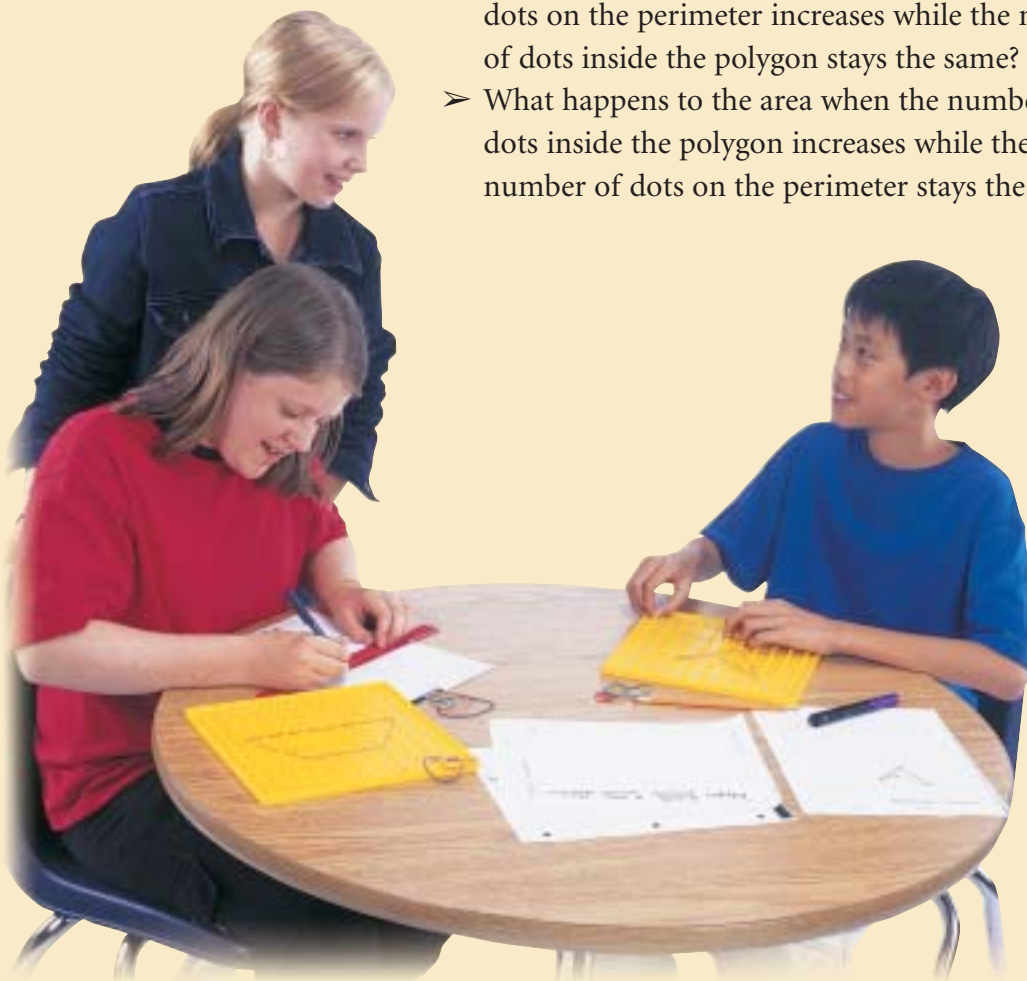
- Repeat this process for 10 or more different polygons. Include triangles, parallelograms, trapezoids, pentagons, hexagons, and so on.
- Choose two measurements: for example, *Number of dots on the perimeter* and *Number of dots inside*. Draw a scatter plot of the data. What trends do you see in the graph? Explain.

- What patterns do you see in the table?
How can you use the patterns to write an expression for the area of a polygon in terms of the number of dots inside the polygon and the number of dots on the perimeter?

- Use variables.
Let the number of dots on the perimeter be p .
Let the number of dots inside be i .
Write a formula for the area, A , of a polygon in terms of p and i .
This relationship is called **Pick's Theorem**.

Take It Further

- What happens to the area when the number of dots on the perimeter increases while the number of dots inside the polygon stays the same?
- What happens to the area when the number of dots inside the polygon increases while the number of dots on the perimeter stays the same?



UNIT

8

Working with Percents

Stores offer goods on sale to encourage you to spend your money.

What is the sale price of each item in the picture?

How did you calculate the sale price?

What do you need to add to find the price you pay?

What You'll Learn

- Relate decimals, fractions, and percents.
- Solve problems that involve fractions, decimals, and percents.
- Multiply decimals.
- Divide decimals.
- Draw circle graphs by hand.

Why It's Important

When you buy something, you pay sales tax. To be able to calculate the sales tax is useful. Then you know if you have enough money to buy the item.



Ice Skates
\$50.00



Skis
\$200.00



Running Shoes
\$150.00



Hockey
Sweater
\$80.00



Half price
Sale!
Everything
50% off
tag price!

Key Words

- percent
- percent circle

Skills You'll Need

Writing a Fraction As a Decimal

To write a fraction as a decimal, try to write an equivalent fraction with denominator 100.

When we cannot write an equivalent fraction, we use a calculator to divide.



Example 1

Convert each fraction to a decimal.

a) $\frac{3}{5}$

b) $\frac{7}{8}$

Solution

a) $\frac{3}{5}$

Write an equivalent fraction.

$$\frac{3}{5} \xrightarrow{\times 20} \frac{60}{100}$$

$$\frac{60}{100} = 0.60$$

So, $\frac{3}{5} = 0.60$

b) $\frac{7}{8}$

We cannot write an equivalent fraction with denominator 100.

Use a calculator.

$\frac{7}{8}$ means $7 \div 8$.

Key in: 7 $\boxed{\div}$ 8 $\boxed{=}$ to display 0.875

$$\frac{7}{8} = 0.875$$

Some conversions from fractions to decimals are worth remembering.

Try to remember these:

$$\frac{1}{2} = 0.5 \quad \frac{1}{4} = 0.25 \quad \frac{1}{5} = 0.2 \quad \frac{1}{8} = 0.125 \quad \frac{1}{10} = 0.1 \quad \frac{1}{100} = 0.01$$

You can use these conversions to write other fractions as decimals.

For example, since $\frac{1}{10} = 0.1$, then $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, and so on.



Check

1. Write each fraction as a decimal. Use mental math.

a) $\frac{3}{4}$

b) $\frac{2}{5}$

c) $\frac{6}{10}$

d) $\frac{68}{100}$

2. Write each fraction as a decimal.

a) $\frac{5}{8}$

b) $\frac{3}{16}$

c) $\frac{3}{8}$

d) $\frac{7}{16}$

Percent

Percent means per hundred. One whole, or 1, is 100%.

So, 70% means $\frac{70}{100}$, 4% means $\frac{4}{100}$, and 100% means $\frac{100}{100}$, or 1.

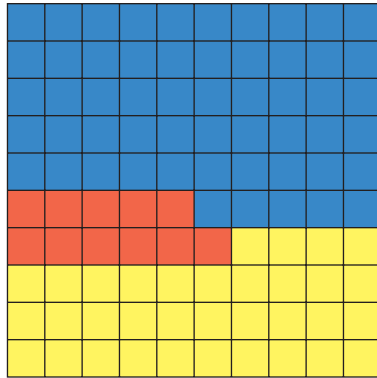
Example 2

What percent of this hundred chart is shaded each colour?

a) Red

b) Blue

c) Yellow



Solution

a) There are 11 red squares out of 100 squares.

That is: $\frac{11}{100} = 11\%$

b) There are 55 blue squares out of 100 squares.

That is: $\frac{55}{100} = 55\%$

c) There are 34 yellow squares out of 100 squares.

That is: $\frac{34}{100} = 34\%$

✓ Check

Use a hundred chart.

3. Shade:

a) 15% red

b) 26% yellow

c) 43% green

d) 10% blue

4. What percent of the hundred chart in question 3 is not shaded?

8.1

Relating Fractions, Decimals, and Percents

Focus Relate percent to fractions and decimals.

We see uses of percent everywhere.

What do you know from looking at each picture?

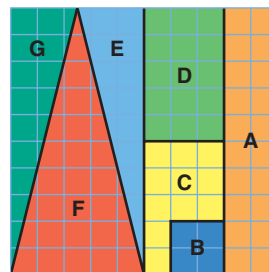


Explore

Work with a partner.

Your teacher will give you a large copy of this puzzle.

Describe each puzzle piece as a fraction, a decimal, and a percent of the whole puzzle.



Reflect & Share

Compare your answers with those of another pair of classmates. If the answers are different, how do you know which are correct?

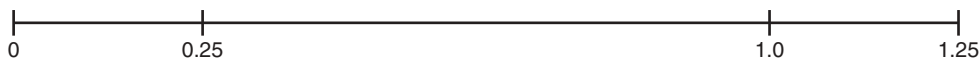
Connect

➤ Since a percent can be written as a fraction, a percent can also be written as a decimal.

We can use number lines to illustrate the relationships.

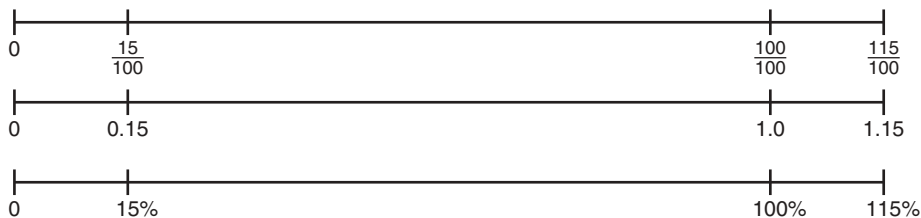
For example:

$$25\% = \frac{25}{100} = 0.25 \qquad 125\% = \frac{125}{100} = 1.25$$



➤ Conversely, a decimal can be written as a percent:

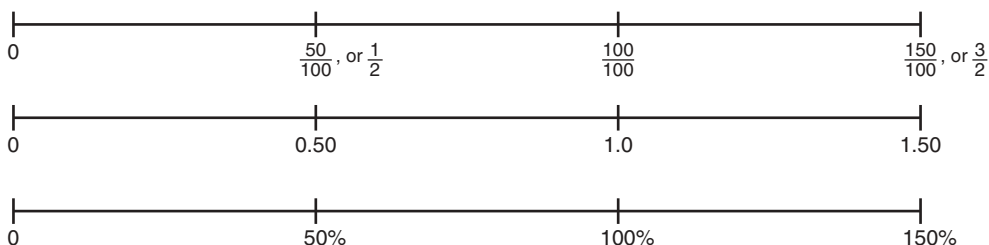
$$0.15 = \frac{15}{100} = 15\% \qquad 1.15 = \frac{115}{100} = 115\%$$



➤ To write a fraction as a percent, first write the fraction with denominator 100.

For example:

$$\frac{1}{2} = \frac{50}{100} = 50\% \qquad \text{and} \qquad \frac{3}{2} = \frac{150}{100} = 150\%$$



When a decimal has 3 digits after the decimal point, we can write it as a fraction with denominator 1000.

➤ Some fractions cannot be written with denominator 100. Use a calculator to divide.

$$\begin{aligned} \frac{5}{8} &= 0.625 \\ &= \frac{625}{1000} && \text{Divide numerator and denominator by 10.} \\ &= \frac{62.5}{100} \\ &= 62.5\% \end{aligned}$$

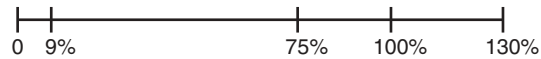
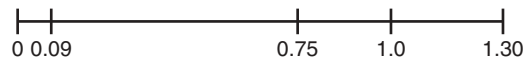
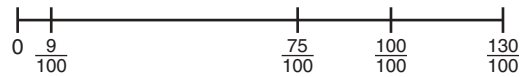
Example

- a) Write each percent as a fraction and as a decimal.
- i) 75% ii) 9% iii) 130%
- b) Write each fraction as a percent and as a decimal.
- i) $\frac{2}{5}$ ii) $\frac{5}{2}$

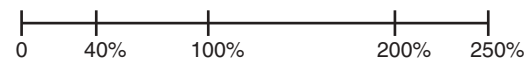
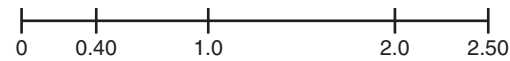
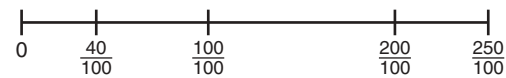
Draw number lines to show how the numbers are related.

Solution

- a) i) $75\% = \frac{75}{100} = 0.75$
 ii) $9\% = \frac{9}{100} = 0.09$
 iii) $130\% = \frac{130}{100} = 1.30$

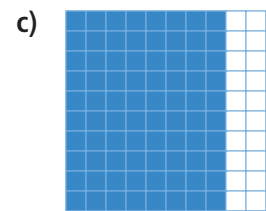
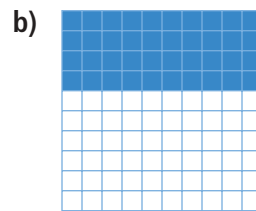
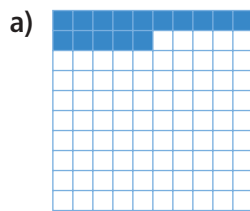


- b) i) $\frac{2}{5} = \frac{40}{100} = 40\%$ ii) $\frac{5}{2} = \frac{250}{100} = 250\%$
- $\frac{40}{100} = 0.40$ $\frac{250}{100} = 2.50$



Practice

1. What percent of each hundred chart is shaded?
 Write each percent as a fraction and a decimal.



2. Write each fraction as a percent.
 Sketch number lines to show how the numbers are related.

a) $\frac{1}{4}$ b) $\frac{3}{10}$ c) $\frac{7}{5}$ d) $\frac{3}{4}$

3. Write each fraction as a decimal and a percent.

a) $\frac{2}{10}$ b) $\frac{3}{50}$ c) $\frac{4}{25}$ d) $\frac{23}{20}$ e) $\frac{14}{10}$



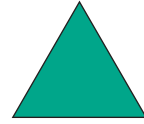
4. Fred had 8 out of 10 on a test. Janet had 82% on the test. Who did better? How do you know?

5. This equilateral triangle is 20% of a larger figure.

Use triangular grid paper.

Draw a figure that shows 100%.

Is there more than one answer? Explain.



6. This orange square represents 25% of a larger figure.

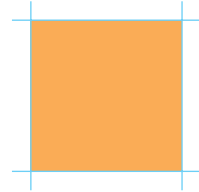
Use 2-cm grid paper.

a) Draw 50% of the larger figure.

b) Draw 75% of the larger figure.

c) Draw the larger figure.

d) Draw 125% of the larger figure.



7. **Assessment Focus** You will need a sheet of paper and coloured pencils.

Divide the paper into these 4 sections.

- 1 blue section that is $\frac{1}{2}$ the page
- 1 red section that is 10% of the page
- 1 yellow section that is 25% of the page
- 1 green section to fill the remaining space

Explain how you did this.

What percent of the page is the green section?

How do you know?

8. What does it mean when someone states, “She gave it 110%”? How can this comment be explained using math?

9. Suppose each pattern is continued on a hundred chart. The numbers in each pattern are coloured red. For each pattern, what percent of the numbers on the chart are red?

Explain your strategy for each pattern.

- | | |
|--------------------------|---------------------|
| a) 4, 8, 12, 16, 20, ... | b) 1, 3, 5, 7, ... |
| c) 2, 4, 8, 16, ... | d) 1, 3, 7, 13, ... |

Number Strategies

How many:

- centimetres in 1 m?
- square centimetres in 1 m²?
- cubic centimetres in 1 m³?

Sketch a picture to show each relationship.

Take It Further

Reflect

Suppose you know your mark out of 20 for an English test. How could you write the mark as a percent?



Showdown

HOW TO PLAY THE GAME:

1. Cut out the cards, then shuffle them. Deal all the cards. Each player stacks his cards face down in a pile.
2. Players turn over the first card in their piles, then compare the numbers. The player whose card shows the greater number wins and takes both cards. Both cards are placed in a “captured pile” next to the winner’s original pile. See *Step 6* when there is a tie.
3. Play continues until all cards are turned up.
4. Each player then shuffles his captured pile of cards and play continues.
5. The game ends when one player has no cards.
6. **Showdown** If there is a tie between two cards, a showdown occurs. Each player takes the next two cards from his pile and places them face down on top of the original card. A third card is then turned over by each player, and these cards are compared. The player whose card shows the greater number takes all cards involved in the showdown. If there is a tie, another showdown occurs until the tie is broken.

YOU WILL NEED

2 sheets of SHOWDOWN playing cards (48 cards); scissors; a calculator

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To have all the cards when the game ends

What strategies did you use to find which number was greater?

Note: If a player is unable to place cards in a showdown because the player has only one card, the player’s last card is the turnover card.



Explore

Work with a partner.

Look at the surface of your desk.



There is probably a textbook on it, and maybe a pencil case, an eraser, or a ruler.

How much of the surface is covered?

How much of the surface is not covered?

Use a calculator and a ruler.

Estimate the percent of your desk's surface that is covered and the percent that is not. Show your work.

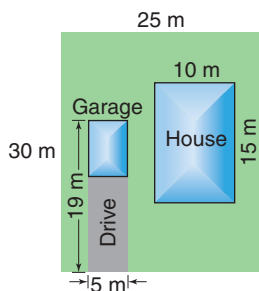
Reflect & Share

Compare your results with those of another pair of classmates.

Discuss the strategies you used to find the percent of surface not covered.

Connect

Here is a sketch of a yard.



Round 245 to 250 to get a "friendly" number. Both 250 and 750 have 250 as a common factor.

- To find the percent of the yard that is covered by the house, garage, and drive, calculate the area covered by

- the house: $15 \text{ m} \times 10 \text{ m} = 150 \text{ m}^2$
- the garage and the drive: $19 \text{ m} \times 5 \text{ m} = 95 \text{ m}^2$

Total area covered is: $150 \text{ m}^2 + 95 \text{ m}^2 = 245 \text{ m}^2$

The area of the yard is: $30 \text{ m} \times 25 \text{ m} = 750 \text{ m}^2$

The fraction of the yard that is covered is: $\frac{245}{750}$

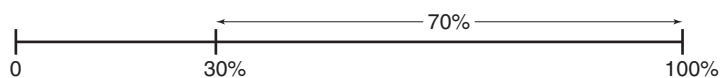
For an estimate of the fraction, round 245 to 250.

The fraction of the yard that is covered is about $\frac{250}{750} = \frac{1}{3}$.

$\frac{1}{3}$ is about 30%.

So, about 30% of the yard is covered by the house, garage, and drive.

- To find the percent of the yard made up of garden and grass:
 The yard is 100%.
 Subtract: $100\% - 30\% = 70\%$
 So, about 70% of the yard is made up of garden and grass.
 We can show these percents on a number line.



We can use mental math to estimate and calculate percent.

Example 1

There are 27 students in the Grade 7 class.
 Five students are left-handed.

- What is the fraction of students who are left-handed?
- Estimate the percent of students who are left-handed.

Solution

a) The fraction of students who are left-handed is $\frac{5}{27}$.

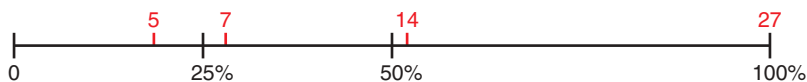
b) *Method 1*

One whole or 100% is 27 students.

50% of 27 students is about 14 students.

25% of 27 students is about 7 students.

So, the percent of students who are left-handed is less than 25%.



Method 2

Approximate $\frac{5}{27}$ to a “friendly” fraction: $\frac{5}{27} \doteq \frac{5}{25}$

$$\frac{5}{25} \begin{array}{c} \xrightarrow{\times 4} \\ = \\ \xrightarrow{\times 4} \end{array} \frac{20}{100} = 20\%$$

Approximately 20% of the students are left-handed.

A “friendly” fraction has a denominator such as 5, 10, 20, 25, 50. A fraction with one of these denominators can be more easily converted to a percent.

Example 2

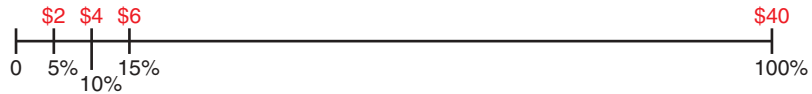
A pair of pants costs \$39.99.
The sales tax is 15%.
Estimate the cost of the pants.



Solution

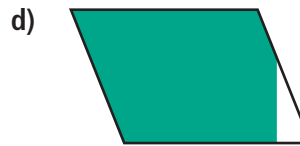
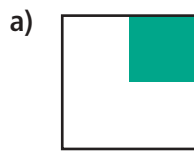
Mental math strategy:
A quick way to find 10% is to move the decimal point 1 place to the left:
10% of 40. = 4.0

Round \$39.99 to \$40.
To find 15%, find 10% and 5%.
10% of \$40 = $0.1 \times \$40 = \4
5% is $\frac{1}{2}$ of 10%.
So, 5% of \$40 = $\frac{1}{2}$ of \$4 = \$2
So, the sales tax is \$4 + \$2 = \$6.
The cost of the pants is about \$40 + \$6 = \$46.
We can show these percents on a number line.



Practice

1. Estimate the percent of each figure that is shaded.



2. Find 10% of each number.

- | | | | |
|-------|-------|-------|--------|
| a) 50 | b) 40 | c) 90 | d) 300 |
| e) 75 | f) 33 | g) 45 | h) 18 |

3. Use your answers to question 2.

Find 5% of each number in question 2.

4. Use your answers to questions 2 and 3.
Find 15% of each number in question 2.
5. This strip is 12 cm long.



Mental Math

How many squares are there on an 8 by 8 checkerboard?

Remember to count squares of all sizes.

Find the length of:

- a) 25% of the strip b) 10% of the strip
c) 20% of the strip d) 150% of the strip

Sketch number lines to illustrate your answers.

6. A pair of shoes costs \$65. The sales tax is 15%. Explain how to find the cost of the shoes using two different methods.

7. Estimate.

- a) 49% of 150 b) 31% of 40
c) 149% of 60 d) 98% of 54
e) 90% of 44 f) 61% of 88
g) 2% of 200 h) 5% of 81

8. Scott estimated that 22% of 160 is approximately 30.

Do you agree with his estimate? Explain.

Sketch number lines to support your answer.

9. There were 341 pine trees and spruce trees in a woodlot.

One hundred twenty-two trees were pine.

- a) What fraction of the trees were spruce?
b) Estimate the percent of spruce trees.



10. In a parking lot, there are 45 North American cars and 21 foreign cars.

- a) What fraction of the cars are foreign?
b) Estimate the percent of cars that are foreign.
c) Estimate the percent of cars that are not foreign.



11. **Assessment Focus** Look at the front and back cover of this textbook, including the spine. Explain how you might estimate the percent of the book cover that is illustrated.

Show all your steps. Provide numbers to support your answer.

- 12.** About 8% of Canada is covered by fresh water.
The area of Canada is approximately 9 970 000 km².
- Estimate the area of Canada covered by fresh water.
 - About how much of Canada is not covered by fresh water?
- 13.** About 23% of Canada is covered by tundra.
Use the data in question 12.
Estimate the area of tundra in Canada.
- 14.** Raji's bedroom floor has an area of 12 m². She estimates that her bed, desk, and bookshelf cover approximately 7.5 m² of floor space.
- What fraction of the floor space is not covered?
 - What percent of the floor space is not covered?
- Sketch number lines to illustrate your answers.

Take It Further

- 15.** Edward estimates he has travelled approximately 80 km.
The total length of Edward's trip is 430 km.
About what percent does Edward still have left to go?

Reflect

Which percents can you find by using mental math or estimation?
Give an example of how to find each percent you name.

Math Link

Your World

Percents are printed on the side of a cereal box to indicate the nutritional value of the contents. The percents relate to every 30-g serving (about 1 cup). Suppose there is a 10% sugar content. Then, in a 30-g serving, there are 3 g of sugar per cup of cereal. Next time you look for a box of your favourite cereal, check out the percents of sugar and fat. Compare them to another brand of cereal. Would your family doctor or dentist approve of your choice? Explain.



Explore

Work with a partner.



How could you find how much you save on an item that originally cost \$48.00?

Find several ways to solve this problem.

Reflect & Share

Compare strategies with another pair of classmates.

Which strategy would you use if the sale was 45% off? Explain.

Connect

A paperback novel originally cost \$7.99.

It is on sale at 15% off.

To find how much you save, calculate 15% of \$7.99.

$$15\% = \frac{15}{100} = 0.15$$

$$\begin{aligned} \text{So, } 15\% \text{ of } \$7.99 &= \frac{15}{100} \text{ of } 7.99 \\ &= 0.15 \times 7.99 \end{aligned}$$

Recall, from Unit 4, how we multiply two decimals with tenths.

We use the same method to multiply two decimals with hundredths.

To multiply: 0.15×7.99 , multiply without the decimal points.

Then insert the decimal point in the answer by estimation or by counting decimal places.

$$\begin{array}{r} 799 \\ \times 15 \\ \hline 3995 \\ 7990 \\ \hline 11985 \end{array}$$

There is a total of 4 decimal places in the question; so, there will be 4 decimal places in the answer.

Estimate:

15% is about 20%, which is $\frac{1}{5}$.

\$7.99 is about \$10.00.



So, 0.15×7.99 is about $\frac{1}{5}$ of 10, which is 2.

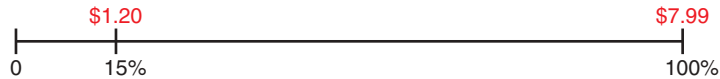
So, $0.15 \times 7.99 = 1.1985$

Round to 2 decimal places.

1.1985 is 1.20, to 2 decimal places.

You save \$1.20 by buying the book on sale.

We can show this on a number line.



Example

A park has an area of 52.6 km^2 .

Sixty-five percent of the park is forest.

The rest of the park is lakes.

What is the area of the lakes?

Solution

65% of the park is forest.

So, $100\% - 65\%$, or 35% of the park is lakes.

The area of the lakes is: 35% of 52.6

$$35\% = \frac{35}{100} = 0.35$$

So, 35% of 52.6 = 0.35×52.6

Multiply: 526×35

$$\begin{array}{r} 526 \\ \times 35 \\ \hline 2630 \\ 15780 \\ \hline 18410 \end{array}$$

Since the area of the park is given to 1 decimal place, we round the answer to 1 decimal place.

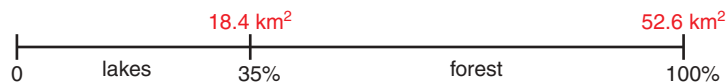
Estimate:

35% of 52.60 is about 50% of 50, which is 25.

So, $0.35 \times 52.6 = 18.41$

35% of 52.6 km^2 is 18.41 km^2 .

The area of the lakes is about 18.4 km^2 .



Practice



1. Calculate.
a) 10% of 27.3 b) 20% of 48.4 c) 1% of 30.6 d) 120% of 81.2
2. Find.
a) 18% of 36 b) 24% of 67 c) 98% of 28 d) 67% of 112
3. Find each percent of \$59.99.
a) 25% b) 75% c) 30% d) 70% e) 80% f) 90%
4. The regular price of a radio is \$60.00. Find the sale price when the radio is on sale for:
a) 25% off b) 30% off c) 40% off
5. Find the cost of each item on sale.
Each item has a 15% sales tax added to the sale price.
a) coat: 55% off \$90 b) shoes: 45% off \$40

Number Strategies

Use Pattern Blocks.

Suppose the red block represents 1 whole.

What does each block represent?

- the blue block
- the green block
- the yellow block

6. Assessment Focus

- a) Calculate each percent of 52.3.
How can you do this by completing only one multiplication?
i) 2% ii) 20% iii) 200%
 - b) Make up a similar example. Choose a number and 3 related percents. Show how you only need to multiply once to find all the percents of the number.
7. How is calculating 25% of \$15.00 the same as calculating 15% of \$25.00? How are the calculations different? Sketch number lines to illustrate your answer.
 8. A garage floor is rectangular.
Its length is 9.0 m and its width is 5.1 m.
The length and width of a Toyota Corolla are 4.5 m and 1.7 m.
What percent of the garage floor is occupied by the car?

Reflect

Choose a percent. Choose an amount of money.
Calculate the percent of the money. Show your work.

Mid-Unit Review

LESSON

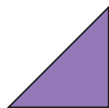
- 8.1 1.** Write each fraction as a decimal and as a percent. Sketch number lines to illustrate.

a) $\frac{4}{5}$ b) $\frac{3}{25}$ c) $\frac{118}{50}$ d) $\frac{7}{20}$

- 2.** Write each fraction as a decimal and as a percent. Sketch number lines to illustrate.

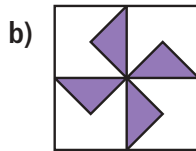
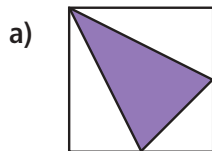
a) $\frac{27}{20}$ b) $\frac{14}{25}$ c) $\frac{15}{8}$ d) $\frac{7}{16}$

- 3.** This right isosceles triangle is 20% of a larger figure.



Draw a figure that represents 100%.

- 8.2 4.** Estimate the percent of each figure that is shaded.



- 5.** In a piggy bank, there were 21 pennies, 32 nickels, and 13 dimes.
- What fraction of the coins were nickels?
 - Estimate the percent of coins that were nickels.
 - How much money was in the piggy bank?
 - Estimate the percent of money that was in dimes.

- 6.** Find 10% of each number.
a) 28 b) 66 c) 35 d) 180

- 7.** Use the results of question 6. Find 60% of each number in question 6.

- 8.** Use estimation. Find an approximate percent for each fraction.

a) $\frac{14}{17}$ b) $\frac{21}{30}$ c) $\frac{118}{60}$ d) $\frac{172}{80}$ e) $\frac{2}{21}$

- 9.** Estimate.

a) 14% of 98 b) 61% of 52
c) 76% of 202 d) 98% of 134

- 8.3 10.** a) Find:

- 5% of \$1.00
- 30% of \$1.00
- 130% of \$1.00
- 20% of \$1.00

- b) Sketch a number line to illustrate your answers to part a.

- 11.** Toni wants to buy a shirt. The original price is \$85.00, but it is on sale for 30% off. Toni will pay 15% sales tax. How much will the shirt cost?

- 12.** How is calculating 85% of \$40.00 the same as calculating 40% of \$85.00? How are the calculations different? Sketch number lines to illustrate your answers.

Focus Use circle graphs to display data and solve problems.

You have drawn a circle graph using a computer.

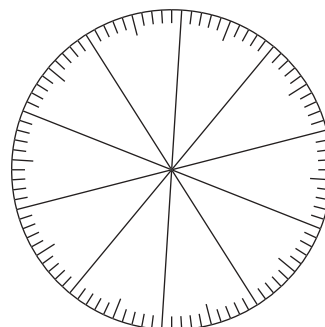
Now that you can calculate percents, you can draw a circle graph by hand.

This is a **percent circle**.

The circle is divided into 100 congruent sectors.

Each sector is 1% of the whole circle.

You can draw a circle graph on a percent circle.



Explore

Work with a partner.

Your teacher will give you a percent circle.

Everyone in the class writes on the board the number of siblings he or she has.

Copy this table.

0 Siblings	1 Sibling	2 Siblings	More than 2 Siblings

Record the data.

Use the percent circle.

Draw a circle graph to display the data.

Write 2 questions you can answer by looking at the graph.

Reflect & Share

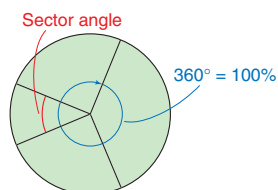
Trade questions with another pair of classmates.

Use your graph to answer your classmates' questions.

Compare graphs. If they are different, try to find out why.

How did you use fractions and percents to draw a circle graph?

Connect



Recall that a circle graph shows how parts of a set of data compare with the whole set.

Each sector of a circle graph represents a percent of the whole circle.

The whole circle represents 100% and has a central angle of 360° .



This table shows the 2003 top 10 Ladies' Professional Golf Association (LPGA) money winners' place of birth.

Asia	Australia	Europe	North America
Kung	Teske	Sorenstam	Daniel
Pak			Inkster
Park			Jones
Han			Ochoa

To draw a circle graph to show what percent of the top 10 were born in each place, follow these steps.

Step 1

Write the number of players born in each place as a fraction of 10, then as a percent.

$$\begin{aligned} \text{Asia: } \frac{4}{10} = 0.4 = \frac{40}{100} = 40\% & \quad \text{Australia: } \frac{1}{10} = 0.1 = \frac{10}{100} = 10\% \\ \text{Europe: } \frac{1}{10} = 0.1 = \frac{10}{100} = 10\% & \quad \text{North America: } \frac{4}{10} = 0.4 = \frac{40}{100} = 40\% \end{aligned}$$

The area of the circle represents all the golfers. All the sector angles add to 360° .

Step 2

To find the sector angle for each place of birth, multiply each decimal by 360° .

Round to the nearest degree, when necessary.

Asia 40%: $0.40 \times 360^\circ = 144^\circ$

Australia 10%: $0.10 \times 360^\circ = 36^\circ$

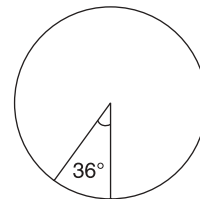
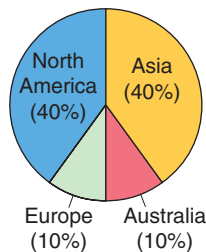
Europe 10%: same as Australia, so 36°

North America 40%: same as Asia, so 144°

To check, add the angles. The sum should be 360° .

$$\begin{array}{r} 144^\circ \\ 36^\circ \\ 36^\circ \\ + 144^\circ \\ \hline 360^\circ \end{array}$$

Top 10 LPGA Winners' Place of Birth



Step 3

Construct a circle.

Use a protractor to construct each sector angle.

Start with the smallest angle. Draw a radius.

Measure 36° .

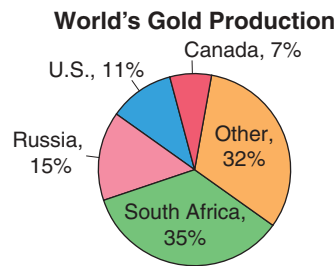
Start the next sector where the previous sector finished.

Label each sector with its name and percent.

Write a title for the graph.

Example 1

This circle graph shows the world's gold production.



In 2002, the world's gold production was approximately 2300 t. About how much gold would have been produced in each country?

a) Canada

b) South Africa

Solution

a) 7% is produced in Canada. b) 35% is produced in South Africa.

$$\begin{aligned}\text{Mass of gold} &= 7\% \times 2300 \\ &= 0.07 \times 2300 \\ &= 161\end{aligned}$$

About 161 t would have been produced in Canada.

$$\begin{aligned}\text{Mass of gold} &= 35\% \times 2300 \\ &= 0.35 \times 2300 \\ &= 805\end{aligned}$$

About 805 t would have been produced in South Africa.

Example 2

These four oceans have an area of approximately 337 million km².

a) Draw a circle graph to represent the data in the table.

b) Find the area of the largest ocean.

Ocean	Percent of Total Area
Atlantic	25
Arctic	3
Indian	22
Pacific	50

Solution

a) Each area is written as a percent of the total area of the oceans.

The area of the circle represents the total area of the oceans.

To find the sector angle for each ocean, multiply each percent by 360°.

$$\begin{aligned}\text{Atlantic: } 25\% \text{ of } 360^\circ &= 0.25 \times 360^\circ \\ &= 90^\circ\end{aligned}$$

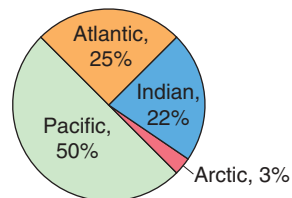
$$\begin{aligned}\text{Arctic: } 3\% \text{ of } 360^\circ &= 0.03 \times 360^\circ \\ &= 10.8^\circ \\ &\doteq 11^\circ\end{aligned}$$

Round each angle to the nearest degree, when necessary.

$$\begin{aligned} \text{Indian: } 22\% \text{ of } 360^\circ &= 0.22 \times 360^\circ \\ &= 79.2^\circ \\ &\approx 79^\circ \end{aligned}$$

$$\begin{aligned} \text{Pacific: } 50\% \text{ of } 360^\circ &= 0.50 \times 360^\circ \\ &= 180^\circ \end{aligned}$$

Areas of Oceans



Construct a circle.

Use a protractor to construct each sector angle.

Label each sector with its name and percent.

Write a title for the graph.

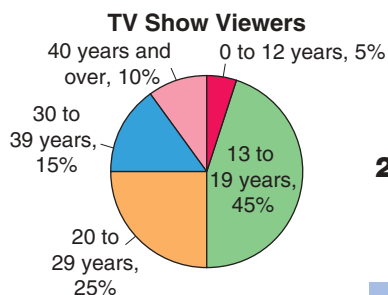
- b) The largest ocean is the Pacific Ocean.

Its area is:

$$\begin{aligned} 50\% \text{ of } 337\,000\,000 \text{ km}^2 &= 0.50 \times 337\,000\,000 \text{ km}^2 \\ &= 168\,500\,000 \text{ km}^2 \end{aligned}$$

The largest ocean has an area about 168 500 000 km².

Practice



1. This circle graph shows the ages of viewers of a TV show. One week, approximately 250 000 viewers tuned in.

How many viewers were in each age group?

- a) 13 to 19 b) 20 to 29 c) 40 and over

2. a) Can the data in each table below be displayed in a circle graph? Explain.

i)

ii)

Canadians, Educational Attainment, 2001

0 to 8 years of elementary school	10%
Some secondary school	17%
Graduated from high school	20%
Some post secondary education	9%
Post-secondary certificate or diploma	28%
University degree	16%


Canadian Households with These Appliances, 2000

Automobiles	64%
Cell phone	42%
Dishwasher	51%
Internet	42%

- b) For the data that could not be displayed in a circle graph, state what type of graph you would use to display it.

3. The table shows the number of Grade 7 students with each eye colour at Northern Public School.

Eye Colour	Number of Students
Blue	11
Brown	24
Green	9
Grey	6



- a) Calculate the percent of students with each eye colour.
 b) Draw a circle graph to represent the data.

4. In a telephone survey, 400 people voted for their favourite radio station.

Radio Station	Votes
MAJIC99	88
EASY2	?
ROCK1	120
HITS2	100

- a) How many people chose EASY2?
 b) Write the number of people who voted for each station as a percent of the total number surveyed.
 c) Draw a circle graph to display the results of the survey.

5. **Assessment Focus** Choose some labels from canned food, cereal boxes, or other foods.

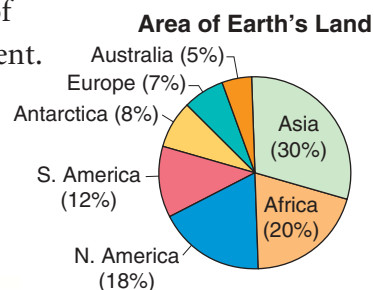
- a) List the nutritional information on each label.
 b) Calculate the mass of each nutrient as a percent of the total mass.
 c) Find the mass of each nutrient in 50 g of the food.
 d) Draw a circle graph to display the percent of each nutrient. Show your work.

Calculator Skills

How many times can you multiply 2 by itself until the product gets too big for your calculator display? Write the greatest product as a power of 2.

Take It Further

6. This circle graph shows the percent of Earth's land occupied by each continent. The area of North America is approximately 220 million km². Use the percents in the circle graph. Find the approximate area of each of the other continents.



Reflect

What do you need to know about percents to be able to draw a circle graph? Include an example in your answer.

Explore

Work with a partner.

Suppose you pay \$15.00 for a shirt. How could you find out what the original price was?

Draw a diagram to model this problem.

Show several strategies for solving the problem.

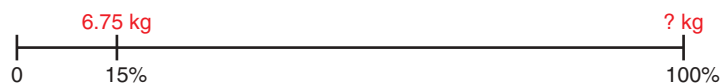


Reflect & Share

Compare your strategies with those of another pair of classmates. Which strategies can you use to find the original price if you paid 35% of the original price? Explain.

Connect

Jenny wanted to know her brother Alan's mass. Alan told Jenny that 15% of his mass is 6.75 kg.



Jenny knows what 15% is. She wants to find 100%.

15% is 6.75.

So, 1% is: $\frac{6.75}{15}$

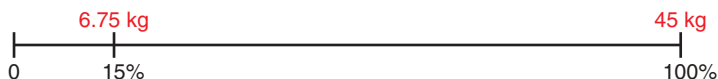
Use long division: $6.75 \div 15$

1% is 0.45.

So, 100% is: $0.45 \times 100 = 45$

Alan's mass is 45 kg.

$$\begin{array}{r} 0.45 \\ 15 \overline{)6.75} \\ \underline{60} \\ 75 \\ \underline{75} \\ 0 \end{array}$$



Example

This sign appeared in a shop window.

Eric pays \$58.50 for a jacket.

What is the list price of the jacket?



Solution

Use a calculator or long division to find $\frac{\$58.50}{65}$.

65% of the list price is \$58.50.

So, 1% of the list price is: $\frac{\$58.50}{65} = \0.90

And, 100% is: $\$0.90 \times 100 = \90.00

The list price of the jacket is \$90.00.



Practice

- 25% is 1.25 m.
 - What is 50%?
 - What is 100%?Sketch a number line to show your answer.
- 35% is 4.2 kg.
 - What is 1%?
 - What is 100%?Sketch a number line to show your answer.
- 45% is 13.5 cm.
 - What is 1%?
 - What is 100%?Sketch a number line to show your answer.
- The principal reported that 75% of the total number of families attended the school's Fun Fair. Three hundred sixty families went to the fair. How many families have children at the school?
- A hockey team played 30 games. It won 60% of the games. How many games did the team lose?
- Grace has read 30% of a book. She has read 72 pages. How many pages are in the book?



Number Strategies

Add or subtract.

- $\frac{3}{5} + \frac{3}{6}$
- $\frac{4}{5} - \frac{1}{2}$
- $\frac{3}{4} + \frac{4}{3}$
- $\frac{5}{6} - \frac{1}{4}$

7. Paco ate 25% of a 16-slice pizza and placed the rest in the fridge. Santos ate 25% of the leftovers.
- a) How many pieces were left?
 - b) What percent of the original pizza remained?
8. This year, 40 more children joined the local soccer club than last year. This is a 10% increase. How many students played in the club last year?
9. **Assessment Focus** Anika wants to buy a blouse. The original price is \$75.00. It is on sale for 30% off. Anika will pay 15% sales tax.
- a) How much will the blouse cost?
 - b) Does it make any difference to the cost in each situation?
 - i) The 30% is taken off before the 15% tax is added.
 - ii) The 15% tax is added before the 30% is taken off.Explain. Draw number lines to show your thinking.
10. Write your own problem that involves working with percents. Solve your problem. Show your work.

Take It Further

11. A pair of shoes in a clearance store went through a series of reductions. The regular price was \$125. The shoes were reduced by 20%. Three weeks later, the shoes were reduced by a further 20%. Later in the year, the shoes were advertised for sale at $\frac{3}{4}$ off the ticket price. Sean wants to buy the shoes. He has to pay 15% sales tax.
- a) Sean has \$40.00. Can Sean buy the shoes? Explain.
 - b) If your answer to part a is yes, how much change does Sean get?
- Sketch number lines to illustrate your work.
12. A box was $\frac{3}{4}$ full. The box fell on the floor. Thirty marbles fell out. This was 20% of the marbles that were in the box. How many marbles were in a full box?



Reflect

How does a good understanding of percents help you outside the classroom? Give an example.

Choosing a Strategy

Strategies

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.

Use a calculator when you need to.

1. Write 720 as the product of consecutive whole numbers. Find all the possible ways.

2. A digital clock shows this time.

Seven minutes past 7 is a palindromic time.

- a) List all the palindromic times between noon and midnight.
- b) What is the shortest time between two palindromic times?
3. I am a perfect square less than 400. The sum of my digits is also a perfect square. Which number could I be? Find all possible numbers.
4. The trail around Lake Pender is 20 km long. How long do you think it would take you to walk around the lake? Explain.



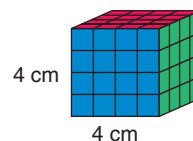
5. Jack's heart rate is 72 beats/min. How long will it take for Jack's heart to beat one million times?

6. A 4-cm cube is painted red on the top and bottom faces, blue on the front and back faces, and green on the side faces.

The cube is cut into 1-cm cubes.

How many of the small cubes will have:

- a) 3 colours of paint?
- b) 2 colours?
- c) 1 colour?
- d) no paint?
7. Use dot paper. Draw a quadrilateral with at least two sides equal. How many different quadrilaterals can you draw?





8. The decimals 0.1, 0.2, 0.3, up to 1.0, are written on ten cards.
How many sets of three cards have a sum of 2?

9. Use 1-cm grid paper.
Draw a 6 cm by 4 cm rectangle.
Shade $\frac{1}{4}$ red.
Shade $\frac{1}{3}$ of the remainder blue.
Shade $\frac{1}{2}$ of the unshaded area green.
What fraction is unshaded?

10. A small company consists of the owner and 3 employees.
The owner earns \$10 000 a month. The employees earn \$4000, \$4000, and \$6000 per month.

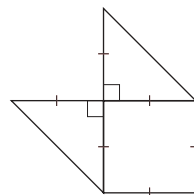
a) The employees ask the owner for a pay raise.

Should they use the mean, median or mode salary when they present their case? Explain.

b) Should the owner use the mean, median or mode when she explains she cannot give the employees a raise? Explain.

11. The area of a rectangular field is 3496 m².
Its perimeter is 260 m.
What are the dimensions of the field?

12. This figure comprises a square and two halves of a congruent square.



Arrange the parts of the figure to produce each new figure.

a) a rectangle **b)** a parallelogram **c)** a trapezoid **d)** a triangle

Sketch your answers.



Sports Trainer

Sports trainers use scientific research and scientific techniques to maximize an athlete's performance. An athlete may be measured for percent body fat, or percent of either fast- or slow-twitch muscle fibre.

A trainer may recommend the athlete eat pre-event meals that contain a certain percent of carbohydrate, or choose a "sports drink" that contains a high percent of certain minerals. The trainer creates and monitors exercise routines. These enable the athlete to attain a certain percent of maximum heart rate, speed, or power.

Most sports drinks contain minerals. Research shows that the most effective sports drink has a magnesium to calcium ratio of 1:2. The body absorbs about 87% of magnesium in a drink, and about 44% of calcium in a drink. One sports drink lists 100 mg of calcium per scoop. About how much magnesium is there in 1 scoop?



What Do I Need to Know?

 Here are some fractions, decimals, and percents you should know.

$$1 = 1.0 = 100\%$$

$$\frac{1}{2} = 0.5 = 50\%$$

$$\frac{1}{4} = 0.25 = 25\%$$

$$\frac{1}{5} = 0.2 = 20\%$$

$$\frac{1}{10} = 0.1 = 10\%$$

$$\frac{1}{100} = 0.01 = 1\%$$

What Should I Be Able to Do?

For extra practice, go to page 445.

LESSON

8.1 1. Write each fraction as a decimal and as a percent.

a) $\frac{8}{20}$

b) $\frac{14}{5}$

c) $\frac{14}{25}$

d) $\frac{7}{10}$

e) $\frac{9}{8}$

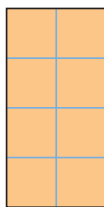
f) $\frac{17}{10}$

2. Write each number two other ways.

a) 0.18 b) 0.3

c) 80% d) $\frac{3}{8}$

3. This rectangle is 40% of a larger figure.



Use grid paper.

a) Draw a figure that shows 100%.

b) Draw a figure that shows 120%.

8.2 4. Estimate.

a) 39% of 250

b) 41% of 89

c) 19% of 60

d) 91% of 46

e) 97% of 64

f) 59% of 98

g) 3% of 300

h) 4% of 92

5. A DVD costs \$29.99.

The sales tax is 15%.

Estimate the cost of the DVD.

8.3 6. There are 35 students in a Grade 7 class. On one day, 20% of the students were at a sports meet. How many students were in class?

7. The regular price of a DVD player is \$120.00. What is the sale price in each case?

a) 25% off

b) 30% off

c) 40% off

d) 50% off

e) 60% off

f) 45% off

8. The regular price for a mountain bike is \$640.00.

It is on sale for 30% off.

The sales tax is 15%.

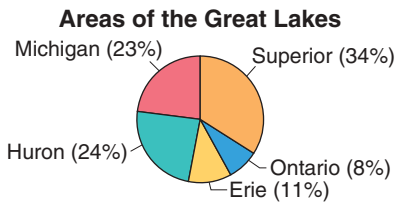
a) Jamie bought the bike at the regular price.

What did it cost her?

b) Sam bought the bike on sale.

How much did Sam save?

- 8.4** 9. This circle graph shows the surface area of the Great Lakes.



- Which lake has a surface area about $\frac{1}{4}$ of the total area?
 - Explain why Lake Superior has that name.
 - The total area of the Great Lakes is about 244 000 km². Find the surface area of Lake Erie.
- 10.** These tables show energy resources and electricity generation in Canada.

Primary Energy Resources in Canada, 2001	
Coal	11%
Hydro-electric	27%
Natural gas	24%
Nuclear	6%
Oil	32%

Electricity Generation by Fuel Type, 2001	
Coal	18%
Hydro-electric	61%
Natural gas	4%
Nuclear	13%
Oil	4%

- Draw a circle graph to display the data in each table.
- What do you know from looking at the two graphs?

- 11.** Here are 25 players on the Toronto Maple Leafs roster for the 2003/2004 season. The table shows each player's place of birth.

USSR	Canada	Europe	U.S.
Antropov	Belak	Berg	Fitzgerald
Mogilny	Belfour	Kaberle	Johnson
	Berehowsky	Nolan	Kidd
	Domi	Pilar	Klee
	Marchment	Ponikarovsky	
	McCabe	Reichel	
	Nieuwendyk	Renberg	
	Perrott	Sundin	
	Roberts		
	Stajan		
	Tucker		

- Draw a circle graph to show what percent of the team was born in each place.
- What if a U.S.-born player was traded for a USSR-born player? How would the graph change?

- 8.5** **12.** Eighty percent of Areyana's height is 140 cm. How tall is Areyana? Draw a number line to illustrate your answer.

- 13.** At Lakehead Elementary School, 280 students participate in a walk for charity. That is 70% of the students in the school. How many students attend the school?

Practice Test

1. How can 25% of one item be different from 25% of another? Explain.
2. The strip below is 25% of a longer strip.
 - a) What is the length of a strip that is 80% of the longer strip?
 - b) Draw the longer strip.



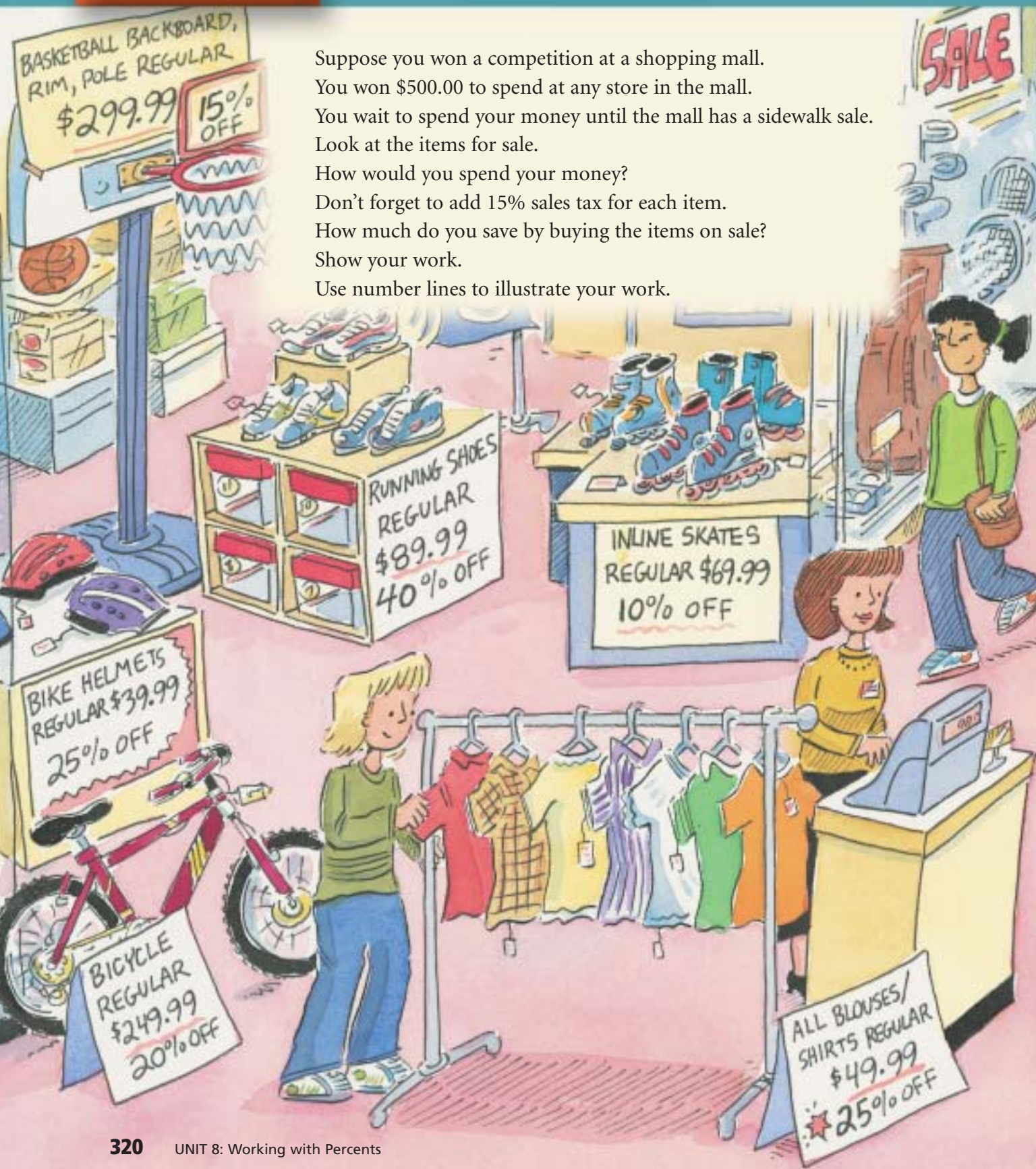
3. Find.
 - a) 600% of 40
 - b) 60% of 40
 - c) 6% of 40
 What patterns do you see in the answers?

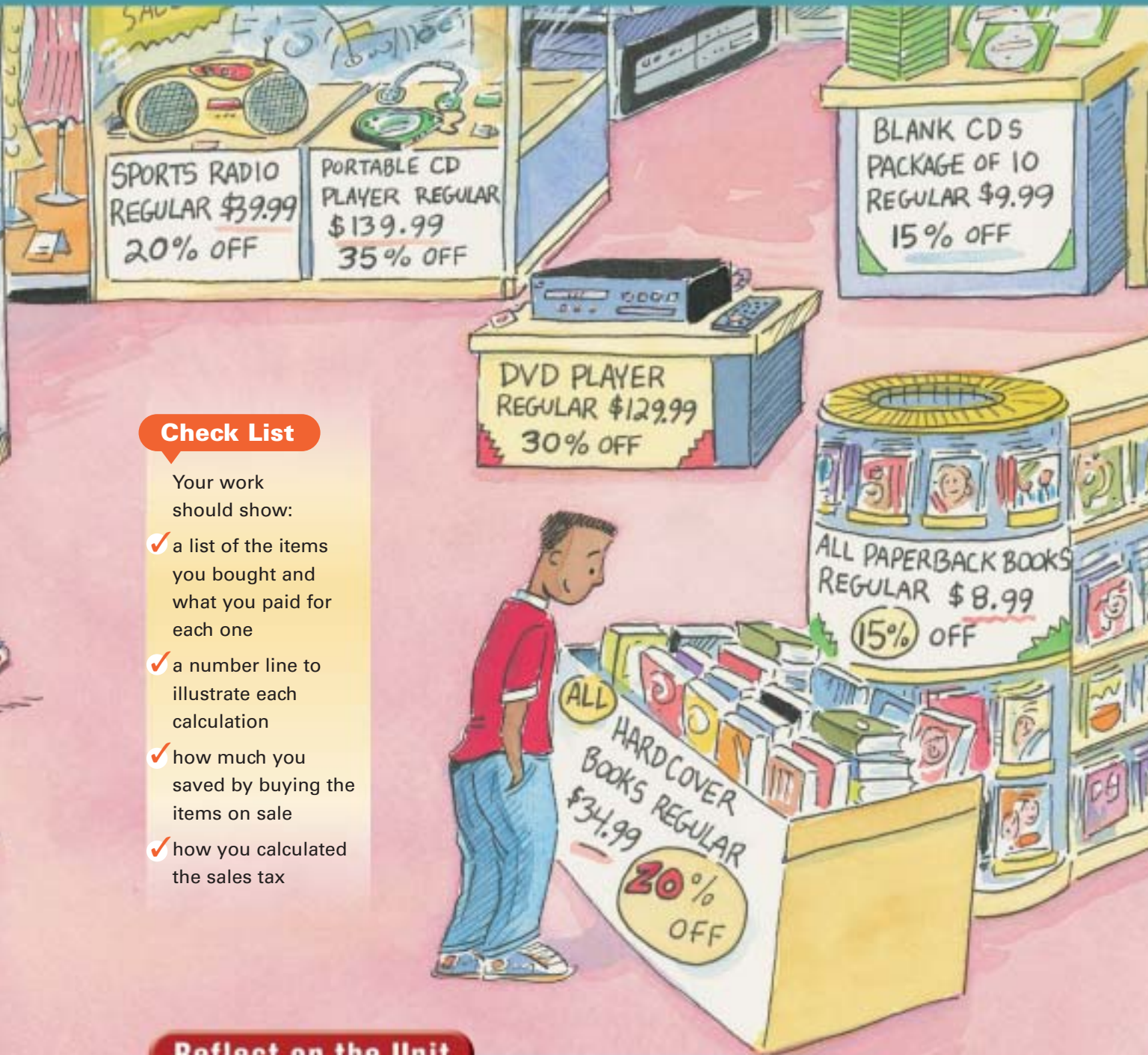
4. The regular price of a pair of shoes is \$75.00. The shoes are on sale for 25% off. The sales tax is 15%.
 - a) What is the sale price of the pair of shoes?
 - b) What do the shoes cost, with sales tax?

Type of Land Cover in Canada	
Type	Percent of Total Area
Forest and taiga	45
Tundra	23
Wetlands	12
Fresh water	8
Cropland and rangeland	8
Ice and snow	3
Human use	1

5. The table shows the type of land cover in Canada, as a percent of the total area.
 - a) Draw a circle graph to display these data.
 - b) Did you need to know the area of Canada to draw the circle graph? Explain.
6. This spring, 26 dogs were adopted from the local animal shelter. This is 130% of the number of dogs that were adopted last spring. How many dogs were adopted last spring? Draw a number line to illustrate your answer.

Suppose you won a competition at a shopping mall. You won \$500.00 to spend at any store in the mall. You wait to spend your money until the mall has a sidewalk sale. Look at the items for sale. How would you spend your money? Don't forget to add 15% sales tax for each item. How much do you save by buying the items on sale? Show your work. Use number lines to illustrate your work.





Check List

Your work should show:

- ✓ a list of the items you bought and what you paid for each one
- ✓ a number line to illustrate each calculation
- ✓ how much you saved by buying the items on sale
- ✓ how you calculated the sales tax

Reflect on the Unit

What have you learned about percents and how they are used?

UNIT

- 1** **1.** Use grid paper.
Draw a picture that shows a square root of each number.
a) 49 b) 16 c) 144
- 2** **2.** To make a pot of tea for 4 people, you need 3 tea bags.
a) How many tea bags are needed for each number of people?
i) 8 ii) 12 iii) 16 iv) 20
b) How many tea bags are needed for 6 people? Explain.
- 3** **3.** Students in a Grade 7 class are filling shoe boxes with toys for children in other countries. A shoe box measures 30 cm by 18 cm by 16 cm.
a) Find the volume of a shoe box. The students fill 24 shoe boxes. Eight shoe boxes are packed into a larger box.
b) What could the dimensions of this larger box be?
c) What are the most likely dimensions of the larger box? Justify your choice.
- 4** **4.** a) Multiply: 3.6×2.4
b) Which other pair of factors has the same product as the product in part a)?
How many different pairs of factors can you find?
Explain your work.

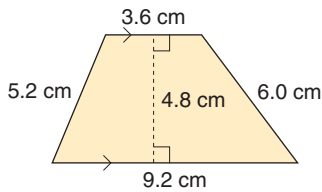
- 5** **5.** To celebrate his birthday, Justin and his friends played miniature golf. Here are their scores: 29, 33, 37, 24, 41, 38, 48, 26, 36, 33, 40, 29, 36, 22, 31, 38, 42, 35, 33
a) Draw a stem-and-leaf plot. It was a par 36 course. This means that a good golfer takes 36 strokes to complete the course.
b) How many scores were under par? At par? Over par?
c) What was the range of the scores?
d) Calculate the mean, median, and mode scores.
- 6.** Some Grade 7 students were surveyed about the average number of hours they spend reading each week, and their overall English mark out of 100. Here are the data:

Time (h)	Mark	Time (h)	Mark
0	55	0	60
0.5	65	0.5	62
1	70	1	84
1	68	1.5	75
1.5	78	2	83
2	85	2	77
2	88	2	81
2.5	91	2.5	79
2.5	80	3	90

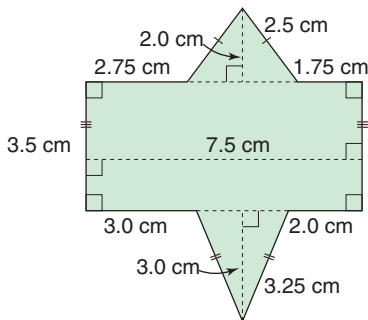
- a) Draw a scatter plot. What trends do you see?
b) Do you think a student will get a better English mark if he reads more? Explain.

- 6** **7.** A parallelogram has base 2.4 cm and height 3.9 cm.
- What is the area of the parallelogram?
 - What is the area of a triangle with the same base and height as the parallelogram?

- 8.** Find the area and perimeter of this trapezoid.

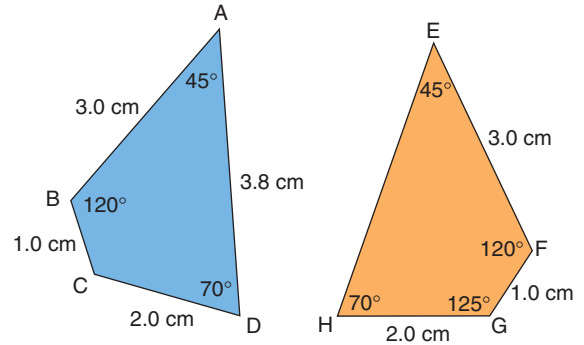


- 9.** Find the area and perimeter of this figure.



- 7** **10.** Use dot paper. Draw two congruent figures. Trade figures with a classmate. Explain how you know your classmate's figures are congruent.

- 11.** These quadrilaterals are congruent.



- What is the length of EH?
- What is the measure of $\angle BCD$? Justify your answers.

- 12.** Use grid paper.

- Choose a figure that tiles the plane. Create a design with your figure.
- Choose one figure as Figure A. Describe the position of another figure as a transformation image of Figure A. Do this for 5 different figures.

- 8** **13.** The regular price of a scooter is \$89.99. The scooter is on sale for 20% off.
- What is the sale price of the scooter?
 - There is 15% sales tax. What would a person pay for the scooter?

- 14.** How is calculating 30% of \$70 the same as calculating 70% of \$30? Sketch number lines to illustrate your answer.

UNIT

9

Integers



Pacific



Mountain

Canada has 6 time zones.

- What time is it where you are now?
- You want to call a friend in Newfoundland.

What time is it there?

- In the province or territory farthest from you, what might students be doing now?

What other questions can you ask about this map?



What You'll Learn

- Use integers to describe real-life situations.
- Model integers with coloured tiles.
- Compare and order integers.
- Add integers.
- Subtract integers.

Why It's Important

- We use integers to compare temperatures with the temperature at which water freezes.
- Integers extend the whole number work from earlier grades.





Central



Eastern



Atlantic



Newfoundland



Key Words

- positive number
- negative number
- integer
- positive integer
- negative integer
- opposite integers
- zero pair



Skills You'll Need

Mental Math Strategies for Addition and Subtraction

Example

Add or subtract.

Use mental math.

a) $38 + 17$

b) $111 - 64$

Solution

a) $38 + 17$

Make a friendly number. Subtract from one number and add to the other.

$$\begin{aligned} \text{So, } 38 + 17 &= \underbrace{38 + 2} + \underbrace{17 - 2} \\ &= 40 + 15 \\ &= 55 \end{aligned}$$

Subtract 2 from 17.

Add 2 to 38.

b) $111 - 64$

Subtract the ones, then subtract the tens.

Subtract 4: $111 - 4 = 107$

Then subtract 60: $107 - 60 = 47$

So, $111 - 64 = 47$

Count back by 1s

to subtract 4.

✓ Check

Use mental math.

1. Add or subtract. What strategy did you use?

a) $22 + 88$

b) $69 - 29$

c) $93 + 38$

d) $132 - 85$

2. In a magic square every row, column, and diagonal has the same sum.

Copy and complete each magic square. Explain how you did it.

a)

	1	6
	5	7
4		2

b)

17	10	
	14	
13	18	

We measure temperature in degrees Celsius. The highest temperature in Prince George, BC, on April 27, 2004, was 12°C .

12 is a **positive number**.

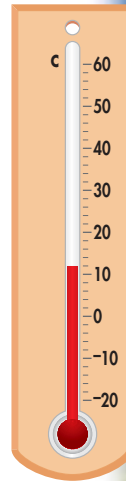
12°C is the difference between the temperature of the air and the temperature at which water freezes, 0°C .

We may not be able to use a positive number to express a difference.

For example, the temperature inside a freezer is 4°C less than the freezing point of water. We represent this difference as -4°C .

-4 is a **negative number**.

We say “negative 4.”



When we talk about temperatures below zero, we say “minus 4,” rather than “negative 4.”

Explore



Work with a partner.

Use a positive or a negative number to represent each situation.

- eight degrees above zero
- ten degrees below zero
- parking three levels below ground level
- two points ahead in a soccer game
- a loss of sixteen dollars
- taking four steps backward

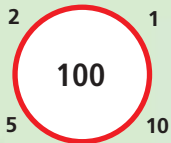
Reflect & Share

Compare your answers with those of another pair of classmates. For each situation, how did you decide whether to use a positive number or a negative number?

Connect

Mental Math

Use any math operation. Use the numbers around the circle. Write as many ways as you can to make the number in the circle.

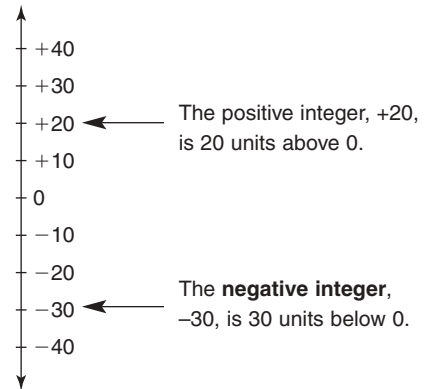


Positive and negative whole numbers are called **integers**.

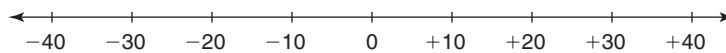
We put + in front of a number to indicate it is a **positive integer**.

We can show integers on a number line.

The number line may be vertical, like a thermometer.



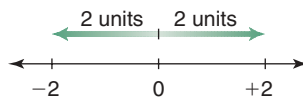
A number line may be horizontal.



Opposite integers are the same distance from 0 but are on opposite sides of 0.

For example, +2 and -2 are opposite integers.

They are the same distance from 0 and are on opposite sides of 0.



+4 and -4 are also opposite integers, as are +21 and -21.

Practice

- Mark each integer on a number line.
 - +1
 - 5
 - 2
 - +9
- Write the opposite of each integer.
 - +3
 - 1
 - 8000
 - +10
- Use a positive or negative integer to represent each situation.
 - thirty-five degrees Celsius below zero in Yellowknife
 - a weather balloon 28 000 m above Earth's surface
 - diving 35 m below the ocean's surface
 - earning \$500
- Use each integer below to describe a situation.
 - +4
 - 5
 - +120
 - 8500

Use question 3 as a model.

5. Describe two situations in which you might use negative and positive integers. Write integers for your situations.



6. **Assessment Focus** Statistics Canada reported these data about Canada's population.



Years	Births	Deaths	Immigration	Emigration
1961-66	2 249 000	731 000	539 000	280 000
1996-2001	1 704 000	1 095 000	1 051 000	270 000

- a) Which numbers can be represented by positive integers? Negative integers? Explain.
- b) For each column of data in the table, find the difference in the numbers. Write the difference as a positive or negative integer. Explain your choice of integer.
- c) Choose one time period. Use a number line and integers to explain the relationship between births and deaths.
- d) Choose one time period. Use integers to explain the relationships between immigration and emigration.

Take It Further

7. Changes in stock prices on the Stock Exchange are written as positive or negative integers.
- a) Express each change as an integer.
- The value of Apple Computers increased \$2.
 - Palm Tech dropped from \$25 to \$22.
 - MDS started the day at \$13 and ended the day at \$12.
 - Steve bought Global stock at \$10 and sold it for \$15.
- b) Look at the two stock prices in part a, ii. How are the prices related?
- c) Look at the two stock prices in part a, iv. How are the prices related?
- d) How can an integer be used to show the relationship between two prices of a stock? Explain.

Find the price of a stock in the financial section of a newspaper. Follow the price every day for one month. How are integers used to show how the stock price changes?

Reflect

Suppose you read a situation that can be described with integers. What clues do you look for to help you decide whether to use a positive or negative integer? Use examples in your explanation.

9.2

Comparing and Ordering Integers

Focus Use a number line to order and compare integers.

Elevation is the height above or below sea level. Elevation influences climate and how people live. For example, crops will not grow at elevations above 5300 m.



Explore

Work with a partner.

You will need an atlas or Internet access.

Here are some examples of extreme elevations around the world:

Vinson Massif, Antarctica	4897 m above sea level
Dead Sea, Israel/Jordan	411 m below sea level
Bottom of Great Slave Lake	458 m below sea level
Mt. Nowshak, Afghanistan	7485 m above sea level
Challenger Deep, Pacific Ocean	10 924 m below sea level

Research to find at least 4 more extreme elevations in Canada.

Two should be above sea level, and two should be below sea level.

Order *all* the elevations from least to greatest.

Reflect & Share

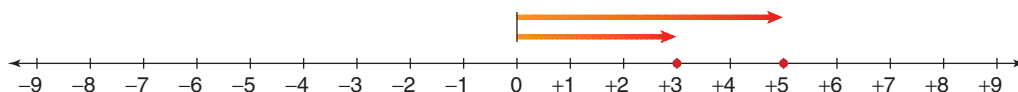
What strategies did you use to order the elevations?

What other ways could you display these data to show the different elevations?

Connect

We use the symbols $>$ and $<$ to show order.

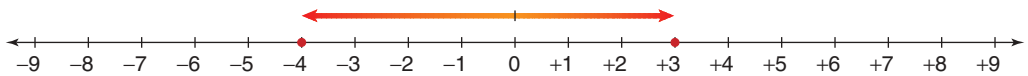
The symbol points to the lesser number.



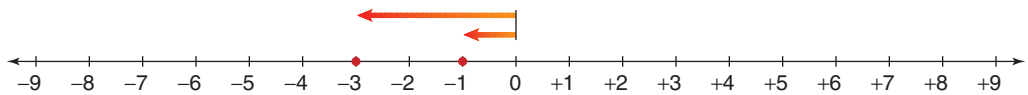
$+5$ is to the right of $+3$ on a number line.

$+5$ is greater than $+3$, so we write: $+5 > +3$

$+3$ is less than $+5$, so we write: $+3 < +5$



+3 is to the right of -4 on a number line.
 +3 is greater than -4 , so we write: $+3 > -4$
 -4 is less than $+3$, so we write: $-4 < +3$



-3 is to the left of -1 on a number line.
 -3 is less than -1 , so we write: $-3 < -1$
 -1 is greater than -3 , so we write: $-1 > -3$

Example

For the integers $0, +1, -2, +3, -5$:

- Use a number line to order the integers from least to greatest.
- Write the opposite of each integer.
Show the opposites on a number line.
- Order the opposites from least to greatest.

Solution

- Mark each integer on a number line.



The integers increase from left to right.

So, the integers from least to greatest: $-5, -2, 0, +1, +3$

- The opposite of an integer is its mirror image reflected in a vertical line drawn through 0 on a number line.

The opposite of -5 is $+5$. The opposite of -2 is $+2$.

Since 0 is the reference point, 0 is its own opposite.

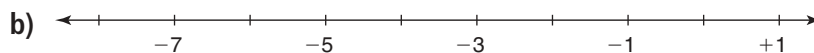
The opposite of $+1$ is -1 . The opposite of $+3$ is -3 .



- So, the opposite integers from least to greatest: $-3, -1, 0, +2, +5$

Practice

1. Copy each number line. Fill in the missing integers.



2. Order the integers in each set from least to greatest.

- a) +5, +13, +1 b) -3, -5, -4 c) +4, -2, +3

3. Order the integers in each set from greatest to least.

- a) +4, +1, +8 b) -7, -5, -3 c) 0, +4, -4

4. Order the integers in each set from least to greatest.

- a) +5, -5, +4, +2, -2 b) -8, -12, +10, 0, -10
c) +41, -39, -41, -15, -25 d) +1, -1, +2, -2, +3

5. Order the integers in each set from greatest to least.

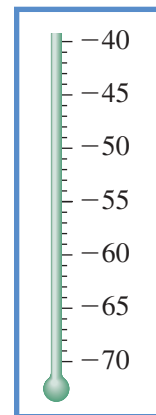
- a) +14, -25, -30, +3, -10 b) 0, +1, +2, -1, -2
c) -29, +27, -11, -4, +6 d) -7, +8, -9, +10, -11

6. This table shows the coldest temperatures ever recorded in 6 provinces and territories.

a) Draw a thermometer like the one shown.

Mark each temperature on it.

Province or Territory	Coldest Temperature °C
Alberta	- 61
Manitoba	- 53
Nova Scotia	- 47
Nunavut	- 64
Ontario	- 58
Quebec	- 54



A real thermometer does not show negative signs. We include them for better understanding.

b) Order the temperatures in part a from least to greatest. How can you use your thermometer to do this?

7. Copy and complete by placing $<$ or $>$ between the integers. Use a number line if it helps.

- a) $+5 \square +10$ b) $-5 \square -10$ c) $-6 \square 0$
d) $-5 \square -4$ e) $+100 \square -101$ f) $-80 \square -40$

Calculator Skills

Tina bought 6 packages of markers for her friends. Each package cost \$4.95. The cashier charged 15% tax. Tina left the store with \$13.42. How much money did Tina start with?

8. Assessment Focus

Look at the integers in the box.

	+4	
-8		-5
	+9	
0		+8

- a) Which integers are:
- greater than 0?
 - between -3 and $+3$?
 - greater than -10 and less than -5 ?
 - less than $+1$?
- b) What other questions can you ask about these integers? Write down your questions and answer them.

9. On January 18, 2002, the temperature in Charlottetown, Prince Edward Island, was -21°C ; in Sydney, Nova Scotia, it was -23°C ; in Point Lepreau, New Brunswick, it was -22°C . Which place was the warmest? Coldest? How do you know?

Take It Further

10. a) Draw a number line from -6 to $+6$. Find the integer that is:
- halfway between -6 and $+6$
 - halfway between -5 and $+1$
 - halfway between -5 and -1
 - 1 less than $+3$
 - 3 more than -4
 - 4 less than -1
- b) Explain why the answer for part a, ii is different from the answer for part a, iii.
- c) -3 is halfway between two integers on a number line. Draw a number line and mark the two integers on it.

11. One day, the temperature in Wabash Lake, Newfoundland, was -41°C ; in Pelly Bay, Nunavut, it was -51°C . The temperature in Churchill, Manitoba, was halfway between these temperatures. What was the temperature in Churchill?

12. Copy each pattern. Extend the pattern for 3 more terms. Describe each pattern in words.

- a) $-5, -3, -1, +1, \dots$ b) $+7, +4, +1, -2, \dots$
c) $-20, -18, -16, -14, \dots$ d) $-5, -10, -15, -20, \dots$

Reflect

When two integers have different signs, how can you tell which is greater? When two integers have the same sign, how can you tell which is greater?

9.3

Representing Integers

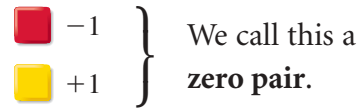
Focus Use coloured tiles to represent integers.

We can use coloured tiles to represent integers.

One yellow tile  can represent +1.

One red tile  can represent -1.

A red tile and a yellow tile combine to model 0:



Explore



Work with a partner.

You will need coloured tiles.

How many different ways can you use coloured tiles to model each number?

➤ 0, +1, +2, +3, +4

➤ -1, -3, -5, -6

Draw a picture to show the tiles you used for each way you found.

Reflect & Share

Compare your models with those of another pair of classmates.

Look at all the models that represent one integer.

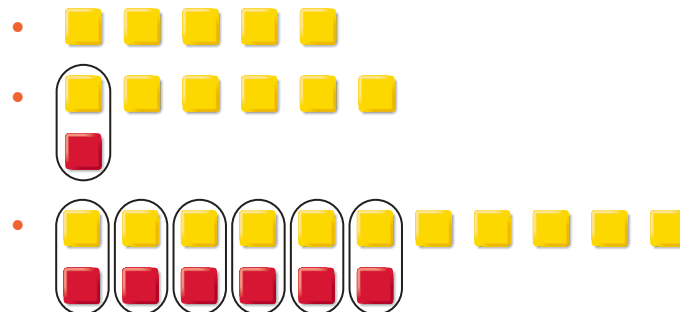
How do you know that all the models represent that integer?

Connect

We can model any integer in many ways.

Each set of tiles below models +5.

Each pair of 1 yellow tile and 1 red tile makes a zero pair. The pair models 0.



Example

Use coloured tiles to model -4 three different ways.

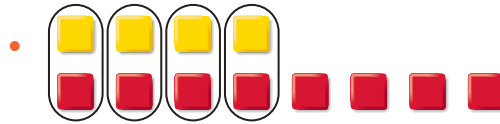
Solution

Start with 4 red tiles to model -4 .

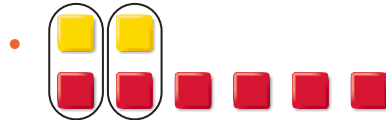
Add different numbers of zero pairs.

Each set of tiles below models -4 .

Adding 4 zero pairs does not change the value.



Adding 2 zero pairs does not change the value.

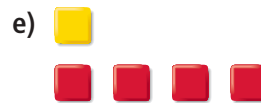
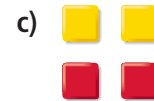
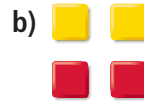


Adding 7 zero pairs does not change the value.



Practice

1. Write the integer modelled by each set of tiles.



2. Use yellow and red tiles to model each integer. Draw the tiles.

a) -6

b) $+7$

c) $+6$

d) -2

e) $+9$

f) -4

g) 0

h) $+10$

3. Work with a partner. Place 10 yellow and 10 red tiles in a bag.

a) Pull out a handful of tiles.

Tell the integer that the tiles model.

b) Have your partner tell what other set of tiles could model this integer.



Mental Math

Sarah ran a marathon in 3 h and 41 min. The race started at 7:45 a.m. At what time did Sarah finish?

4. Assessment Focus

- Choose an integer between -9 and $+6$.
Use coloured tiles to model the integer.
- How many more ways can you find to model the integer?
Create a table to order your work.
- What patterns can you find in your table?
- Explain how the patterns in your table can help you model an integer between -90 and $+60$.

- Suppose you have 10 yellow tiles.
How many red tiles would you need to model $+2$?
How do you know?
 - Suppose you have 100 yellow tiles.
How many red tiles would you need to model $+2$?
How do you know?

Math Link

Sports

In golf, a hole is given a value called **par**.

Par is the number of strokes a good golfer takes to reach the hole.

A score of $+2$ means a golfer took 2 strokes more than par, or 2 strokes over par.

A score of -1 means a golfer took 1 stroke fewer than par, or 1 stroke under par.

Some scores have special names.

A score of $+1$ is a bogey.

A score of -1 is a birdie.

A score of -2 is an eagle.

In a golf tournament, the golfer with the fewest strokes wins the game.

Reflect

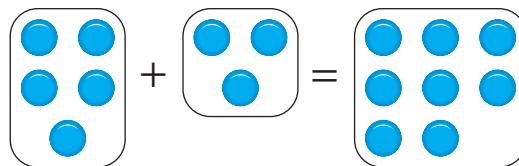
Explain how it is possible to use different sets of red and yellow tiles to model the same integer.

9.4

Adding Integers with Tiles

Focus Use coloured tiles to add integers.

Recall that when you add two numbers, such as $5 + 3$, you combine 5 counters with 3 counters to obtain 8 counters.



You can add two integers in a similar way. You know that $+1$ and -1 combine to make a zero pair. We can combine coloured tiles to add integers.

Explore



Work with a partner.

You will need coloured tiles.

- Choose two different positive integers. Add the integers. Draw a picture of the tiles you used. Write the addition equation.
- Repeat the activity for a positive integer and a negative integer.
- Repeat the activity for two different negative integers.

Reflect & Share

Share your equations with another pair of classmates.

How did you use the tiles to find a sum of integers?

How can you predict the sign of the sum?

Connect

- To add two positive integers: $(+5) + (+4)$
Model each integer with tiles.

+5:

+4:

This is an addition equation. →

Combine the tiles. There are 9 yellow tiles.

They model +9.

$$\text{So, } (+5) + (+4) = +9$$

- To add a negative integer and a positive integer: $(-6) + (+9)$
Model each integer with tiles. Circle zero pairs.



There are 6 zero pairs.

There are 3 yellow tiles left.

They model +3.

$$\text{So, } (-6) + (+9) = +3$$

- To add two negative integers: $(-3) + (-7)$
Model each integer with tiles.



Combine the tiles. There are 10 red tiles.

They model -10.

$$\text{So, } (-3) + (-7) = -10$$

Example

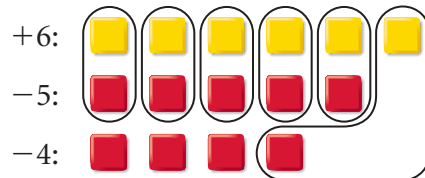
Add. $(+6) + (-5) + (-4)$

Solution

$(+6) + (-5) + (-4)$

Model each integer with tiles.

Circle zero pairs.



There are 3 red tiles left.

They model -3.

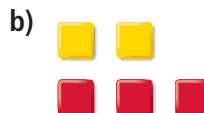
$$\text{So, } (+6) + (-5) + (-4) = -3$$

Practice

Use coloured tiles.

1. What sum does each set of tiles model?

Write the addition equation.



2. What sum does each set of tiles model?

How do you know you are correct?

- a) 3 yellow tiles and 2 red tiles
- b) 3 yellow tiles and 4 red tiles
- c) 2 red tiles and 2 yellow tiles

3. Use coloured tiles to represent each sum. Find each sum.

Sketch the tiles you used.

- a) $(+2) + (-2)$
- b) $(-4) + (+4)$
- c) $(+5) + (-5)$

4. Add. Sketch coloured tiles to show how you did it.

- a) $(+2) + (+3)$
- b) $(-3) + (+4)$
- c) $(-4) + (-1)$
- d) $(+1) + (-1)$
- e) $(-3) + (-4)$
- f) $(+5) + (-2)$

5. Add.

- a) $(+4) + (+3)$
- b) $(-7) + (+5)$
- c) $(-4) + (-5)$
- d) $(+8) + (-1)$
- e) $(-10) + (-6)$
- f) $(+4) + (-13)$

6. Represent each sentence with integers, then find each sum.

- a) The temperature is -3°C and rises 4 degrees Celsius.
- b) Ravinder earned \$5 and spent \$3.

7. Write 3 integer addition problems.

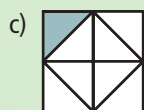
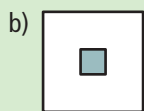
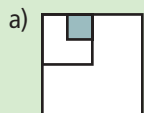
Trade problems with a classmate.

Solve your classmate's problems with coloured tiles.



Number Strategies

About what fraction of the large square is shaded in each diagram?



8. Add. Sketch coloured tiles to show how you did it.

a) $(+1) + (+2) + (+3)$ b) $(+2) + (-1) + (+3)$

c) $(-3) + (-1) + (-1)$ d) $(+4) + (-3) + (+1)$

e) $(-4) + (+1) + (-2)$ f) $(-5) + (-3) + (-2)$

9. Copy and complete.

a) $(+5) + \square = +8$ b) $\square + (-3) = -4$ c) $(+3) + \square = +1$

d) $(-5) + \square = -3$ e) $(+2) + \square = +1$ f) $\square + (-6) = 0$

10. Assessment Focus

a) Add: $(+3) + (-7)$

b) Suppose you add the integers in the opposite order: $(-7) + (+3)$. Does the sum change?

Use coloured tile drawings and words to explain the result.

c) How is $(-3) + (+7)$ different from $(+3) + (-7)$? Explain.

11. In a magic square every row, column, and diagonal has the same sum. Copy and complete each magic square. How did you do it?

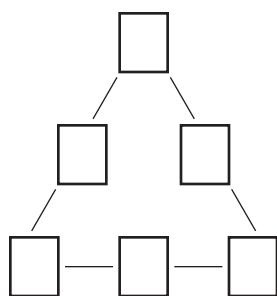
a)

+3		+1
	0	
-1		

b)

-1		+1
	-2	
		-3

Take It Further



12. Add.

a) $(+4) + (+1) + (+1) + (+1)$

b) $(-3) + (+2) + (-1) + (+1)$

c) $(+3) + (-2) + (-1) + (-1)$

13. Copy the triangle at the left.

Place the numbers $-6, -5, -4, -3, -2, -1$ in the triangle so the sums of the sides are the same.

Explain how you solved the problem.

Reflect

Explain how to add two integers when the integers have:

- the same signs
- opposite signs

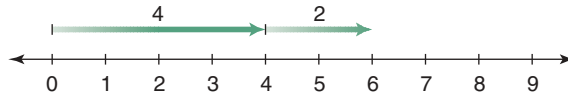
9.5

Adding Integers

Focus Add integers using number lines and patterns.

We can show the addition of whole numbers on a number line.

$$4 + 2 = 6$$

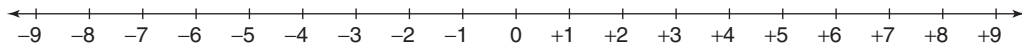


We can also show the addition of integers on a number line.

Explore

Work with a partner.

You will need copies of this number line.



- Choose two different positive integers.
Use a number line to add them.
Write the addition equation.
- Repeat the activity for a positive integer and a negative integer.
- Repeat the activity for two different negative integers.

Reflect & Share

Trade your addition problems with another pair of classmates.

Answer your classmates' problems.

Compare answers.

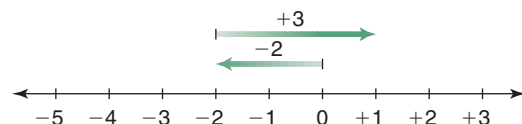
If the answers do not agree, decide who is correct. Explain.

Connect

Start at -2 .

Move 3 units right.

- To add a positive integer, move right (in the positive direction).
 $(-2) + (+3)$
 $(-2) + (+3) = +1$

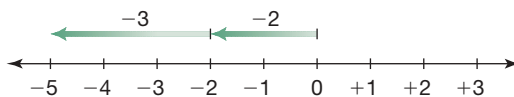


Start at -2 .
Move 3 units left.

➤ To add a negative integer, move left (in the negative direction).

$$(-2) + (-3)$$

$$(-2) + (-3) = -5$$



We can use patterning to add integers.

Example 1

Add. $(+5) + (-2)$

Solution

$$(+5) + (-2)$$

Think of a pattern. Start with facts you know.

Continue the pattern until you reach the fact you want.

$$(+5) + (+2) = +7$$

$$(+5) + (+1) = +6$$

$$(+5) + (0) = +5$$

$$(+5) + (-1) = +4$$

$$(+5) + (-2) = +3$$

Example 2

Sandra and Joe buy and sell CDs at a flea market.

In August, they bought 14 CDs for \$5 each.

They sold 6 CDs for \$9 each.

a) Did Sandra and Joe make money or lose money in August?

b) How much money did they make or lose?

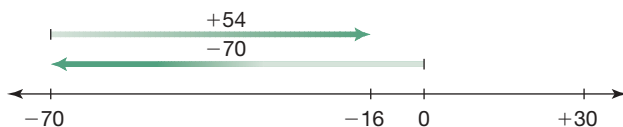
Solution

Expenses: $14 \times \$5 = \70

Income: $6 \times \$9 = \54

a) Since the expenses are greater than the income, Sandra and Joe lost money in August.

b) Draw a number line.



$$(-70) + (+54) = -16$$

In August, Sandra and Joe lost \$16.



Practice

Use a number line when it helps.

1. Add.

a) $(+1) + (+3)$

b) $(-1) + (+3)$

c) $(-3) + (+1)$

d) $(-1) + (-3)$

2. Find each sum. Use a pattern when you can.

a) $(+4) + (+2)$

b) $(+5) + (-3)$

c) $(-4) + (-2)$

d) $(-8) + (+2)$

3. a) Reverse the order of the integers in question 2, then add.

b) Compare your answers to the answers in question 2.

What do you notice?

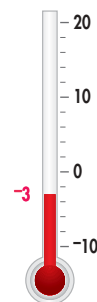
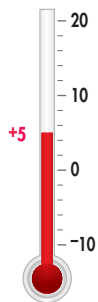
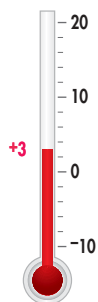
c) Make a general statement about your observations.

4. Look at these thermometers. Find each temperature after:

a) it falls 4°C

b) it falls 7°C

c) it rises 6°C



5. a) The temperature rises 7°C , then drops 2°C .

What is the overall change in temperature?

b) Adrian loses \$4, then earns \$8.

Did Adrian gain or lose overall?

c) The value of a stock went up \$3, then down \$2.

What was the final change in the value of the stock?

6. Add.

a) $(+12) + (+3)$

b) $(+13) + (-7)$

c) $(-5) + (-10)$

d) $(-5) + (+8)$

e) $(-8) + (-7)$

f) $(+4) + (-10)$



Mental Math

Which expression does not belong? Why?

$12 \times 4 \times 50$

$25 \times 10 \times 10$

24×100

$99 \times 20 + 420$

80×30

$90 \times 30 - 300$



Take It Further

7. Add.
- a) $(+30) + (+10)$ b) $(+20) + (-10)$
c) $(-35) + (-5)$ d) $(-15) + (+18)$
e) $(-82) + (+79)$ f) $(-58) + (-22)$
8. **Assessment Focus** Use an example to explain why each statement is true.
Use a number line to support your explanations.
- a) The sum of two opposite integers is 0.
b) The sum of two positive integers is always positive.
c) The sum of two negative integers is always negative.
d) The sum of a negative integer and a positive integer is sometimes negative and sometimes positive.
9. Add.
- a) $(+4) + (+3) + (-6)$ b) $(-2) + (-4) + (+1)$
c) $(-5) + (+3) + (-4)$ d) $(+6) + (-8) + (+2)$
e) $(+12) + (-3) + (-2)$ f) $(-5) + (-8) + (-10)$
10. The temperature in North Bay is 23°C .
The temperature falls 7°C , then rises 12°C .
What is the final temperature?
How did you find out?
Which model did you use?
11. Susanna earned \$24, spent \$7, earned \$12 more, and spent \$10 more. Express her earnings and spendings using integers.
How much money does Susanna have left over from her earnings and spendings?
12. Copy and complete.
- a) $(+10) + \square = +25$ b) $(-10) + \square = -25$
c) $(+20) + \square = +15$ d) $(-20) + \square = -15$
e) $(+35) + \square = +17$ f) $(-35) + \square = -17$

Reflect

You have used three models to add integers.
Which model do you prefer? Why?

Mid-Unit Review

LESSON

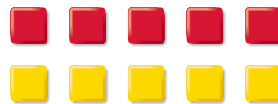
- 9.1 1.** Draw a number line. Show each integer on the number line:
 $+3, -2, -5, 0$
- 2.** Use an integer to represent each situation.
- a) 12°C below zero
 - b) a golf score of 3 strokes above par
 - c) 10 m above sea level
 - d) a drop of \$2 in the price of a stock
 - e) \$25 earned
 - f) a mountain elevation of 1500 m
- 9.2 3.** Order these integers from least to greatest.
- a) $+4, -3, -2, +1, -4$
 - b) $+18, +50, 0, -50, -17$
- 9.3 4.** Use red and yellow tiles to model each integer two different ways. Draw the tiles.
- a) -5 b) 0
 - c) $+8$ d) -1
 - e) $+3$ f) -7
- 9.4 5.** Use coloured tiles to add. Draw pictures of the tiles you used.
- a) $(+4) + (-1)$
 - b) $(-3) + (-2)$
 - c) $(-5) + (+1)$
 - d) $(+6) + (+3)$
 - e) $(-4) + (-8)$
 - f) $(+4) + (+8)$
- 9.5 6.** Use a number line to add.
- a) $(+3) + (+2)$ b) $(-5) + (-1)$
 - c) $(-10) + (+8)$ d) $(+6) + (-5)$
 - e) $(-8) + (+8)$ f) $(-5) + (+12)$
- 7.** Use patterns to add.
- a) $(+6) + (-3)$ b) $(-2) + (+8)$
 - c) $(+5) + (-9)$ d) $(-4) + (+9)$
- 8.** Add.
- a) $(+4) + (+1) + (-2)$
 - b) $(-3) + (-1) + (-4)$
 - c) $(-5) + (+1) + (+3)$
 - d) $(+6) + (-5) + (-8)$
- 9. a)** Puja earned \$50, was given \$10, and spent \$20.
How much did Puja then have?



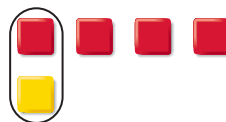
- b) The temperature starts at $+5^{\circ}\text{C}$, goes up 2°C , then drops 10°C . What is the final temperature?
 - c) The population of a city was 124 000, then it dropped by 4000 people. What was the population then?
- 10. a)** Add. $(+4) + (-5)$
- b)** Find another pair of integers that has the same sum as part a. Do this 3 times.

To add integers, we combine groups of tiles.
To subtract integers, we do the reverse:
we “take away” tiles from a group.

Recall that equal numbers of
red and yellow tiles model 0.
For example, $(-5) + (+5) = 0$



Adding 0 to a set of tiles does
not change its value.
For example, $(-3) + 0 = -3$



Explore

Work with a partner.
You will need coloured tiles.
Use tiles to subtract. Sketch the tiles you used in each case.

- $(+5) - (+3)$
- $(+5) - (-3)$
- $(-3) - (+5)$
- $(-3) - (-5)$

Reflect & Share

Compare your results with those of another pair of classmates.
Did they draw the same sets of tiles? Explain.
When you subtracted, how did you know how many tiles to use
to model each integer?

Connect

To use tiles to subtract integers, we model the first integer, then take
away the number of tiles indicated by the second integer.

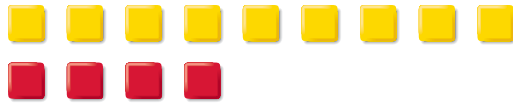
We can use tiles to subtract: $(+5) - (+9)$

Model +5.

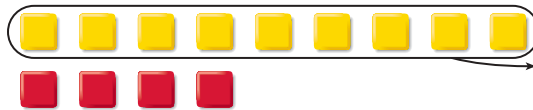


There are not enough tiles to take away $+9$.
To take away $+9$, we need 4 more yellow tiles.

We add zero pairs without changing the value.
Add 4 yellow tiles and 4 red tiles. They represent 0.



By adding 0, the integer the tiles represent has not changed.
Now take away the 9 yellow tiles.



Since 4 red tiles remain, we write: $(+5) - (+9) = -4$

Example

Use tiles to subtract.

a) $(-2) - (-6)$ b) $(-6) - (+2)$ c) $(+2) - (-6)$

Solution

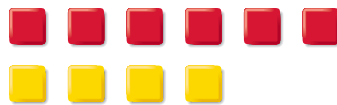
a) $(-2) - (-6)$

Model -2 .

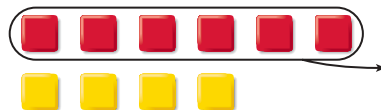


There are not enough tiles to take away -6 .
To take away -6 , we need 4 more red tiles.

We add zero pairs without changing the value.
Add 4 red tiles and 4 yellow tiles.



Now take away 6 red tiles.



Since 4 yellow tiles remain, we write: $(-2) - (-6) = +4$

b) $(-6) - (+2)$

Model -6 .



There are no yellow tiles to take.

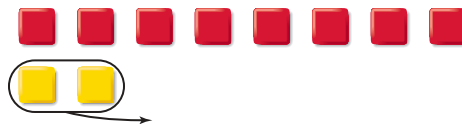
We need 2 yellow tiles to take away.

We add zero pairs.

Add 2 yellow tiles and 2 red tiles.



Now take away 2 yellow tiles.



Since 8 red tiles remain, we write: $(-6) - (+2) = -8$

c) $(+2) - (-6)$

Model $+2$.

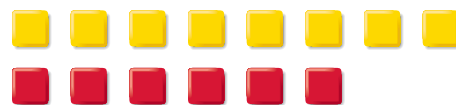


There are no red tiles to take.

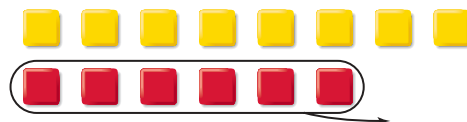
We need 6 red tiles to take away.

We add zero pairs.

Add 6 red tiles and 6 yellow tiles.



Now take away 6 red tiles.



Since 8 yellow tiles remain, we write: $(+2) - (-6) = +8$

Notice the results in the *Example*, parts b and c.

When we reverse the order in which we subtract two integers, the answer is the opposite integer.

$$(-6) - (+2) = -8$$

$$(+2) - (-6) = +8$$

Practice

- Use tiles to subtract. Draw pictures of the tiles you used.
 - $(+7) - (+4)$
 - $(-2) - (-2)$
 - $(-9) - (-6)$
 - $(+4) - (+2)$
 - $(-8) - (-1)$
 - $(+3) - (+3)$
- Use tiles to subtract.
 - $(-1) - (-4)$
 - $(+3) - (+8)$
 - $(-4) - (-11)$
 - $(+7) - (+8)$
 - $(-4) - (-6)$
 - $(+1) - (+10)$
- Subtract.
 - $(-4) - (-1)$
 - $(+8) - (+3)$
 - $(-11) - (-4)$
 - $(+8) - (+7)$
 - $(-6) - (-4)$
 - $(+10) - (+1)$
- Subtract.
 - $(+4) - (-7)$
 - $(-2) - (+8)$
 - $(-9) - (+5)$
 - $(+6) - (-8)$
 - $(-3) - (+6)$
 - $(-5) - (+7)$
- Subtract.
 - $(+4) - (+5)$
 - $(-3) - (+5)$
 - $(-4) - (+3)$
 - $(-1) - (-8)$
 - $(+8) - (-2)$
 - $(+4) - (-7)$
- Write 3 integer subtraction problems.
Trade problems with a classmate.
Solve your classmate's problems.
- Use coloured tiles to subtract each pair of integers.
 - $(+3) - (+1)$ and $(+1) - (+3)$
 - $(-3) - (-2)$ and $(-2) - (-3)$
 - What do you notice about each pair of problems in part a?
- $(+5) - (-2) = +7$
Predict the value of $(-2) - (+5)$.
Explain the reasoning for your choice.
- Assessment Focus** Use integers.
Write a subtraction problem that would give each answer.
How many problems can you write each time?
 - $+2$
 - -3
 - $+5$
 - -6



Calculator Skills

Assume the \div key on your calculator does not work.

Find two ways to use your calculator to divide: $888 \div 24$

10. Here is a magic square:

0	+5	-2
-1	+1	+3
+4	-3	+2

- a) Subtract +4 from each entry.
Is it still a magic square? Why?
- b) Subtract -1 from each entry.
Is it still a magic square? Why?
- c) Make up your own magic square.

Take It Further

11. a) Find two integers with a sum of -1 and a difference of +5.
b) Create and solve a similar integer problem.

12. Evaluate.

- a) $(+4) + (+1) - (+3)$ b) $(+1) - (+2) - (-1)$
c) $(-3) - (+1) + (+4)$ d) $(-2) - (-4) + (-1)$
e) $(+2) - (+1) - (+4)$ f) $(+1) - (+2) + (+1)$

13. Copy and complete.

- a) $(+4) - \square = +3$
b) $(+3) - \square = -1$
c) $\square - (+1) = +4$

14. Which expression in each pair has the greater value?

Explain your reasoning.

- a) i) $(+3) - (-1)$ ii) $(-3) - (+1)$
b) i) $(-4) - (-5)$ ii) $(+4) - (+5)$

Reflect

Here are 4 types of subtraction problems:

- (negative integer) - (negative integer)
- (negative integer) - (positive integer)
- (positive integer) - (positive integer)
- (positive integer) - (negative integer)

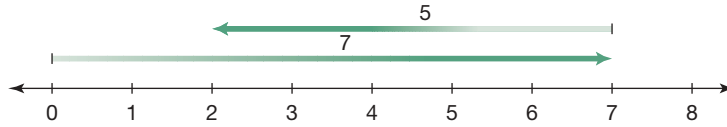
Write a problem for each type of subtraction.

Show how you use tiles to solve each problem.

Focus Subtract integers using number lines and patterns.

Just as we can show addition of whole numbers on a number line, we can show subtraction of whole numbers on a number line.

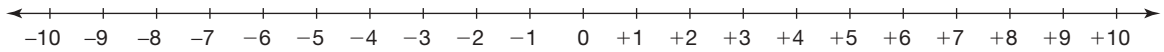
$$7 - 5 = 2$$



Explore

Work with a partner.

You will need copies of this number line.



➤ Use the number line to subtract.

$$(+6) - (-4) \quad (-3) - (+5)$$

$$(-5) - (-4) \quad (+7) - (+10)$$

Check your answers using coloured tiles.

➤ Subtract.

$$(+7) - (+2)$$

$$(-7) - (-2)$$

$$(+7) - (-2)$$

$$(-7) - (+2)$$

Add.

$$(+7) + (-2)$$

$$(-7) + (+2)$$

$$(+7) + (+2)$$

$$(-7) + (-2)$$

What do you notice about the answers in each row?

What patterns do you see in each subtraction and addition?

Check your pattern using other integers.



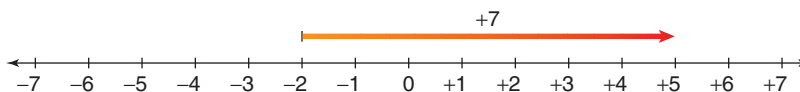
Reflect & Share

Compare your answers with those of another pair of classmates.

How can you use addition to subtract two integers?

- To subtract two whole numbers, such as $5 - 2$, we think, “What do we add to 2 to get 5?”
We add 3 to 2 to get 5; so, $5 - 2 = 3$

We can do the same to subtract two integers.
For example, to subtract: $(+5) - (-2)$
Think: “What do we add to -2 to get $+5$?”



We add $+7$ to -2 to get $+5$; so, $(+5) - (-2) = +7$
We also know that $(+5) + (+2) = +7$.
We can look at other subtraction equations and related addition equations.

$(+9) - (+4) = +5$	$(+9) + (-4) = +5$
$(-9) - (-4) = -5$	$(-9) + (+4) = -5$
$(-9) - (+4) = -13$	$(-9) + (-4) = -13$
$(+9) - (-4) = +13$	$(+9) + (+4) = +13$



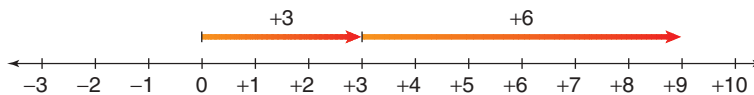
In each case, the result of subtracting an integer is the same as adding the opposite integer.

For example,

$(-9) - (+4) = -13$	$(-9) + (-4) = -13$
$\underbrace{\hspace{2cm}}$ \downarrow Subtract +4.	$\underbrace{\hspace{2cm}}$ \downarrow Add -4.

- To subtract an integer, we add the opposite integer.
For example, to subtract: $(+3) - (-6)$
Add the opposite: $(+3) + (+6)$

The opposite of -6 is $+6$.



So, $(+3) - (-6) = +9$

Example 1

Use a number line to subtract.

a) $(+2) - (+9)$

b) $(+2) - (-9)$

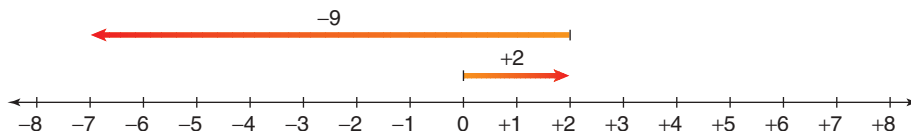
Solution

a) To subtract: $(+2) - (+9)$

Add the opposite: $(+2) + (-9)$

Use a number line.

$$(+2) + (-9) = -7$$

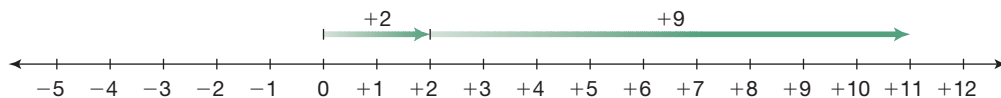


b) To subtract: $(+2) - (-9)$

Add the opposite: $(+2) + (+9)$

Use a number line.

$$(+2) + (+9) = +11$$



Example 2

On April 27, 2004, the highest temperature in Windsor was $+5^{\circ}\text{C}$ and the highest temperature in Yellowknife was -2°C .

What is the difference in temperature? Interpret the answer.

Solution

The difference between $+5$ and -2 can be written in two ways:

$$(+5) - (-2)$$

$$(-2) - (+5)$$

Add the opposite: $(+5) + (+2)$

Add the opposite: $(-2) + (-5)$

$$(+5) + (+2) = +7$$

$$(-2) + (-5) = -7$$

The difference in temperature is $+7^{\circ}\text{C}$.

The difference in temperature is -7°C .

The temperature in Windsor is 7°C greater than the temperature in Yellowknife.

The temperature in Yellowknife is 7°C less than the temperature in Windsor.

Practice

1. Rewrite using addition.

- a) $(+6) - (+4)$ b) $(-5) - (+4)$ c) $(-2) - (-3)$
 d) $(+4) - (-2)$ e) $(+1) - (+1)$ f) $(+1) - (-1)$

2. Use a number line to subtract.

Use coloured tiles to check your answers.

- a) $(+2) - (+1)$ b) $(+4) - (-3)$ c) $(-4) - (-1)$
 d) $(-5) - (+2)$ e) $(-2) - (-6)$ f) $(-3) - (-7)$

3. a) Reverse the order of the integers in question 2, then subtract.

b) How are the answers different from those in question 2? Explain.

4. Use a number line to subtract.

- a) $(+10) - (+5)$ b) $(+7) - (-3)$ c) $(-8) - (+6)$
 d) $(-10) - (+5)$ e) $(-4) - (+4)$ f) $(-4) - (-4)$

5. Find the difference between:

- a) a temperature of $+3^{\circ}\text{C}$ and -5°C
 b) a temperature of -15°C and -10°C
 c) an elevation of 5 m above sea level and one of 2 m below sea level
 d) a golf score of 1 below par and one of 3 below par

6. The table shows the average afternoon temperatures in January and April for four Canadian cities.

Find the difference between the temperatures in April and January for each city. Show your work.

	City	January Temperature	April Temperature
a)	Calgary	-4°C	$+13^{\circ}\text{C}$
b)	Iqaluit	-22°C	-10°C
c)	Toronto	-3°C	$+12^{\circ}\text{C}$
d)	Victoria	$+7^{\circ}\text{C}$	$+13^{\circ}\text{C}$



Calculator Skills

Find each answer.

- $15 \times (32 + 41)$
- $198 - 104 \div 13$
- $14 \times 15 - 16 \times 9$

7. Use patterns to subtract.

a) Subtract: $(+2) - (+5)$

Start the pattern with $(+6) - (+5) = +1$.

b) Subtract: $(+7) - (-3)$

Start the pattern with $(+7) - (+4) = +3$.

c) Subtract: $(-3) - (+7)$

Start the pattern with $(+8) - (+7) = +1$.

8. Assessment Focus

a) Subtract: $(-6) - (+11)$

b) Suppose we subtract the integers in the opposite order:

$(+11) - (-6)$

How does the answer compare with the answer in part a?

Use number lines or patterns to explain.

c) How is $(+6) - (-11)$ different from $(-6) - (+11)$?

Explain.

9. This table shows the mean daily maximum temperatures in degrees Celsius for Rankin Inlet for each month.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
-28	-28	-23	-12	-2	+8	+16	+14	+6	-3	-16	-24

a) What is the median?

b) What is the range?

c) What is the mode?

10. Evaluate.

a) $(+4) - (+2) - (+1)$

b) $(-2) - (+1) - (-4)$

c) $(-1) + (-2) - (+1)$

d) $(+5) - (+1) + (-2)$

e) $(+10) - (+3) - (-5)$

f) $(-7) - (+1) + (-3)$

11. Show three ways that $+4$ can be written as the difference of two integers.

Reflect

Choose two integers: one positive, one negative.

Add the integers. Subtract the integers.

Explain how the subtraction of two integers is related to addition.



Choosing a Strategy

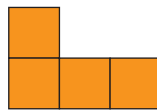
Strategies

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.



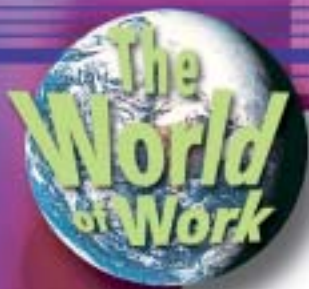
1. A textbook is opened at random. The product of the page numbers is 23 256. What is the number on the left hand page?
2. Ms. Pantuso has a bag of candies that she wants to share equally among the students in her class.
On Monday, all 20 students were present. If she had shared the candies, she would have had 7 candies left over.
On Tuesday, two boys were absent. If she had shared the candies, she would again have had 7 left over.
On Wednesday, the two boys were still absent and one girl was absent. Ms. Pantuso shared all the candies among the students with none left over.
 - a) What was the least number of candies Ms. Pantuso could have had?
 - b) How many candies did each student receive?
3. Briony wants to print copies of her new brochure.
The local print shop charges 15¢ a copy for the first 25 copies, 12¢ a copy for the next 50 copies, and 8¢ a copy for any additional copies.
How much would Briony pay for each number of copies?
 - a) 60 copies
 - b) 240 copies
4. What is the greatest number of 5 cm by 3 cm rectangles that can be cut from a 20 cm by 10 cm sheet of cardboard?
5. Mr. Anders estimates it will take him 4 h to build a wall. His assistant would take 6 h to build the wall.
 - a) What fraction of the wall could each person build in one hour?
 - b) Suppose they worked together. What fraction could they build in one hour?
 - c) How long would it take them to build the wall?
6. The mean of five numbers is 13.
The mode is 16.
The median is 14.
Write two sets of numbers that satisfy these conditions.

- 7.** A rectangular prism has a square base, and a volume of 36 cm^3 .
 What are the possible whole number dimensions of the prism?
- 8.** Copy this figure on grid paper.
 Add one square to make the figure symmetrical. How many solutions can you find?



- 9.** Eight students wrote a math test. Their mean mark was 54%.
 Another student wrote the test and scored 99%.
 What is the mean mark for the nine students?
- 10.** Write 45 as the sum of consecutive integers.
 How many solutions can you find?
- 11. a)** The product of two whole numbers is 1000.
 No number contains a zero.
 What are the numbers?
 How many different answers can you find?
- b)** The product of three whole numbers is 1000.
 No number contains a zero.
 What are the numbers?
 How many different answers can you find?
- 12.** A book contains 124 pages numbered from 1 to 124.
 How many times does the digit 7 appear?





Historian

The modern historian does a lot of ‘detective’ work, especially when reading old documents or journals that describe timelines. Adding or subtracting days, months, or years from a known date is a regular part of the work. The historian must deal with modern timekeeping conventions, such as Common Era (CE) and Before Common Era (BCE). But he must also know that, throughout history, different societies had different ways of describing time. Often it was simply recorded as a period of time before or after another significant date or event.

Suppose you read this entry in an 18th century explorer’s journal dated May 21:

**A fortnight is
2 weeks.**

“We have been following this river for a fortnight. Three days into our journey we came upon a great waterfall blocking our path and had to portage around it. But the river has been calm since.”

How would you be able to calculate what date the waterfall was discovered?

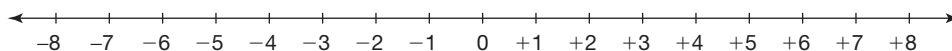
Suppose you knew that Augustus Caesar died in 14 CE at the age of 77. Can you calculate the year of his birth? Explain.



What Do I Need to Know?

✓ Integer Models

- You can use a number line to show integers.



Negative numbers are to the left of 0. Positive numbers are to the right of 0.

- Coloured tiles model integers.

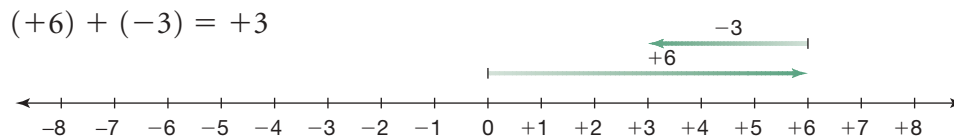


✓ Adding Integers

- You can use tiles to add integers.



- You can use a number line to add integers.



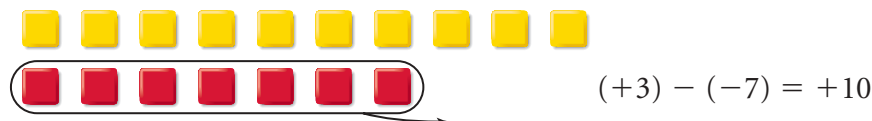
✓ Subtracting Integers

- You can use tiles to subtract integers: $(+3) - (-7)$

We need enough red tiles to take away 7 of them.

Model +3:

Since there are not enough tiles to take away -7 , add 7 yellow tiles and 7 red tiles. Now take away 7 red tiles. There are 10 yellow tiles left.



- You can also subtract by adding the opposite:

$$\begin{aligned} (-5) - (-8) &= -5 + (+8) \\ &= +3 \end{aligned}$$

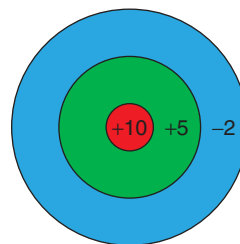
LESSON

- 9.1** 1. Draw a number line. Mark each integer on the number line:
+3, -5, +1, -2
2. Use an integer to represent each situation.
- a golf score of 2 strokes under par
 - 250 m below sea level
 - 32°C
 - a loss of \$125
 - an increase of \$3 in the monthly cost of cable television
- 9.2** 3. Order these integers from least to greatest:
+200, -55, +150, -3, -54
- 9.4** 4. Use tiles to add or subtract.
- 9.6** a) $(-1) + (+3)$
b) $(+3) + (-4)$
c) $(-2) - (+3)$
d) $(-1) - (-3)$
- 9.5** 5. Use a number line to add or subtract.
- 9.7** a) $(-1) + (+3)$
b) $(+6) + (-4)$
c) $(-4) - (+6)$
d) $(-5) - (-3)$
6. When you add two positive integers, their sum is always a positive integer. When you subtract two positive integers, is their difference always a positive integer? Explain.
7. At midnight in North Bay, the temperature was -5°C . During the next 24 h, the temperature rose 12°C , then dropped 9°C . What was the final temperature? Show your work.
- 9.6** 8. Use tiles or a number line to subtract.
- 9.7** a) $(+4) - (+1)$
b) $(+5) - (-1)$
c) $(+2) - (-2)$
d) $(-4) - (+1)$
e) $(-6) - (-2)$
f) $(-10) - (-5)$
g) $(-4) - (-2)$
9. Use tiles or a number line. Find the difference between:
- a temperature of $+5^{\circ}\text{C}$ and -7°C
 - an elevation of -100 m and $+50$ m
 - a golf score of 1 over par and 2 under par
10. Subtract.
- $(+3) - (+1)$
 - $(-5) - (-2)$
 - $(+100) - (+60)$
 - $(-100) - (+60)$
11. a) Find 5 pairs of integers with a sum of -6 .
b) Find 5 pairs of integers with a difference of -3 .

Practice Test

- We measure time in hours.
Suppose 12 noon is represented by the integer 0.
 - Which integer represents 1 p.m. the same day?
 - Which integer represents 10 a.m. the same day?
 - Which integer represents 12 midnight the same day?
 - Which integer represents 10 p.m. the previous day?
- Order all the integers in question 1 from least to greatest.
- Evaluate.
 - $(+5) + (-8)$
 - $(-3) - (+7)$
 - $(-9) + (-1)$
 - $(-4) + (+10)$
 - $(-6) - (-2)$
 - $(+12) - (-11)$
- Without calculating the sum, how can you tell if the sum of two integers will be:
 - zero?
 - negative?
 - positive?Include examples in your explanations.
- Here is a different type of dartboard.

A player throws 3 darts at the board.
His score is the sum of the integers in the areas his darts land.
Assume all 3 darts hit the board.
 - How many different scores are possible?
 - Find each score.
- The lowest temperature possible is approximately -273°C .
The temperature at which water boils is 100°C .
What is the difference in these temperatures?



TIME ZONES



The map shows the world's time zones. Greenwich, in London, England, is the reference point, or the zero for the time zones.

Its time is called UTC, or Coordinated Universal Time.

London, England, is also in this time zone.

The positive and negative integers on the map show the changes in time from UTC.

The 2008 Summer Olympics will be held in Beijing, China.

1. The local start times of some Olympic events are given. If family members want to watch these events live, in Sudbury (the same time zone as Toronto), what time should they “tune in”?
 - a) 200-m backstroke at 2:00 p.m.
 - b) 100-m dash at 7:00 p.m.
 - c) gymnastics at 11:00 p.m.
 - d) middleweight boxing at 8:00 a.m.

- An event is broadcast live in Montreal at 9:00 p.m.
What time is it taking place in Beijing?
- Two pen pals plan to meet in Beijing for the Olympics.
Atsuko lives in Tokyo, Japan.
She can get a direct flight to Beijing that takes 4 h.
Paula lives in Sydney, Australia, and her direct flight takes 13 h.
What time does each girl leave her country to arrive in Beijing at 6 p.m., Beijing time?

Check List

Your work should show:

- ✓ how you used integers to solve each problem
- ✓ the strategies you used to add and subtract integers
- ✓ correct calculations
- ✓ a clear explanation of how integers relate to time zones



- Olympic funding depends on money from North American television networks. What problems will the organizers of the Beijing Olympics encounter when they plan the times for events?
- Make up your own problem about the time zone map.
Solve your problem.

Show your work. Show how you can use integers to solve each problem.

Reflect on the Unit

Choose a positive integer and a negative integer.

How many different ways can you add them? Subtract them?

Show your work.

If you used number lines, patterns, or coloured tiles, draw pictures to show how you used them.

UNIT

10

Patterning and Algebra

Students in a grade 7 class were raising money for charity. Some students had a “bowl-a-thon.”

This table shows the money that one student raised for different bowling times.

Time (h)	Money Raised (\$)
1	8
2	16
3	24
4	32
5	40
6	48

- What patterns do you see in the table?
- Suppose you drew a graph of the data. What might the graph look like?

What You'll Learn

- Describe, extend, and explain patterns.
- Use patterns to make predictions.
- Show patterns as graphs.
- Write and evaluate algebraic expressions.
- Read and write equations.
- Solve equations.

Why It's Important

- Extending a pattern is a useful problem-solving strategy.
- Using algebra is an efficient way to describe a pattern.





Key Words

- ordered pairs
- algebraic expression
- evaluate
- equation
- solve an equation

Order of Operations

Recall this order of operations.

Brackets

Perform operations inside the brackets.

Divide and multiply

Do in order from left to right.

Add and subtract

Do in order from left to right.

Example

Simplify.

a) $6 + 3 \times 4$

b) $(5 + 3) \div 2$

c) $18 \div 3 \times 2$

Solution

$$\begin{array}{l} \text{a) } 6 + 3 \times 4 = 6 + 12 \\ \phantom{\text{a) } 6 + 3 \times 4} = 18 \end{array}$$

Multiply first.
Then add.

$$\begin{array}{l} \text{b) } (5 + 3) \div 2 = 8 \div 2 \\ \phantom{\text{b) } (5 + 3) \div 2} = 4 \end{array}$$

Add in brackets first.
Then divide.

$$\begin{array}{l} \text{c) } 18 \div 3 \times 2 = 6 \times 2 \\ \phantom{\text{c) } 18 \div 3 \times 2} = 12 \end{array}$$

Divide first.
Then multiply.

✓ Check

1. Simplify.

a) $5 \times 7 + 2$

b) $5 \times (7 + 2)$

c) $13 + 2 \times 8$

d) $13 + 2 - 8$

e) $13 + 8 \div 2$

f) $13 - 8 \div 2$

g) $12 \div 4 + 2$

h) $12 \div (4 + 2)$

i) $12 + 4 \div 2$

2. a) Simplify.

i) $3 + 4 \times 2 + 5$

ii) $3 \times 4 + 2 \times 5$

iii) $3 \times (4 + 2) \times 5$

iv) $3 \times (4 + 2) + 5$

v) $3 \times 4 \times 2 + 5$

vi) $3 + 4 + 2 \times 5$

b) All the expressions in part a have the same numbers and the same operations. Why are the answers different?

Graphing on a Coordinate Grid

$(3, 2)$ is an **ordered pair**. It tells the position of a point on a grid.

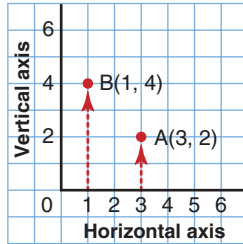
In an ordered pair, the first number is the horizontal distance from the origin, O .

The second number is the vertical distance from the origin.

We use a letter to label a point.

To plot point $A(3, 2)$, start at 3 on the horizontal axis, then move up 2.

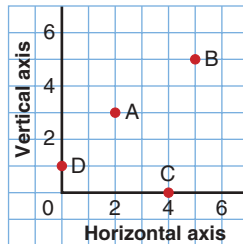
To plot point $B(1, 4)$, start at 1 on the horizontal axis, then move up 4.



The numbers in an ordered pair are also called the **coordinates** of a point.

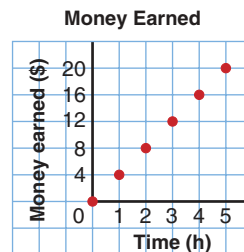
✓ Check

3. Write the ordered pair for each point on the grid.



4. On graph paper, draw a grid.
Plot and label these points:
 $A(2, 9)$, $B(5, 3)$, $C(8, 8)$, $D(0, 10)$

5. The graph shows the money earned for the time worked.
- How much money was earned in 4 h?
 - How long will it take to earn \$12.00?
 - What is the hourly rate of pay?



Focus Extend patterns and use them to make predictions.

Explore

Work with a partner.

➤ Suppose this pattern continues.



Describe the pattern.

What is the next figure in the pattern?

What is the 17th figure?

How can you find out without drawing 17 figures?

➤ Suppose this pattern continues.



Figure 1

Figure 2

Figure 3

Figure 4

Describe the pattern.

What is the next figure in the pattern?

How many dots will there be in the 15th figure?

How can you find out without drawing 15 figures?

Reflect & Share

How are the two patterns the same?

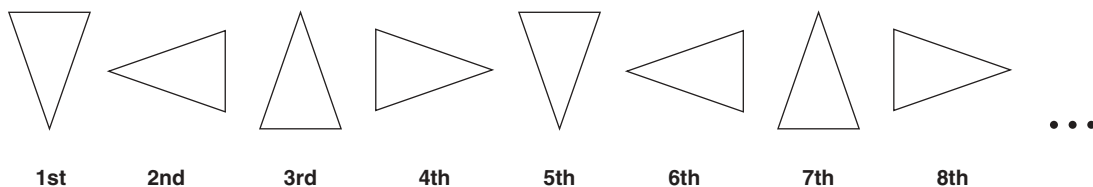
How are they different?

Compare your answers with those of another pair of classmates.

Did you use the same strategies to answer the questions? Explain.

Connect

Here is a pattern of triangles.



1st

2nd

3rd

4th

5th

6th

7th

8th

The pattern is made up of different positions of an isosceles triangle. To get the next term each time, rotate the triangle a $\frac{1}{4}$ turn clockwise about its centre.

The core of the pattern is 4 triangles.

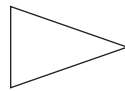
The 5th term is the same as the 1st term.

The 6th term is the same as the 2nd term, and so on.

To find any term in the pattern, we find which of the first 4 terms it matches.

Think of multiples of 4: 4, 8, 12, 16, 20, 24, 28, ...

All the 4th, 8th, 12th, 16th, 20th, 24th, 28th, ... terms have the triangle pointing to the right.



To find the 99th term, we find the closest multiple of 4.

100 is a multiple of 4, so the 100th term is the same as the 4th term.

The 99th term will be the same as the 3rd term:



The 99th term is the triangle pointing up.

We can use the same ideas to make predictions with number patterns.

Example

Each pattern continues.

- i) Describe each pattern.
 - ii) Write the next 3 terms.
 - iii) Find the 50th term.
- a) 4, 7, 10, 13, ... b) 1, 4, 9, 16, ...

Solution

- a) 4, 7, 10, 13, ...
- i) The pattern begins with 4.
To get the next term, add 3 each time.
 - ii) The next 3 terms are: $13 + 3 = 16$
 $16 + 3 = 19$
 $19 + 3 = 22$
- The next 3 terms are 16, 19, 22.

iii) Since the terms increase by 3 each time, compare the pattern with multiples of 3.

Pattern: 4, 7, 10, 13, 16, 19, 22, ...

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, ...

Each term in the pattern is 1 more than a multiple of 3.

So, the terms in the pattern are multiples of 3, plus 1.

The 1st term: $1 \times 3 + 1 = 4$

The 2nd term: $2 \times 3 + 1 = 7$

The 3rd term: $3 \times 3 + 1 = 10$

The 4th term: $4 \times 3 + 1 = 13$, and so on

• •
• •
• •

The 50th term: $50 \times 3 + 1 = 151$

The 50th term is 151.

b) 1, 4, 9, 16, ...

i) The pattern begins with 1.

To get the 2nd term, add 3.

To get the 3rd term, add 5.

To get the 4th term, add 7.

To get each successive term, increase the number you add by 2 each time.

ii) The next 3 terms are: $16 + 9 = 25$

$$25 + 11 = 36$$

$$36 + 13 = 49$$

The next 3 terms are 25, 36, 49.

iii) To use the same method to get the 50th term, we would need to know the 49th term and what to add.

So, we look at the pattern a different way.

The pattern is: 1, 4, 9, 16, 25, 36, 49, ...

These are the perfect squares: $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, \dots$

So, the 50th term is: $50^2 = 2500$

The 50th term is 2500.

Recall that a perfect square is the product of a number and itself.

Practice

1. This pattern continues.



- Describe the pattern.
- Sketch the next 3 terms.
- Sketch the 29th term and the 49th term.

2. This pattern continues.



- Describe the pattern.
- Sketch the next 3 terms.
- Describe the 18th term and the 38th term.
Sketch them if you can.

3. For each pattern:

- Describe the pattern.
- Write the next 3 terms.
- Find the 40th term. Explain how you found it.

- a) 6, 9, 12, 15, ... b) 6, 10, 14, 18, ... c) 6, 11, 16, 21, ...

4. There is a pattern in the patterns in question 3.

Write the first 5 terms of the next pattern. Justify your answer.

5. For each pattern:

- Describe the pattern.
- Write the next 3 terms.
- Find the 20th term.

- a) 2, 4, 8, 16, ... b) 3, 6, 12, 24, ...

6. Look at the patterns below.

How are they the same? How are they different?

- Write the next 3 terms for each pattern.
- Write the 20th term in each pattern.

- a) 2, 5, 10, 17, 26, ... b) 0, 3, 8, 15, 24, ...



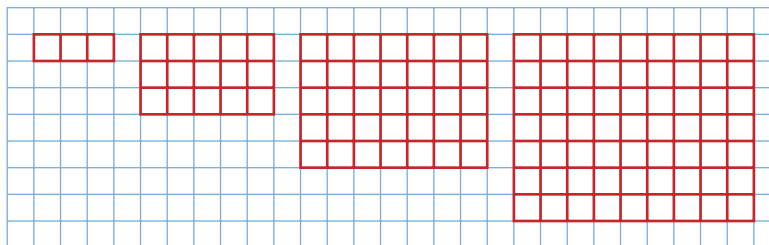
Mental Math

Estimate each product.
Order the products from
least to greatest.

- 24.8×3.2
- 35.23×2.89
- 13.21×5.78
- 5.5×15.5

What strategies did
you use?

7. This pattern continues.

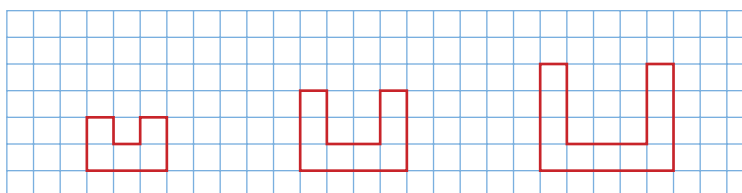


- a) Describe the pattern.
b) Sketch the next 3 terms.
c) Describe the 17th term and the 37th term.
Sketch them if you can.
8. **Assessment Focus** Create two different number patterns.
Each pattern must contain the numbers 12 and 32.
Describe each pattern in words.
Write the next 4 terms in each pattern.

Take It Further

9. a) Describe this pattern. The pattern continues.
b) How many squares would be in the 10th figure?
c) What are the perimeters of the 4th figure and 5th figure?
d) What is the perimeter of the 10th figure?
e) How many squares are there in the figure with perimeter 72?
f) Could you make one of the figures using exactly 27 small squares? Explain.

Use a geoboard and
geoband if they help.



10. Create two different number patterns that contain the numbers
16, 20, and 25. Write the first 5 terms in each pattern.

Reflect

Describe how you identify a number pattern.
Use two different examples from this lesson.

Explore

Work with a partner.

You will need grid paper.

One CD costs \$10.

- Copy and complete this table.
Find the cost for up to 10 CDs.
- Graph the data in the table.

Number of CDs	Cost (\$)
0	
2	
4	
6	
8	
10	

Reflect & Share

Describe the patterns in the table.

How are these patterns shown in the graph?

Connect

We can use a table and a graph to illustrate number patterns.

Recall this pattern from *Section 10.1, Example, page 369*.

Term 1: 4

Term 2: 7

Term 3: 10

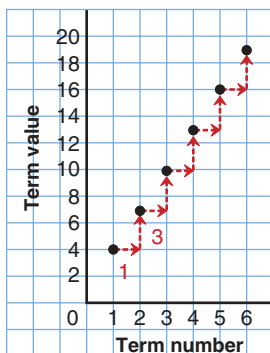
Term 4: 13

Term 5: 16

Term 6: 19

We write these terms in a table.

Term Number	Term Value
1	4
2	7
3	10
4	13
5	16
6	19



We plot these data on a graph.

The *Term number* is plotted on the horizontal axis.

The *Term value* is plotted on the vertical axis.

The graph is a set of points that lie on a straight line.

To get from one point to the next, move 1 unit right and 3 units up.

Moving 1 unit right is the same as adding 1 in the first column to get the next term number.

Moving 3 units up is the same as adding 3 in the second column to get the next term value.

We can use a table related to an Input/Output machine to make a pattern.

Example 1

- a) Complete the table for this pattern:
Multiply each number by 2, then add 3.
- b) Graph the pattern.
Explain how the graph shows the pattern.

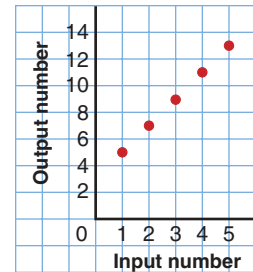
Input	Output
1	
2	
3	
4	
5	

Solution

Input	Output
1	5
2	7
3	9
4	11
5	13

- a) Multiply each Input number by 2, then add 3.
- $$1 \times 2 + 3 = 5$$
- $$2 \times 2 + 3 = 7$$
- $$3 \times 2 + 3 = 9$$
- $$4 \times 2 + 3 = 11$$
- $$5 \times 2 + 3 = 13$$

- b) The points lie on a straight line.
To get from one point to the next, move 1 unit right and 2 units up.
Moving 1 unit right is the same as adding 1 to an Input number to get the next Input number.
Moving 2 units up is the same as adding 2 to an Output number to get the next Output number.



Example 2

- a) Describe the patterns in this table.
- b) Use the patterns to extend the table 3 more rows.
- c) Graph the table in part b. Explain how the graph shows the patterns.

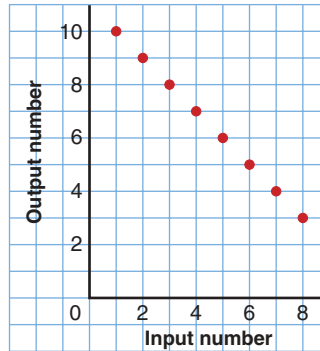
Input	Output
1	10
2	9
3	8
4	7
5	6

Solution

- a) The numbers in the Input column start at 1 and increase by 1 each time.
The numbers in the Output column start at 10 and decrease by 1 each time.
The sum of matching Input and Output numbers is 11.

- b) The next 3 Input numbers are 6, 7, 8.
The next 3 Output numbers are 5, 4, 3.

- c) The graph is a set of points that lie on a straight line.



Input	Output
1	10
2	9
3	8
4	7
5	6
6	5
7	4
8	3

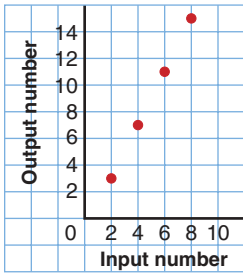
The line goes down to the right.
As the Input numbers increase from 1 to 8,
the Output numbers decrease from 10 to 3.

Practice

Input	Output
1	
2	
3	
4	
5	

- Copy and complete this table for each pattern.
 - Multiply each Input number by 3.
 - Add 2 to each Input number.
 - Multiply each Input number by 3, then add 2.
 - Add 2 to each Input number, then multiply by 3.
- Copy and complete this table for each pattern.
 - Divide each Input number by 10.
 - Subtract 3 from each Input number.
 - Divide each Input number by 10, then subtract 3.

Input	Output
100	
80	
60	
40	
20	



3. Look at this graph.
- Make an Input/Output table for the graph.
 - What patterns do you see in the table?
 - Extend the table 3 more rows. Explain how you did this.

4. The students at a school sell pins at a school fair to raise money for charity.

The students charge \$1.50 per pin.

- Copy and complete this table.
- Graph the data in the table in part a.
- Suppose you know how many pins were sold.
How can you find how much money was collected:

Number of Pins	Money Collected
10	
20	
30	
40	
50	
60	

- by using the table?
- by using the graph?

5. For each table:
- Describe the pattern in the Output column.
 - How can you find an Output number when you know an Input number?
 - Write the next 3 rows in each table.

a)

Input	Output
2	6
4	8
6	10
8	12
10	14

b)

Input	Output
2	6
4	12
6	18
8	24
10	30

c)

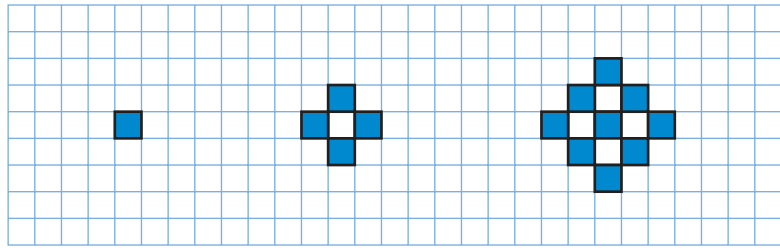
Input	Output
2	6
4	10
6	14
8	18
10	22

6. **Assessment Focus** Mr. Francis is planning a school picnic.
- Mr. Francis estimates he needs 2 sandwiches for each student, plus 3 extras. Make a table for the number of sandwiches needed for 5, 10, 15, 20, 25, 30 students.
 - Mr. Francis estimates he needs 1 drink for each student, plus 5 extras. Make a table for the number of drinks needed for up to 30 students.
 - Draw graphs for the tables in parts a and b.
Explain how each graph shows the patterns in the tables.



Take It Further

7. a) Copy this pattern on grid paper.
Extend the pattern to show the next 2 figures.



- b) Copy and complete this table for the first 5 figures.

Figure Number	Number of Blue Squares

- c) Draw a graph to show the data in the table.
d) How is this graph different from other graphs you have drawn?
e) How could you use the graph or the table to find the number of blue squares in the 9th figure?

Calculator Skills

Three numbers have a product of 24.5. One number is 0.25. What might the other two numbers be?

Reflect

Explain how a pattern in words can be represented by a table and a graph.

Use an example in your explanation.



Math Link

History

The word "algebra" comes from the Arabic world "al-jabr." This word appeared in the title of one of the earliest algebra texts, written around the year 825 by al-Khwarizmi. He lived in what is now Uzbekistan.

Explore



Work with a partner.

Tehya won some money in a competition.

She has two choices as to how she gets paid.

Choice 1: \$20 per week for one year

Choice 2: \$400 cash now plus \$12 per week for one year

Which method would pay Tehya more money? Explain.

For what reason might Tehya choose the method that pays less?

Reflect & Share

Work with another pair of classmates.

For each choice, write a rule you can use to calculate the total money

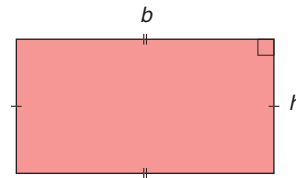
Tehya has received at any time during the year.

Connect

Recall how we used variables in the formulas for the area and perimeter of a rectangle.

Area: $A = bh$

Perimeter: $P = 2(b + h)$



In these formulas, b represents the length of the base and h represents the height.

We can also use a variable to represent a number in an expression.

For example, we know there are 100 cm in 1 m.

There are 2×100 cm in 2 m.

There are 3×100 cm in 3 m.

To write an expression for the number of centimetres in any number of metres, we say there are $n \times 100$ cm in n metres.

n is a variable.

n represents any number we choose.

We can choose any letter as a variable.
The letters n and x are frequently used.
The expression $n \times 100$ is written $100n$.
 $100n$ is an **algebraic expression**.

Variables are written in italics so they are not confused with units of measurement.

Here are some other algebraic expressions, and their meanings.
In each case, n represents the number.

- Three more than a number: $3 + n$ or $n + 3$
- Seven times a number: $7n$
- 8 less than a number: $n - 8$
- Twenty divided by a number: $\frac{20}{n}$

Example 1

A car travels at an average speed of 50 km/h.
How far will the car travel in: a) 3 h? b) t hours?

Solution

$t \times 50$ is equal to $50 \times t$,
which is written $50t$.

- In 1 h, the car travels 50 km.
- a) In 3 h, the car travels: $3 \times 50 \text{ km} = 150 \text{ km}$
b) In t hours, the car travels: $t \times 50 \text{ km} = 50t$ kilometres

Example 2

Write an algebraic expression for each statement.

- a) the amount of money earned at \$5/h
b) the perimeter of a square
c) eight more than three times a number
d) double a number and subtract 5

Solution

We often choose a letter to remind us what the variable represents.

t for time
 s for side length
 n for number

- For each statement, choose a variable to represent the number.
- a) Let t hours represent the time worked.
Then, the amount earned is $5 \times t$, or $5t$ dollars.
- b) Let s centimetres represent the side length of the square.
Then, the perimeter in centimetres is $4 \times s$, or $4s$ centimetres.
- c) Let n represent the number.
Three times the number: $3n$
Then add 8: $3n + 8$
- d) Let x represent the number.
Double the number means 2 times the number: $2x$
Then subtract 5: $2x - 5$

Practice

If you use x as a variable, write it so it is not confused with a multiplication sign, \times .



Remember that the fraction bar indicates division.

- Write an algebraic expression for each statement.
 - six more than a number
 - a number multiplied by eight
 - a number decreased by six
 - a number divided by four
- A person earns \$4/h baby-sitting. Find the money earned for each time.
 - 5 h
 - 8 h
 - t hours
- Find the area of a rectangle for each length and width.
 - length: 8 cm; width: 6 cm
 - length: 10 cm; width: 5 cm
 - length: l centimetres; width: w centimetres
- A person walks at an average speed of 5 km/h. Find the distance walked in each time.
 - 2 h
 - 5 h
 - t hours
- Write each algebraic expression in words. Use the words “a number” in place of the variable.
 - $n + 8$
 - $6a$
 - $\frac{p}{5}$
 - $k - 11$
 - $27 - n$
 - x^2
- Write an algebraic expression for each statement.
 - Double a number and add three.
 - Subtract five from a number, then multiply by two.
 - Subtract one-half of a number from 17.
 - Divide a number by seven, then add six.
 - A number is subtracted from twenty-eight.
 - Twenty-eight is subtracted from a number.
- Write each expression in words.
 - $(40 - 3)r$
 - $40 - 3r$
 - How are the expressions and statements in part a similar? Different?

Number Strategies

The regular price of a pair of running shoes is \$74.99. What is the sale price in each case?

- 50% off
- 25% off
- 30% off
- 70% off

8. a) Write an algebraic expression for each statement.
- three more than a number
 - a number added to three
 - three less than a number
 - a number subtracted from three
- b) How are the expressions in part a alike?
How are they different? Explain.
9. Write each expression in words.
- a) $6h + 5$ b) $\frac{(n-3)}{4}$ c) $\frac{t}{4} + 12$
d) $3(x-3)$ e) $32 - \frac{w}{5}$ f) $\frac{w}{5} - 32$

10. Assessment Focus

- a) Use the cards below to make an algebraic expression for each statement. Write the expression.



- nine times a number subtract 4
 - the sum of four times a number and nine
 - a number plus five
 - nine more than one-quarter of a number
- b) Create two more expressions using the cards.
Write each expression in words.
Show your work.
11. A pizza with cheese and tomato toppings costs \$8.00. There is a cost of \$1 for each extra topping. Write an algebraic expression for the cost of a pizza with e extra toppings.



Reflect

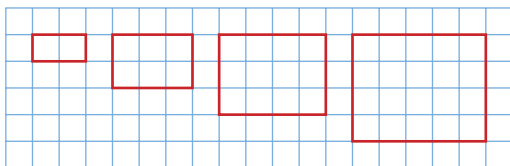
Why do we write algebraic expressions? How are they useful?

Mid-Unit Review

LESSON

- 10.1 1.** For each pattern:
- Describe the pattern.
 - Write the next 3 terms.
 - Find the 10th term.
- 5, 8, 11, 14, ...
 - 14, 25, 36, 47, ...
 - 1, 8, 27, 64, ...
 - 1, 3, 7, 15, 31, ...

- 2. a)** Describe this pattern.



The pattern continues.

- Write the dimensions of the next 2 rectangles.
- Write the areas of the first 5 rectangles.
- What are the dimensions and area of the 19th rectangle?
- Which rectangle has area 110 square units?

- 10.2 3. a)** Complete the table in the next column for this pattern: Multiply each term by 4, then subtract 1.
- b)** Graph the pattern. Explain how the graph shows the pattern.

Input	Output
1	
2	
3	
4	
5	
6	

- 4. a)** Describe the pattern in the Output column.
- b)** How can you find an output number when you know an input number?

Input	Output
1	8
2	11
3	14
4	17
5	20
6	23

- Write the next 3 rows in the table.
- Graph the table. Explain how the graph shows the pattern.

- 10.3 5.** Write an algebraic expression for each statement.
- eleven more than a number
 - four less than a number
 - a number divided by three
 - a number multiplied by nine
 - the sum of five times a number, and 2
 - seventeen more than two times a number

- 6.** Write each expression in words. Use the words “a number” in place of each variable.

a) $n + 3$ **b)** $21 - h$ **c)** $9n$ **d)** $\frac{a}{4}$

Focus Substitute a number for a variable in an algebraic expression.

Explore



Work with a partner.

Ms. Prasad plans to hold a party for a group of her friends.

The cost of renting a room is \$25.

The cost of food is \$3 per person.

Which algebraic expression gives the total cost, in dollars, of the party for n people?

$$3 + 25n \qquad 28n \qquad 28 + n \qquad 25 + 3n$$

Check your answer by finding the cost for 10 people.

Reflect & Share

Compare your answer with that of another pair of classmates.

How did you decide which expression was correct?

How does the expression change in each of the following cases?

- The cost of food doubles.
- The rent of the room doubles.

Connect

When we replace a variable with a number, we **substitute** a number for the variable.

When we replace a variable with a number in an algebraic expression, we **evaluate** the expression. That is, we find the value of the expression for a particular value of the variable.

Recall the work you did in *Unit 6 Measuring Perimeter and Area*. You substituted numbers for variables in the formulas for area and perimeter.

We use the same method to evaluate algebraic expressions.

Example

Write each algebraic expression in words.

Then evaluate for the value of the variable given.

a) $5k + 2$ for $k = 3$ b) $\frac{(x-3)}{5}$ for $x = 13$

Solution

$5k$ means $5 \times k$.

- a) $5k + 2$ means 5 times a number, then add 2.

Replace k with 3 in the expression $5k + 2$.

Use the order of operations.

$$\begin{aligned}5k + 2 &= 5 \times 3 + 2 && \text{Multiply first.} \\ &= 15 + 2 && \text{Add.} \\ &= 17\end{aligned}$$

- b) $\frac{(x-3)}{5}$ means subtract 3 from a number, then divide by 5.

Replace x with 13.

$$\begin{aligned}\frac{(x-3)}{5} &= \frac{(13-3)}{5} && \text{Do the operation in brackets first.} \\ &= \frac{10}{5} && \text{Divide.} \\ &= 2\end{aligned}$$

Recall that a variable is a symbol that can represent a set of numbers. If we substitute consecutive numbers in an algebraic expression, we get a pattern.

Use the algebraic expression $3n + 2$.

Substitute $n = 1, 2, 3, 4,$ and 5 .

$$\begin{aligned}\text{When } n = 1, 3n + 2 &= 3(1) + 2 \\ &= 3 + 2 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{When } n = 2, 3n + 2 &= 3(2) + 2 \\ &= 6 + 2 \\ &= 8\end{aligned}$$

$$\begin{aligned}\text{When } n = 3, 3n + 2 &= 3(3) + 2 \\ &= 9 + 2 \\ &= 11\end{aligned}$$

$$\begin{aligned}\text{When } n = 4, 3n + 2 &= 3(4) + 2 \\ &= 12 + 2 \\ &= 14\end{aligned}$$

$$\begin{aligned}\text{When } n = 5, 3n + 2 &= 3(5) + 2 \\ &= 15 + 2 \\ &= 17\end{aligned}$$

Use the values of n as the Input.

Use the values of $3n + 2$ as the Output.

Then, write the patterns in a table.

To get each Output number, multiply the Input number by 3, then add 2.

Input n	Output $3n + 2$
1	5
2	8
3	11
4	14
5	17