Addison Wesley

Math Makes Sense 7

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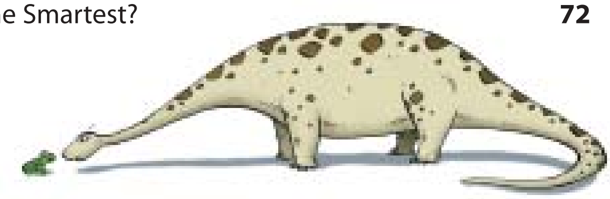


UNIT 3 Geometry and Measurement

Strand
- Geometry and Spatial Sense
- Measurement

UNIT 4 Fractions and Decimals

Strand
- Number Sense and Numeration

UNIT 5 Data Management

Strand • Data Management and Probability

UNIT 6 Measuring Perimeter and Area

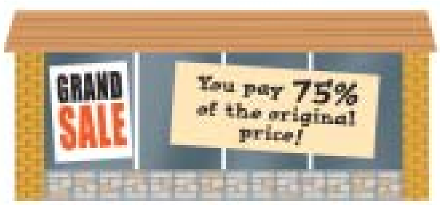
Strand • Measurement
• Patterning and Algebra

UNIT 7 Geometry

Strand • Geometry and Spatial Sense

UNIT 8 Working with Percents

Strand • Number Sense and Numeration
• Data Management and Probability

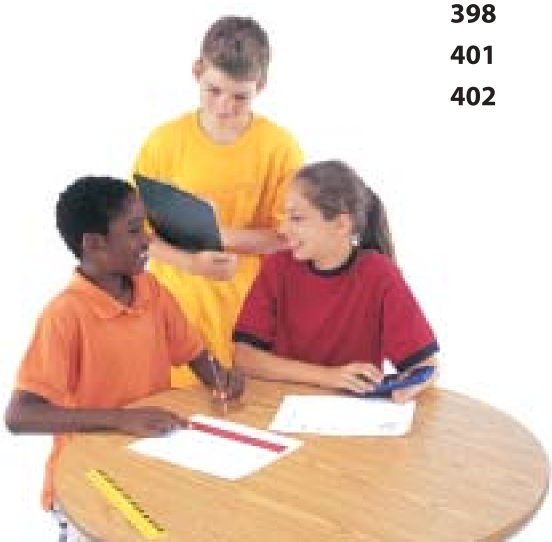


UNIT 9 Integers

Strand • Number Sense and Numeration

UNIT 10 Patterning and Algebra

Strand • Patterning and Algebra



UNIT 11 Probability

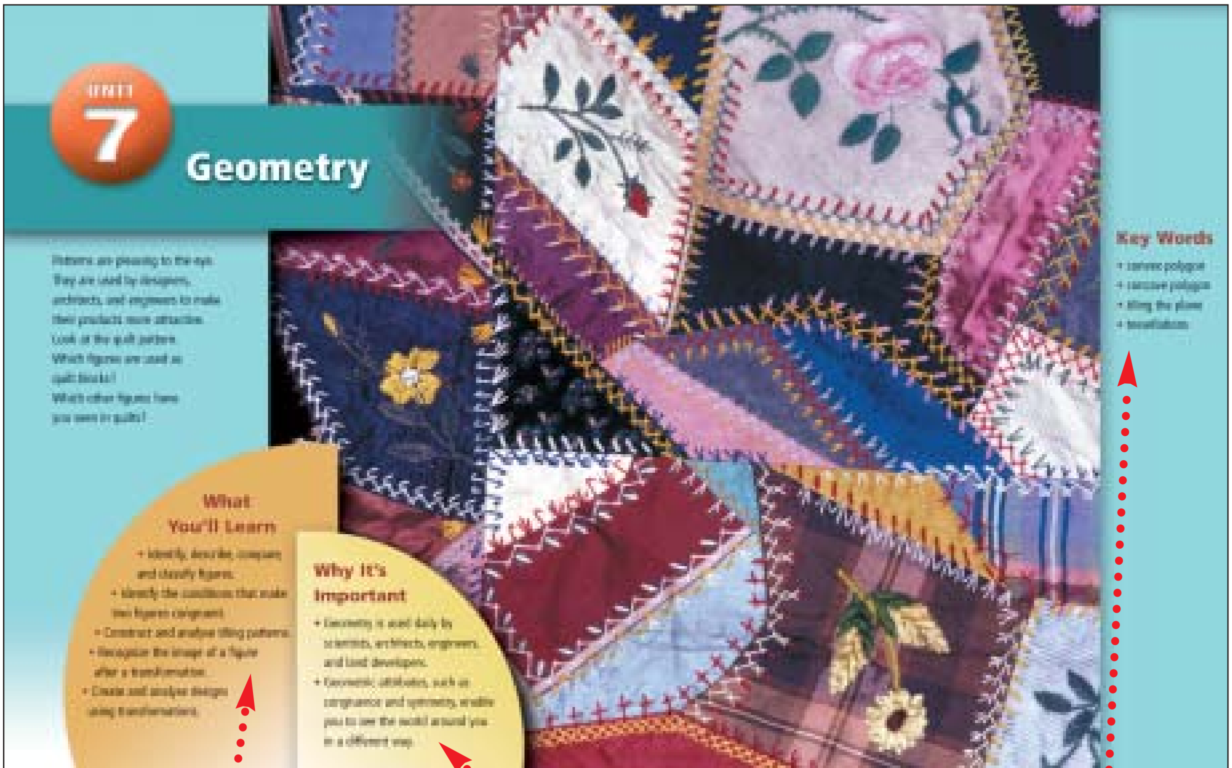
Strand • Data Management and Probability

Welcome to
Addison Wesley Math Makes Sense 7

Math helps you understand your world.

This book will help you improve your problem-solving skills and show you how you can use your math now, and in your future career.

The opening pages of **each unit** are designed to help you prepare for success.

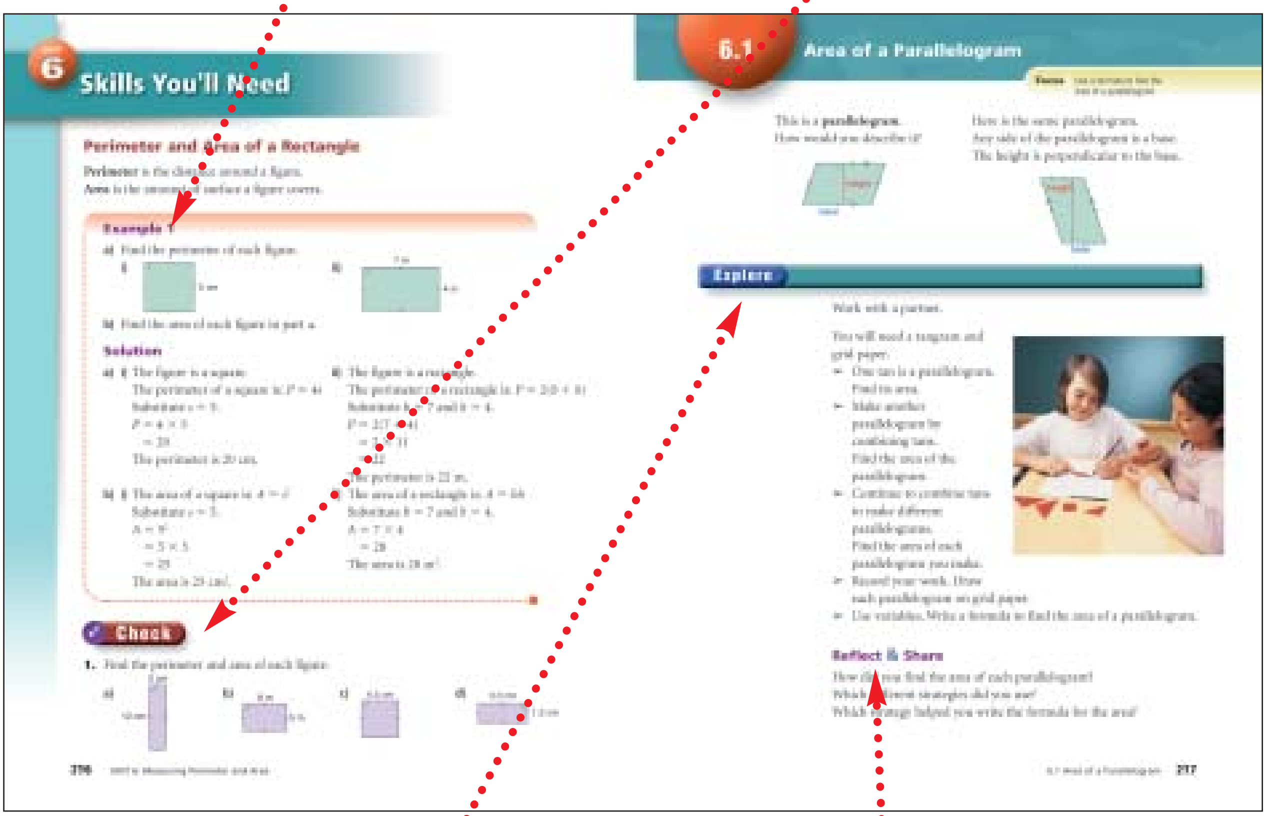


Find out **What You'll Learn** and **Why It's Important**. Check the list of **Key Words**.

Review some of the math concepts you've already met.

Study the **Example**.

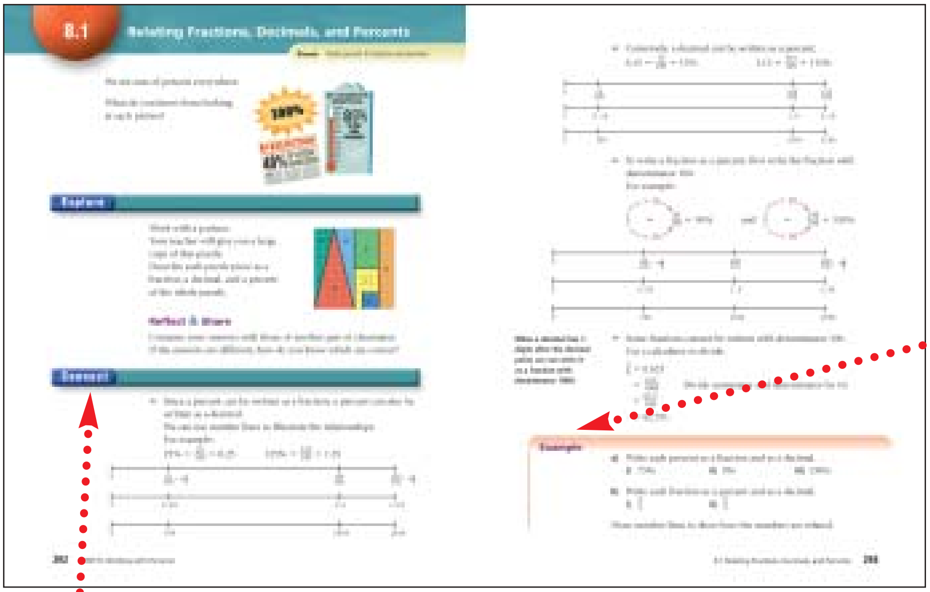
Then try the **Check** questions to review your skills.



In each Lesson:

Explore an idea or problem. You may use materials.

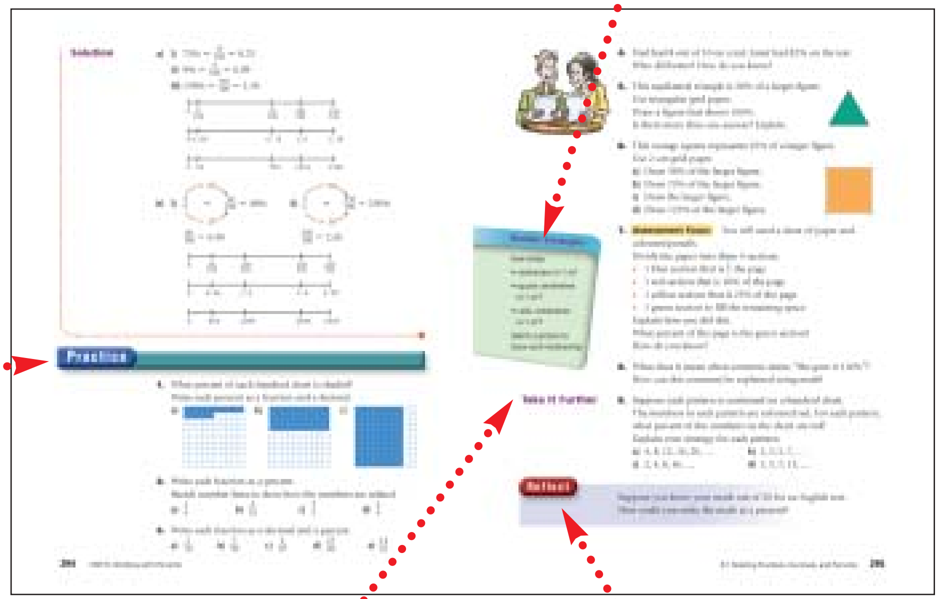
Reflect & Share your results with other students.



Examples show you how to use the ideas.

Connect summarizes the math.

Stay sharp with **Number Strategies**, **Mental Math**, and **Calculator Skills**.

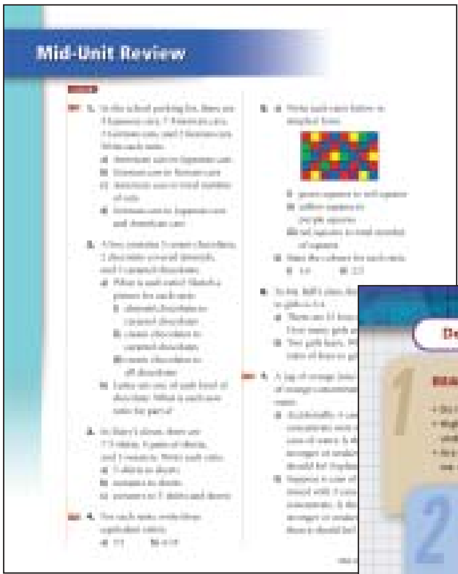


Practice questions reinforce the math.

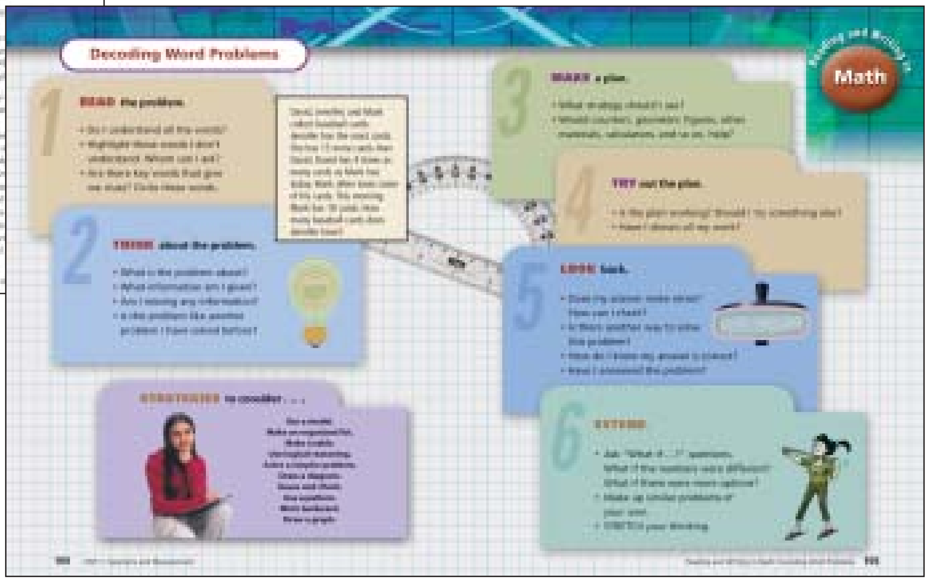
Take It Further questions offer enrichment and extension.

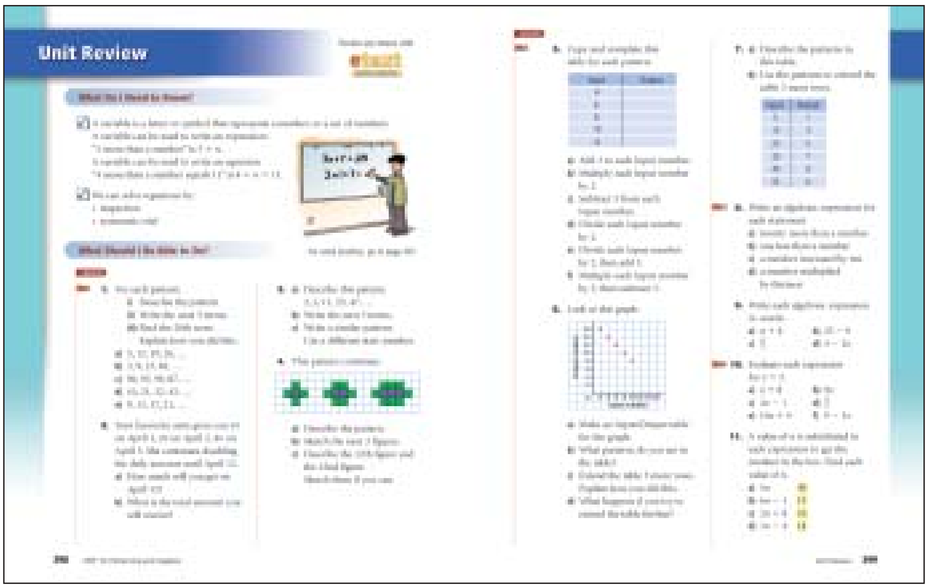
Reflect on the big ideas of the lesson.

Use the **Mid-Unit Review** to refresh your memory of key concepts.



Reading and Writing in Math helps you understand how reading and writing about math differs from other language skills you use. It may present problem solving strategies or problems for you to solve.



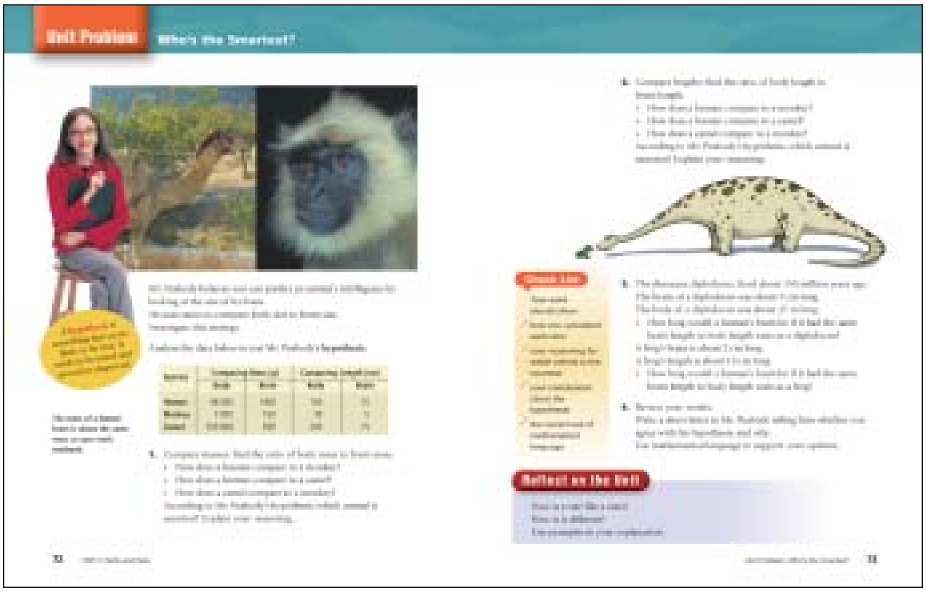


What Do I Need to Know? summarizes key ideas from the unit.

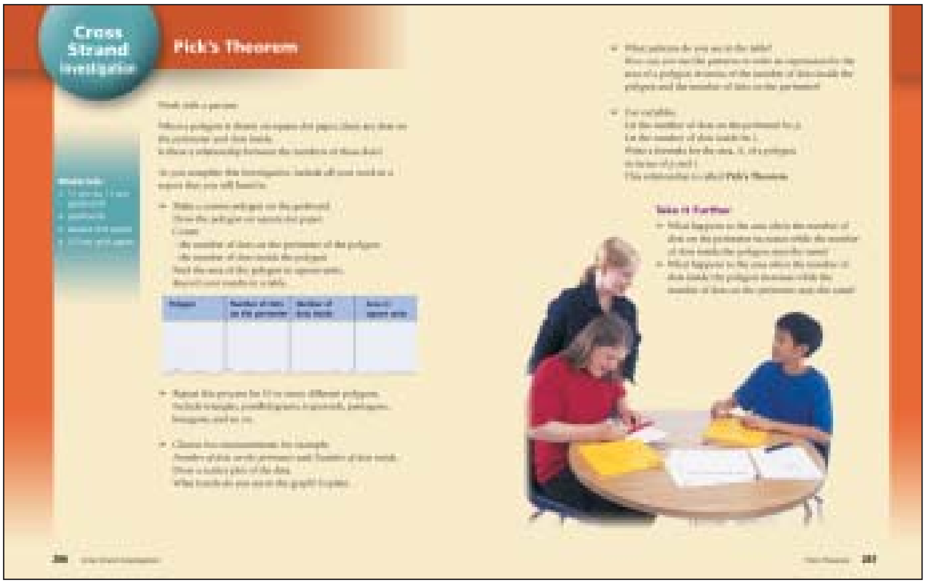
What Should I Be Able to Do? allows you to find out if you are ready to move on. The on-line tutorial **etext** provides additional support.

The **Practice Test** models the kind of test your teacher might give.

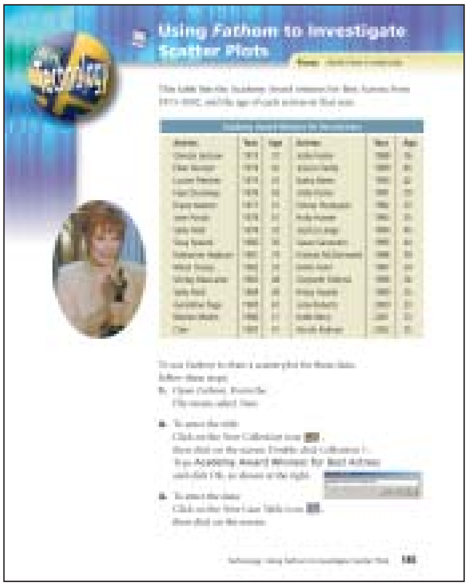
The **Unit Problem** presents problems to solve, or a project to do, using the math of the unit.



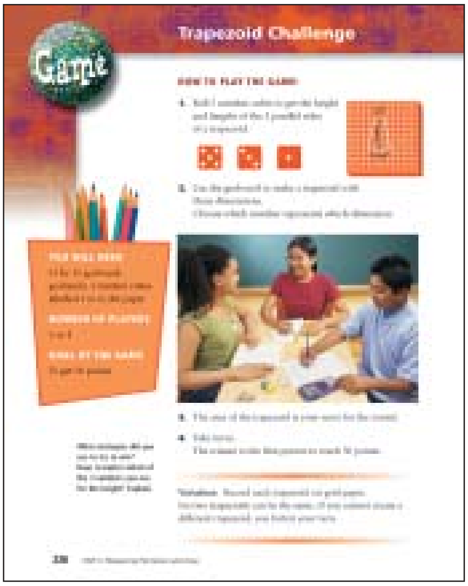
Keep your skills sharp with **Cumulative Review** and **Extra Practice.**



Explore some interesting math when you do the **Cross Strand Investigations**.



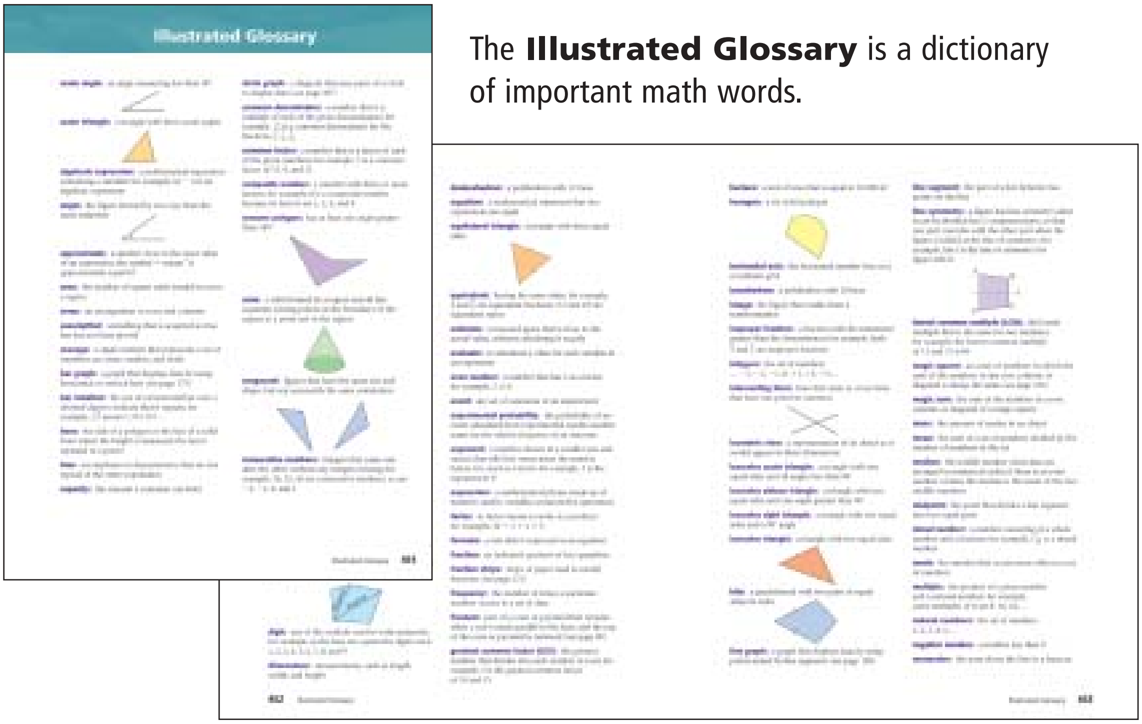
Icons remind you to use **technology**. Follow the step-by-step instructions for using a computer or calculator to do math.


Trapezoid Challenge

Play a **Game** with your classmates or at home to reinforce your skills.

The World of Work describes how people use mathematics in their careers.


Sports Trainer


Illustrated Glossary

The **Illustrated Glossary** is a dictionary of important math words.

Cross Strand Investigation

Making a Booklet

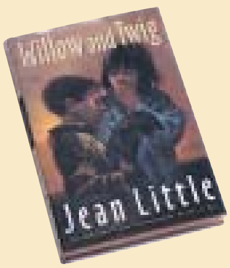
Materials:

- sheets of newsprint measuring 25 inches by 38 inches

Work with a partner.

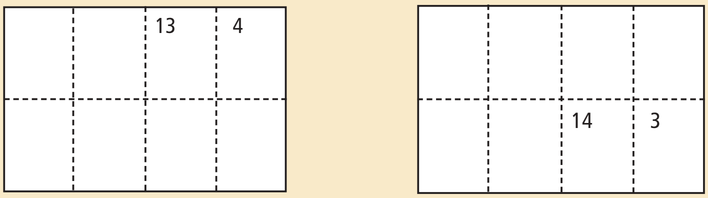
A book is made up of **signatures**. A signature has 16 or 32 pages. A signature is a sheet of paper printed on both sides, in a special arrangement. The sheet is measured in inches. The dimensions for the sheet in this *Investigation* are 25 inches by 38 inches. This is approximately 64 cm by 97 cm. This sheet of paper is then folded into sections of 16 or 32 pages. When the sheet of paper is folded and cut in some places, the pages read in the correct numerical order.

As you complete this *Investigation*, include all your work in a report that you will hand in.



Part 1

Here are both sides of a 16-page signature.
The pages are from 1 to 16.



There are patterns in the numbers on a signature. These patterns help the printer decide which page numbers go on each side of a sheet when it goes on press.

Your challenge will be to find the patterns in the numbers on a signature. How would knowing these patterns help you create a book with more than 32 pages?

➢ How many folds are needed to make a 16-page signature? Fold a 25-inch by 38-inch piece of paper in half, several times, to find out.

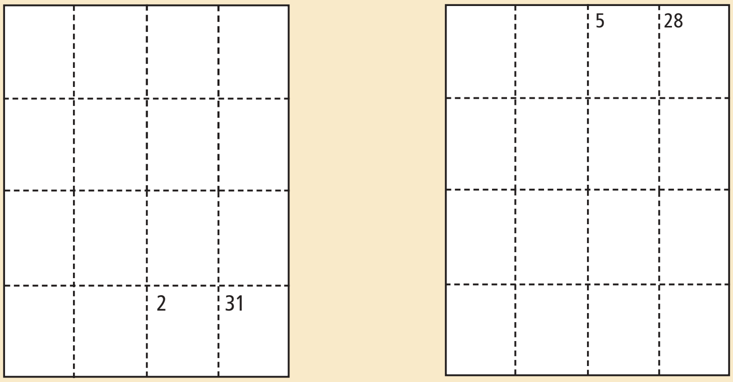
Write the page numbers consecutively on the pages.
Open the sheet of paper.
Where are all the even numbers?
Where are all the odd numbers?
What else do you notice about the page numbers?
What patterns do you see in the page numbers?

➢ The 2nd signature has pages 17 to 32.
Draw a sketch. Show both sides of this signature with page numbers in place.

➢ Look for patterns in the numbers.
How do these patterns compare with those in the 1st signature?

Part 2

Here are both sides of a 32-page signature.



➢ Repeat the steps in *Part 1* for this 32-page signature.
Describe all the patterns you discover.

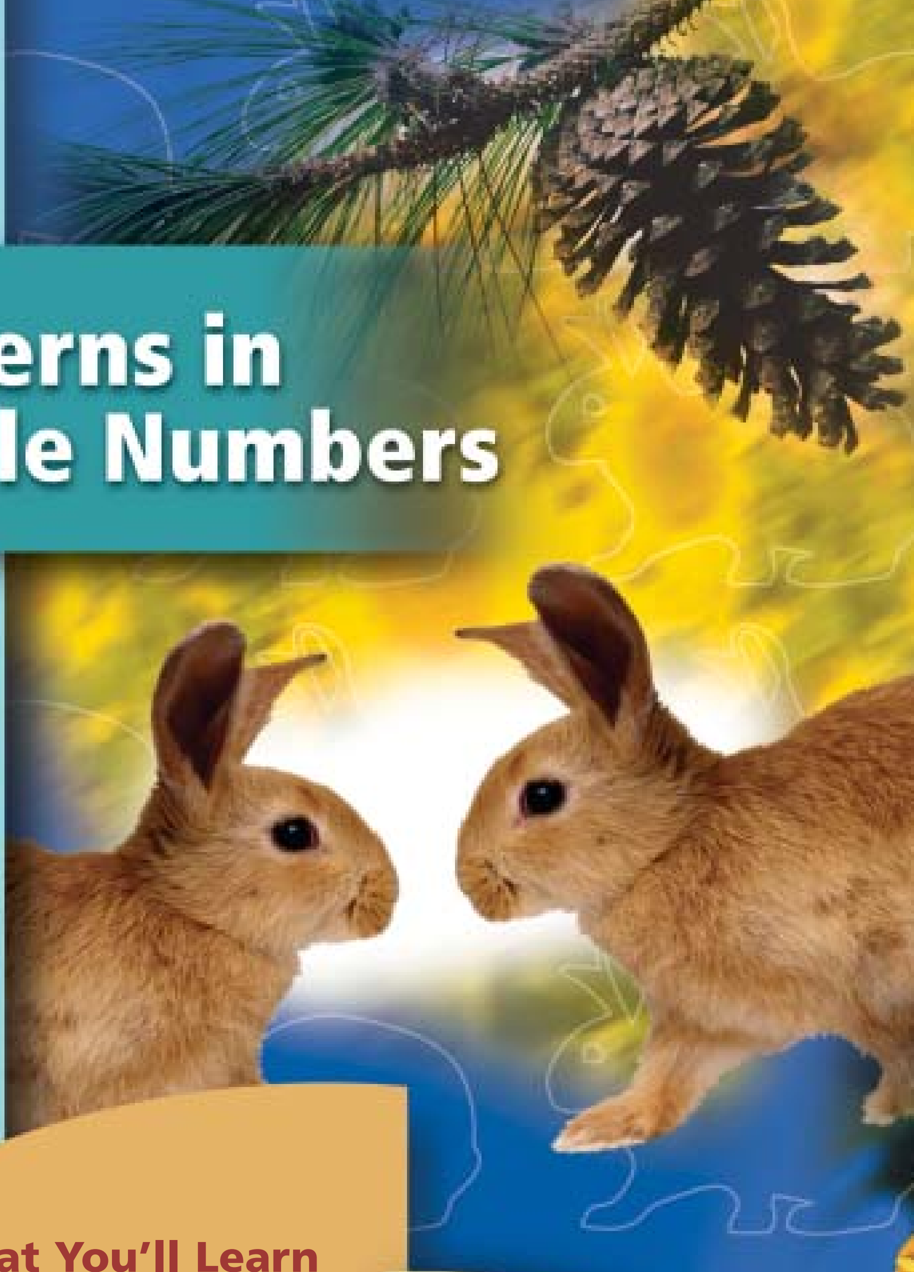
➢ Find some books in the school or class library.
Calculate how many signatures each book might have.

Take It Further

Suppose you have to create a 64-page signature.
How can you use the number patterns in *Parts 1* and *2* to help you create a 64-page signature?

UNIT 1

Patterns in Whole Numbers



There are many patterns you can see in nature. You can use numbers to describe many of these patterns.

At the end of this unit, you will investigate a famous set of numbers, the Fibonacci Numbers. You will explore these numbers, and find how they can be used to describe the breeding of animals, such as rabbits.

Think about the different patterns you learned in earlier grades. Give an example of each type of pattern.

What You'll Learn

- Use mental math, paper and pencil, calculators, and estimation to solve problems.
- Justify your choice of method for calculations.
- Find factors and multiples of numbers.
- Find the greatest common factor and lowest common multiple of two numbers.
- Identify prime and composite numbers.
- Use exponents to represent repeated multiplication.
- Identify and extend number patterns.
- Choose and justify strategies for solving problems.

Why It's Important

- You need to know an appropriate and efficient method for calculations.
- You need to be able to find common denominators for fraction calculations.

Key Words

- factor
- prime number
- composite number
- greatest common factor (GCF)
- multiple
- lowest common multiple (LCM)
- square number
- square root
- exponent form
- base
- exponent
- power
- cube number
- perfect square
- perfect cube

UNIT 1

Skills You'll Need

Rounding

The place-value chart below shows the number 1 234 567.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	4	5	6	7

To round:
Look at the digit to the right of the place to which you are rounding.
Is this digit 5 or greater?
If it is, add 1 to the place digit.
If it is not, leave the place digit as it is.
Change all the digits to the right of the place digit to 0.

Example 1

a) Round 425 to the nearest ten.

b) Round to the nearest thousand.
i) 2471 **ii)** 13 999

Solution

a)

Hundreds	Tens	Ones
4	2	5

This number is 5.

So, add 1 ten to this number to get 3 tens.
Then, replace 5 with 0.

425 rounded to the nearest ten is 430.

b) i)

Thousands	Hundreds	Tens	Ones
2	4	7	1

This number is less than 5.

So, this number does not change.
Replace each number to the right of 2 with 0.

2471 rounded to the nearest thousand is 2000.

ii)

Ten thousands	Thousands	Hundreds	Tens	Ones
1	3	9	9	9

This number is greater than 5.

So, add 1 thousand to this number to get 4 thousands.
Replace each number to the right of 4 with 0.

13 999 rounded to the nearest thousand is 14 000.

✓ Check

1. Round to the nearest ten.

a) 36 **b)** 42 **c)** 75 **d)** 361

2. Round to the nearest hundred.

a) 311 **b)** 789 **c)** 625 **d)** 2356

Multiplying by 10, 100, 1000

To multiply a whole number:
– by 10, write 0 after the number.
– by 100, write 00 after the number.
– by 1000, write 000 after the number.

Example 2

Multiply.

a) 32×10 **b)** 478×100 **c)** 51×1000

d) 32×20 **e)** 47×300

Solution

a) $32 \times 10 = 320$ **b)** $478 \times 100 = 47\,800$ **c)** $51 \times 1000 = 51\,000$

d) $32 \times 20 = 32 \times 2 \times 10$
$= 64 \times 10$
$= 640$

e) $47 \times 300 = 47 \times 3 \times 100$
$= 141 \times 100$
$= 14\,100$

✓ Check

3. Find.
   a) 3×10 b) 1000×5 c) 131×10 d) 100×63

4. Use a place-value chart. Explain why we can write zeros after a number when we multiply by 10, 100, or 1000.

5. Multiply.
   a) 50×72 b) 18×600 c) 4000×33

Mental Math

Example 3

Use mental math.

a) 53×6 b) $308 + 56 - 6$ c) $197 + 452$

Solution

a) 53×6

Think:

$$\begin{aligned} & 50 \times 6 + 3 \times 6 \\ &= 300 + 18 \\ &= 318 \end{aligned}$$

b) $308 + 56 - 6$

Think:

$56 - 6 = 50$

Then: $308 + 50 = 358$

c) $197 + 452$

Think:

$197 = 200 - 3$

Then: $200 + 452 - 3 = 652 - 3$ Count back to subtract.

$= 649$

✓ Check

6. Use mental math.
   a) $4 + 17$ b) $9 + 8$ c) $12 + 6$
   d) $20 + 6$ e) $40 + 30$ f) $17 - 2$
   g) $22 - 4$ h) $70 - 20$ i) $20 - 15$

7. Use mental math. Explain your strategy.
   a) $28 + 13 + 12$ b) $2 \times 29 \times 5$ c) $98 + 327$
   d) $4 \times 981 \times 25$ e) 99×21 f) 62×8

Divisibility Rules

A number is divisible by:

- 2 if the number is even
- 3 if the sum of its digits is divisible by 3
- 4 if the number represented by the last 2 digits is divisible by 4
- 5 if the last digit is 0 or 5
- 6 if the number is divisible by 2 and 3
- 8 if the number represented by the last 3 digits is divisible by 8
- 9 if the sum of the digits is divisible by 9
- 10 if the last digit is 0

Example 4

Which of these numbers is 1792 divisible by?

a) 2 **b)** 3 **c)** 4 **d)** 5 **e)** 6

Solution

a) 1792 is divisible by 2 because 1792 is an even number.

b) $1 + 7 + 9 + 2 = 19$
19 is not divisible by 3, so 1792 is not divisible by 3.

c) The last 2 digits are 92.
$92 \div 4 = 23$
Since the last 2 digits are divisible by 4, 1792 is divisible by 4.

d) 1792 is not divisible by 5 because the last digit is not 0 or 5.

e) Since 1792 is not divisible by 3, 1792 is also not divisible by 6.

✓ Check

8. Which numbers are divisible by 3?
a) 490 **b)** 492 **c)** 12 345

9. Write 4 other numbers greater than 400 that are divisible by 3.

10. Which numbers are divisible by 6?
a) 870 **b)** 232 **c)** 681

11. Which numbers from 1 to 10 is:
a) 660 divisible by? **b)** 1001 divisible by?

1.1 Numbers All Around Us

Focus Justify the choice of strategy when solving problems.

We use numbers to understand and describe our world.


December 6, 1917

Two Ships Collide as Halifax Reels

HALIFAX, NS – Tragedy struck the Halifax harbour shortly after 9:00 yesterday morning when an unmarked munitions vessel collided with another ship and exploded.

To avoid German U-boat attacks the Mont-Blanc was not flying any warning flags, despite carrying 2800 tons of military explosives.

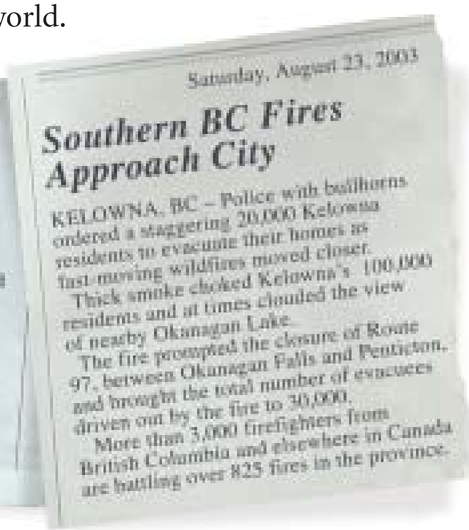

The Halifax Explosion

Did You Know?

At the time of the explosion, the Mont-Blanc was carrying
- 226 797 kg of TNT
- 2 146 830 kg of picric acid
- 56 301 kg of guncotton
- 223 188 kg of benzol

— all highly explosive and dangerous chemicals.

The 1917 explosion was 1/7 the strength of the atomic bomb dropped on Hiroshima.


Saturday, August 23, 2003

Southern BC Fires Approach City

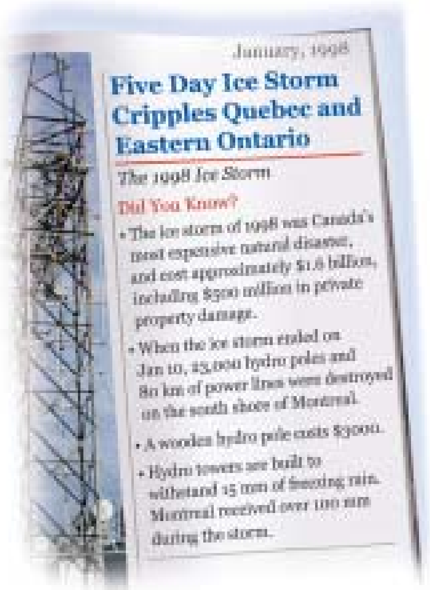
KELOWNA, BC – Police with bullhorns ordered a staggering 20,000 Kelowna residents to evacuate their homes as fast-moving wildfires moved closer.

Thick smoke choked Kelowna's 100,000 residents and at times clouded the view of nearby Okanagan Lake.

The fire prompted the closure of Route 97, between Okanagan Falls and Penticton, and brought the total number of evacuees driven out by the fire to 30,000.

More than 3,000 firefighters from British Columbia and elsewhere in Canada are battling over 825 fires in the province.

Explore


January, 1998

Five Day Ice Storm Cripples Quebec and Eastern Ontario

The 1998 Ice Storm

Did You Know?
- The ice storm of 1998 was Canada's most expensive natural disaster, and cost approximately $1.6 billion, including $500 million in private property damage.
- When the ice storm ended on Jan 10, 23,000 hydro poles and 80 km of power lines were destroyed on the south shore of Montreal.
- A wooden hydro pole costs $3000.
- Hydro towers are built to withstand 45 mm of freezing rain. Montreal received over 100 mm during the storm.

Work on your own.
Read the articles above and at the left.

➢ Which numbers do you think are exact?
Which numbers are estimates? Explain your thinking.

➢ Use the numbers in the articles.
Write a problem you would solve each way:
- using mental math
- by estimating
- using pencil and paper
- using a calculator

➢ Solve your problem.

➢ Trade problems with a classmate.
Solve your classmate's problem.

Reflect & Share

Compare the strategies you used to solve the problems.
- Explain why some strategies work while others may not.
- Is one strategy more effective? Why?

Connect

- When the numbers are easy to handle, use mental math.
- When the problem has too many steps, use a paper and pencil.
- When an approximate answer is appropriate and to check reasonableness, estimate.
- When a more accurate answer is needed and the numbers are large, use a calculator.

Example

The population of Canada was 30 750 000 in July 2000.
Statistics Canada (Stats Can) data show that there were 6367 telephones per 10 000 people in that year.

a) About how many telephones were there in Canada in 2000?
b) Find the exact number of telephones in Canada in 2000.
c) How did Stats Can know there were 6367 phones per 10 000 people? Explain how this answer affects the answer to part b.

Solution

a) Estimate.
Round 30 750 000 to the nearest ten million: 30 000 000
Round 6367 to the nearest thousand: 6000
10 000 people use about 6000 phones.
10 000 000 people use about 6000×1000, or 6 000 000 phones.
30 000 000 people use about $6\,000\,000 \times 3$, or 18 000 000 phones.
There were about 18 million phones in Canada in 2000.

b) Find how many groups of 10 000 people there are in 30 750 000.
That is, $30\,750\,000 \div 10\,000 = 3075$
For each group of 10 000 people, there were 6367 phones.
The number of phones: $3075 \times 6367 = 19\,578\,525$
So, there were 19 578 525 phones in Canada in 2000.

Use a calculator.

c) Stats Can conducted a survey of about 40 000 people.
It asked how many phones each person had.
Stats Can then calculated how many phones per 10 000 people.

To find out exactly how many phones there are in Canada, Stats Can would have to survey the entire population. This is impractical. So, the number of phones in part b is an estimate.

Practice

1. Solve without a calculator.
   a) 72 + 43 b) 123 + 85 c) 672 + 189
   d) 97 − 24 e) 195 − 71 f) 821 − 485
   g) 65 × 100 h) 14 × 75 i) 83 × 25
   j) 780 ÷ 10 k) 724 ÷ 4 l) 245 ÷ 7

2. Use pencil and paper to find each answer.
   a) 6825 + 127 b) 7928 − 815 c) 3614 − 278
   d) 138 × 21 e) 651 ÷ 21 f) 6045 ÷ 15

3. Estimate each answer. Explain the strategy you used each time.
   a) 103 + 89 b) 123 − 19
   c) 72 × 9 d) 418 ÷ 71

Mental Math

Multiply.

3.6 × 1000 3.6 × 100
3.6 × 10 3.6 × 1
3.6 × 0.1 3.6 × 0.01

What patterns do you see in the questions and the answers?

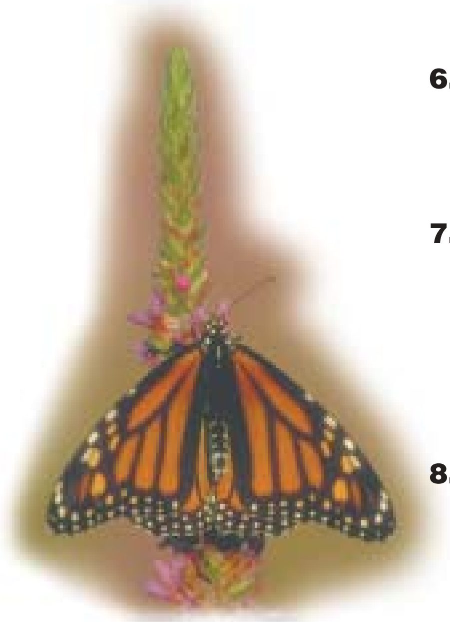
Questions 4 to 7 pose problems about the 1997 Red River Flood in Manitoba. Use mental math, estimation, pencil and paper, or a calculator. Justify your strategy.

4. The 1997 Red River Flood caused over $815 036 000 in damages.
   a) Write this amount in words.
   b) How close to $1 billion were the damages?

5. Pauline Thiessen and fellow volunteers made an average of 10 000 sandwiches every day for 2 weeks to feed the flood relief workers. How many sandwiches did they make?

6. Winnipeg used 6.5 million sandbags to hold back the flood. Each sandbag was about 10 cm thick. About how high would a stack of 6.5 million sandbags be in centimetres? Metres? Kilometres?

7. To help with the flood relief, Joe Morena of St. Viateur Bagels in Montreal trucked 300 dozen of his famous bagels to Manitoba.
   a) How many bagels did he send?
   b) Joe sells bagels for $4.80 a dozen.
   What was the value of his donation?



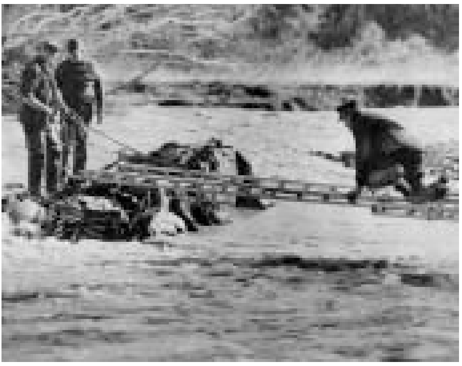
8. The Monarch butterfly migrates from Toronto to El Rosario, Mexico. This is a distance of 3300 km.
   A monarch butterfly can fly at an average speed of 15 km/h.
   How long does the migration flight take?

9. Estimate each answer. Is each estimate high or low? How do you know?

a) $583 + 702$ **b)** $3815 - 576$ **c)** $821 \div 193$ **d)** $695 \div 310$

For questions 10 and 11: Make up a problem using the given data. Have a classmate solve your problems.

10. Sunil earns $7 per hour. He works 4 h per day during the week and 6 h per day on the weekends.



11. In October 1954, Hurricane Hazel blew through Toronto, Ontario. Winds reached 124 km/h, 111 mm of rain fell in 12 h, and over 210 mm of rain fell over 2 days.

12. **Assessment Focus** The table shows the populations of some Canadian provinces in 1999.

Province	Population
NF and Labrador	541 000
PEI	138 000
Nova Scotia	939 800
New Brunswick	755 000
Ontario	11 513 800

The **mean** of a set of numbers is the sum of the numbers divided by how many numbers there are.

a) Do you think these numbers are exact? Explain.
b) Find the total population of the 4 Atlantic provinces.
c) Find the mean population of the Atlantic provinces.
d) Approximately how many times as many people are in Ontario as are in the Atlantic provinces?
e) Make up your own problem about these data. Solve it.

Take It Further

13. Find 2 whole numbers that:
a) have a sum of 10 and a product of 24
b) have a difference of 4 and a product of 77
c) have a sum of 77 when added to 3
Which of parts a to c have more than one answer? Explain.

Reflect

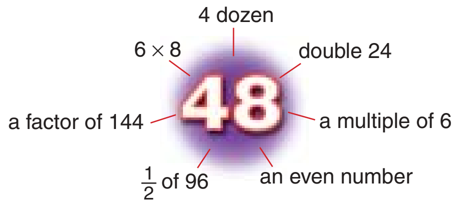
Write an example of a problem you would solve:
- by estimation
- by using a calculator

Justify your choice.

1.2 Factors and Multiples

Focus Generate factors and multiples of given numbers.

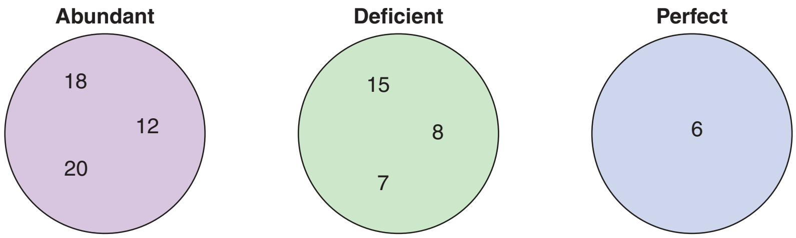
Just as numbers can describe our world, we can use numbers to describe other numbers. We can describe a number by its factors, by the number of its factors, and by the sum of its factors.



Explore

Work with a partner.

➢ Analyse the numbers in the circles.



Use a table to record the factors of each number.
Cross out the number itself.
Find the sum of the remaining factors.

Number	Factors	Sum of Factors

➢ Look for patterns among your results.
Why do you think a number is called "Abundant," "Deficient," or "Perfect"?

➢ Copy the 3 circles. Based on your ideas, place each number from 2 to 30 in the appropriate circle. What do you notice?

➢ Use your observations to predict where 36 and 56 belong.
Check your predictions.

Reflect & Share

Share your results with another pair of students.
What relationships did you see?
How did you describe each type of number?

Connect

➢ Recall that a factor is a number that divides exactly into another number.
For example, 1, 2, 3, and 6 are the factors of 6.
Each number divides into 6 with no remainder.

A **prime number** has only 2 factors, itself and 1.
2, 3, and 5 are prime numbers.
All prime numbers are deficient.

A **composite number** has more than 2 factors.
8 is a composite number because its factors are 1, 2, 4, and 8.
Composite numbers can be deficient, abundant, or perfect.

1 has only one factor, so 1 is neither prime nor composite.

When we find the factors that are the same for 2 numbers, we find **common factors**.

Example 1

Show the factors of 12 and 30 in a Venn diagram.
What is the **greatest common factor** (**GCF**) of 12 and 30?

Solution

Use the divisibility rules.

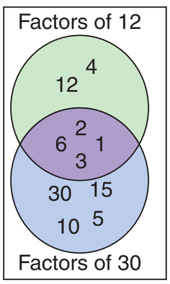
Find pairs of numbers that divide into 12 exactly.

$12 \div 1 = 12$	1 and 12 are factors.
$12 \div 2 = 6$	2 and 6 are factors.
$12 \div 3 = 4$	3 and 4 are factors.

Stop at 3 because the next number, 4, is already a factor.
The factors of 12 are 1, 2, 3, 4, 6, and 12.

Find the factors of 30.

$30 \div 1 = 30$	1 and 30 are factors.
$30 \div 2 = 15$	2 and 15 are factors.
$30 \div 3 = 10$	3 and 10 are factors.
$30 \div 5 = 6$	5 and 6 are factors.



Mark the factors on the Venn diagram.
Place the common factors in the overlapping region.
The GCF of 12 and 30 is 6.

➢ The **multiples** of a number are found by multiplying the number by 1, by 2, by 3, by 4, and so on, or by skip counting.
When we find multiples that are the same for 2 numbers, we find **common multiples**.
We can use a 100 chart to find multiples and common multiples.

Example 2

a) Use a 100 chart to find the common multiples of 12 and 21.
b) Find the **lowest common multiple** (**LCM**) of 12 and 21.

Solution

a)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The multiples of 12 are blue.
The multiples of 21 are shaded green.
The common multiple is blue and shaded green.

b) Multiples of 12 are 12, 24, 36, 48, 60, 72, **84**, 96, …
Multiples of 21 are 21, 42, 63, **84**, …
The LCM is 84.

Practice

1. List 4 multiples of each number.
a) 5 **b)** 7 **c)** 8

2. Find the factors of each number. Explain how you did it.
a) 18 **b)** 20 **c)** 28 **d)** 36 **e)** 37 **f)** 45

3. Find the factors of each number.
a) 50 **b)** 51 **c)** 67 **d)** 75 **e)** 84 **f)** 120

Number Strategies

Three CDs cost $12.99, $8.18, and $7.88 (tax included).

Approximately how much change should there be from $40.00?

4. Is each number prime or composite? How do you know?
a) 18 **b)** 13 **c)** 9 **d)** 19 **e)** 61 **f)** 2

5. Find the GCF of each pair of numbers.
Which strategy did you use?
a) 10, 5 **b)** 12, 8 **c)** 15, 25 **d)** 9, 12 **e)** 18, 15

6. Use a 100 chart. Find the LCM of each pair of numbers.
a) 3, 4 **b)** 2, 5 **c)** 12, 18 **d)** 10, 25 **e)** 27, 18

7. Can a pair of numbers have:
a) more than one common multiple?
b) more than one common factor?
Use a diagram to explain your thinking.

8. Julia and Sandhu bought packages of granola bars.
a) Julia had 15 bars in total. Sandhu had 12 bars in total.
How many bars could there be in one package?
b) What if Julia had 24 bars and Sandu had 18 bars?
How many bars could there be in one package?
Draw a diagram to explain your thinking.

9. **Assessment Focus** Kevin, Alison, and Fred work part-time. Kevin works every second day. Alison works every third day. Fred works every fourth day. Today they all worked together. When will they work together again?
Explain how you know.

10. The numbers 4 and 16 could be called "near-perfect".
Why do you think this name is appropriate?
Find another example of a near-perfect number.
What strategy did you use?

Reflect

What is the difference between a factor and a multiple?
Is a factor ever a multiple? Is a multiple ever a factor?
Use diagrams, pictures, or a 100 chart to explain.

The Factor Game

HOW TO PLAY THE GAME:

YOU WILL NEED
One gameboard;
2 coloured markers

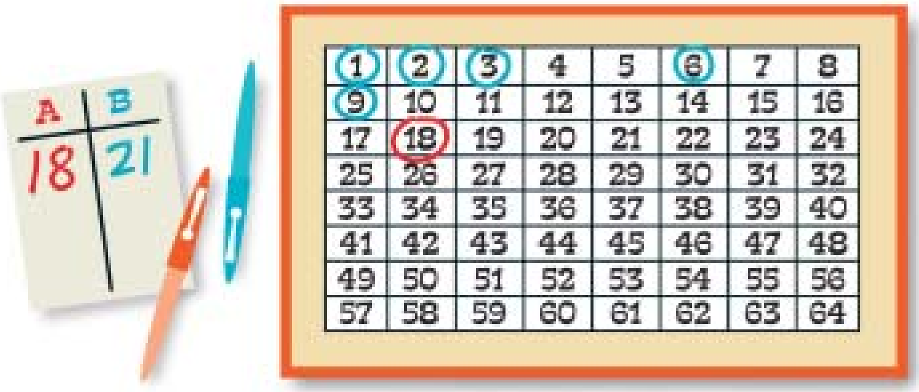
NUMBER OF PLAYERS
2

GOAL OF THE GAME
To circle factors of a number

1. Roll a number cube. The person with the greater number goes first.
2. Player A circles a number on the game board and scores that number. Player B uses a different colour to circle all the factors of that number not already circled. She scores the sum of the numbers she circles.

For example, suppose Player A circles 18.
Player B circles 1, 2, 3, 6, and 9 (18 is already circled)
to score $1 + 2 + 3 + 6 + 9 = 21$ points

3. Player B circles a new number.
Player A circles all the factors of that number not already circled. Record the scores.



4. Continue playing. If a player chooses a number with no factors left to circle, the number is crossed out. The player loses her or his turn, and scores no points.

For example, if player A circled 16, but 1, 2, 4, and 8 have already been circled, he would lose his turn and score no points.

5. The game continues until all numbers have been circled or crossed out. The player with the higher score wins.

What is the best first move? What is the worst first move? Why? How does this game involve factors, multiples, and prime numbers?

1.3 Squares and Square Roots

Focus Find the squares and square roots of whole numbers.

Explore

Work with a partner
This chart shows the number of factors of each whole number.

																							X						X
																							X						X
											X						X		X				X				X		X
											X				X		X		X				X				X		X
					X		X		X		X		X	X	X		X		X	X	X		X		X	X	X		X
			X		X		X	X	X		X		X	X	X		X		X	X	X		X	X	X	X	X		X
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

Look for patterns and relationships in this chart.
Find the factors of the numbers with two factors.
What do you notice?
Describe the numbers with four or more factors.
Describe the numbers that have an odd number of factors.

Reflect & Share

One way to describe a number with an odd number of factors is to call it a **square number**.
Why do you think this name is used?
Draw pictures to support your explanation.

Connect

The factors of a composite number occur in pairs.
For example, $48 \div 2 = 24$ 2 and 24 are factors of 48.
When the quotient is equal to the divisor, the dividend is a square number.
For example, $49 \div 7 = 7$, so 49 is a square number.

dividend **divisor** **quotient**

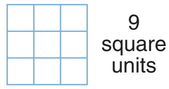
We can get 49 by multiplying the whole number, 7, by itself.

$49 = 7 \times 7$

We write: $7 \times 7 = 7^2$

We say: 7 squared

- One way to model a square number is to draw a square.
  This square has area 9 square units.
  The side length is $\sqrt{9}$, or 3 units.
  We say: A **square root** of 9 is 3.



Other inverse operations are addition and subtraction, multiplication and division

- When we multiply a number by itself, we square the number.
  Squaring a number and taking its square root are inverse operations. That is, they undo each other.

$7 \times 7 = 49$
so, $7^2 = 49$

$\sqrt{49} = \sqrt{7^2}$
$= 7$

Example 1

Find the square of each number.

a) 5 **b)** 15 **c)** 32

Solution

a) $5^2 = 5 \times 5$
$= 25$

b) $15^2 = 15 \times 15$
$= 225$

c) $32^2 = 32 \times 32$
$= 1024$
Use a calculator.

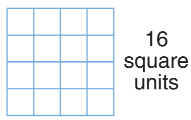
Example 2

Draw a diagram to find a square root of each number.

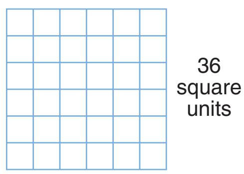
a) 16 **b)** 36

Solution

a) On grid paper, draw a square with area 16 square units.
The side length of the square is 4 units.
So, $\sqrt{16} = 4$



b) On grid paper, draw a square with area 36 square units.
The side length of the square is 6 units.
So, $\sqrt{36} = 6$



Practice

1. Find.
   a) 8^2 b) 16^2 c) 1^2 d) 29^2

2. Find the square of each number.
   a) 4 b) 17 c) 13 d) 52

3. a) Find the square of each number.
   i) 1 ii) 10 iii) 100 iv) 1000

   b) Use the results of part a. Predict the square of each number.
   i) 10 000 ii) 1 000 000

4. Use grid paper. Find a square root of each number.
   a) 16 b) 4 c) 900 d) 144

5. Calculate the side length of a square with each area.
   a) 100 m^2 b) 64 cm^2 c) 81 m^2

6. Order from least to greatest.
   a) $\sqrt{36}$, 36, 4, $\sqrt{9}$ b) $\sqrt{400}$, $\sqrt{100}$, 19, 15

7. Which whole numbers have squares between 50 and 200?

8. **Assessment Focus** Which whole numbers have square roots between 1 and 20? How do you know?

9. A large square room has an area of 144 m^2.
   a) Find the length of a side of the room.
   b) How much baseboard is needed to go around the room?
   c) Each piece of baseboard is 2.5 m long. How many pieces of baseboard are needed?

10. A garden has an area of 400 m^2.
    The garden is divided into 16 congruent square plots.
    What is the side length of each plot?

Number Strategies

Chris had saved \$145.98 by June. He spent \$2.25 in July, \$4.50 in August, and \$9.00 in September. If Chris' spending pattern continues, when will he have less than \$5.00?

Reflect

How can you find the perimeter of a square when you know its area? Use an example to explain.

Mid-Unit Review

LESSON

1.1 **1.** The table shows the most common surnames for adults in the United Kingdom.

Surname	Number
Smith	538 369
Jones	402 489
Williams	279 150

a) Approximately how many adults have one of these three names?
To which place value did you estimate? Explain your choice.

b) Exactly how many more Smiths are there than Jones? Explain.

c) Write your own problem about these data. Solve your problem. Justify your strategy.

2. In one week, Joe worked 23 h cutting grass. He was paid \$9/h. From this money, Joe bought 5 tickets for a football game, at \$15 per ticket, and 2 DVDs for \$21 each, including tax. How much money did Joe have left?

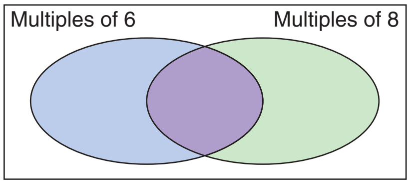
1.2 **3.** Find all the factors of each number.

a) 35 **b)** 24

4. Find 2 factors of each number.

a) 6 **b)** 10
c) 14 **d)** 15
e) 9 **f)** 21

5. Organize the first 8 multiples of 6 and 8 in a Venn diagram.



6. For the numbers 15 and 6, find:

a) the GCF **b)** the LCM

7. **a)** Why is 7 a prime number?
b) Why is 8 not a prime number?

1.3 **8.** Can three consecutive whole numbers all be primes? Justify your answer.

9. A square patio has an area of 81 m^2. How long is each side?

10. Find.

a) $\sqrt{49}$ **b)** 8^2
c) $\sqrt{100}$ **d)** the square of 9

11. Find two squares with a sum of 100.

12. Write 100 as a square number and as a square root of a number.

13. Explain why:
$\sqrt{1} = 1$

1.4 Exponents

Focus Use exponents to represent repeated multiplication.

Explore

Work with a small group.
You will need 65 interlocking cubes.
The edge length of each cube is 1 unit.
The volume of each cube is 1 cubic unit.

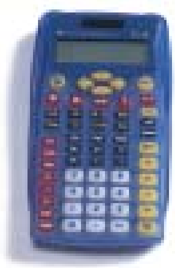
- ➢ How many different ways can you make a larger cube?
- ➢ What is the volume of each larger cube you make? What is its edge length?
- ➢ Use factors to write the volume of each cube.
- ➢ Record your results in a table.

Number of Cubes	Volume (cubic units)	Edge Length (units)	Volume As a Product
1	1	1	$1 \times 1 \times 1$

Reflect & Share

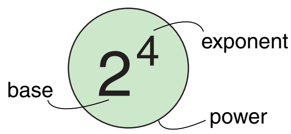
Observe how the volume grows. Describe the growth using pictures or numbers. What other patterns do you see in the table?
Use these patterns to help you write the volumes of the next 3 cubes in the pattern.

Connect



When numbers are repeated in multiplication, we can write them in **exponent form**.

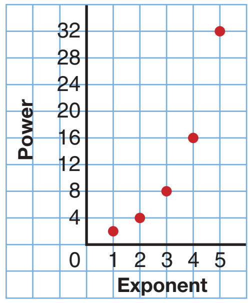
For example, we can write $2 \times 2 \times 2 \times 2$ as 2^4.
2 is the **base**.
4 is the **exponent**.
2^4 is the **power**.



We say: 2 to the power of 4, or
2 to the 4th
2^4 is a power of 2.

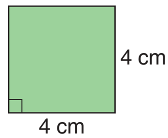
If we graph the power against the exponent, we see how quickly the power gets very large.

Exponent	Power
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
5	$2^5 = 32$



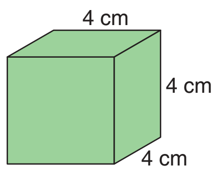
Square numbers and cube numbers are special powers.

➢ A power with exponent 2 is a **square number**.
The area of a square is side length × side length.
This square has side length 4 cm.
Area = 4 cm × 4 cm
= 16 cm^2



Here are 3 ways to write 16:
Standard form: 16
Expanded form: 4 × 4
Exponent form: 4^2
4^2 is a power of 4.
16 is called a **perfect square**.

➢ A power with exponent 3 is a **cube number**.
The volume of a cube is edge length × edge length × edge length.
This cube has edge length 4 cm.
Volume = 4 cm × 4 cm × 4 cm
= 64 cm^3



Here are 3 ways to write 64:
Standard form: 64
Expanded form: 4 × 4 × 4
Exponent form: 4^3
4^3 is a power of 4.
64 is called a **perfect cube**.

Example 1

Write in exponent form.

a) 6×6 b) $5 \times 5 \times 5$ c) 32

Solution

a) $6 \times 6 = 6^2$ b) $5 \times 5 \times 5 = 5^3$

c) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Example 2

Write in expanded form and standard form.

a) 3^5 b) 7^4

Solution

a) 3^5
$= 3 \times 3 \times 3 \times 3 \times 3$
$= 243$

b) 7^4
$= 7 \times 7 \times 7 \times 7$
$= 2401$

A calculator can be used to simplify a power such as 3^5.
For a scientific calculator, the keystrokes are:

3 ^ 5 ENTER or 3 y^x 5 ENTER to display 243

For a non-scientific calculator, use repeated multiplication.
The keystrokes are:

3 × = = = = to display 243

Practice

1. Write the base of each power.

a) 2^4 b) 3^2 c) 7^3 d) 10^5 e) 6^9 f) 8^3

2. Write the exponent of each power.

a) 2^5 b) 3^2 c) 7^1 d) 9^5 e) 8^{10} f) 10^4

3. Write in expanded form.

a) 2^4 b) 10^3 c) 6^5 d) 4^2 e) 2^1 f) 5^4

4. Write in exponent form.

a) $3 \times 3 \times 3 \times 3$ b) $2 \times 2 \times 2$ c) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

d) $10 \times 10 \times 10$ e) 79×79 f) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Calculator Skills

A case of 24 cans of pop costs $7.49 at a local grocery store. A variety store charges customers $1.25 for 1 can. How much more money is the variety store making?

5. Write in exponent form, then in standard form.

a) 5×5 b) $3 \times 3 \times 3 \times 3$
c) $10 \times 10 \times 10 \times 10 \times 10$ d) $2 \times 2 \times 2$
e) $9 \times 9 \times 9$ f) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

6. Write in standard form.

a) 2^4 b) 10^3 c) 3^5 d) 7^3 e) 2^8 f) 4^1

7. Write as a power of 10. How did you do it?

a) 100 b) 10 000 c) 100 000
d) 10 e) 1000 f) 1 000 000

8. Write as a power of 2. Explain your method.

a) 4 b) 16 c) 64 d) 256 e) 32 f) 2

9. What patterns do you see in the pairs of numbers? Which is the greater number in each pair? Explain how you know.

a) 2^3 or 3^2 b) 2^5 or 5^2 c) 3^4 or 4^3 d) 5^4 or 4^5

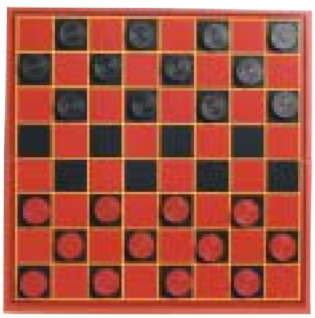
10. Write these numbers in order from least to greatest:
3^5, 5^2, 3^4, 6^3. How did you do this?

11. Simplify.

a) 3^{12} b) 7^3 c) 5^6 d) 4^8 e) 9^8 f) 2^{23}

12. **Assessment Focus**

a) Express each number in exponent form in as many different ways as you can.
i) 16 ii) 81 iii) 64
b) Find other numbers that can be written in exponent form, in more than one way. Show your work.



13. Write in exponent form:

a) the number of small squares on a checkerboard
b) the area of a square with side length 5 units
c) the volume of a cube with edge length 9 units

Reflect

When you see a number, how can you find out if it is a perfect square, or a perfect cube, or neither? Give examples.

Hospital Administrator

The administrator of a busy hospital makes hundreds of decisions every day, many of which involve whole number computations and conversions. Should she purchase more 'standard' 24-tray carts for the orderlies to deliver meals to the new 84-bed hospital wing? The administrator calculates she would need four of these carts to take enough meals to the new wing. In the cart supplier's catalogue, there are also 32-tray carts which are more expensive, but priced within budget. Only three of the 32-tray carts would deliver all the meals, and they would cost less than four of the standard carts.

When the orderlies look at the catalogue picture of the 32-tray cart, they tell the administrator that the cart is too low. It is tiring and slower to bend so far down, so they wouldn't use the lowest eight tray bins. So, it's back to the calculator for the administrator. Which cart would *you* choose now? Give reasons for your answer.

Math Link

Your World

Carpet and tile prices are given per square unit. A paint can label tells the area the paint will cover in square units. Wallpaper is sold in rolls with area in square units.

1.5 Number Patterns

Focus Describe, extend, and explain patterns.

Explore

Work on your own.
Blaise Pascal lived in France in the 17th century.
He was 13 years old when he constructed the triangle below.
This triangle is called Pascal's Triangle.

1	row 1
1 1	row 2
1 2 1	row 3
1 3 3 1	row 4
1 4 6 4 1	row 5
1 5 10 10 5 1	row 6

➢ What patterns do you see in the triangle?
➢ What symmetry do you see in the triangle?

Reflect & Share

Compare your patterns with those of a classmate.
Together, write about three different patterns you see in the triangle.

Connect

Here are some of the patterns in Pascal's Triangle.

➢ Each row begins and ends with 1.
After the second row, each number is the sum of the 2 numbers above it.
To write row 7:
Start with 1.
Add: $1 + 5 = 6$
Add: $5 + 10 = 15$
Add: $10 + 10 = 20$, and so on

	Sum
1	1
1 1	2
1 2 1	4
1 3 3 1	8
1 4 6 4 1	16
1 + 5 + 10 + 10 + 5 + 1	32
1 6 15 20 15 6 1	64

➢ The sum of the numbers in each row is shown above, and in the table on the next page.
From the 2nd row on, the sums can be written as powers.

Row	2	3	4	5	6	7
Sum in standard form	2	4	8	16	32	64
Sum in exponent from	2^1	2^2	2^3	2^4	2^5	2^6

We can use this table to predict the sum of the numbers in any row. All sums are powers of 2.
The exponent is 1 less than the row number.
So, the 10th row has sum: $2^9 = 512$
And the 19th row has sum: $2^{18} = 262\ 144$

➢ The 3rd numbers in each row have this pattern: 1, 3, 6, 10, 15, ...
To get each term in the pattern, we add 1 more than we added before. We can use this to extend the pattern.
The 5th term: 15
The 6th term: $15 + 6 = 21$
The 7th term: $21 + 7 = 28$, and so on

Example

Describe each pattern in words. Write the next 3 terms.

a) 4, 9, 14, 19, … **b)** 1, 3, 9, 27, … **c)** 1, 3, 7, 13, 21, …

Solution

a) 4, 9, 14, 19, …
Start at 4.
Add 5 to get the next number.
The next 3 terms are 24, 29, 34.

b) 1, 3, 9, 27, …
Start at 1.
Multiply by 3 to get the next number.
The next 3 terms are 81, 243, 729.

c) 1, 3, 7, 13, 21, …
Start at 1.
Add 2.
Increase the number added by 2 each time.
The next 3 terms are 31, 43, 57.

Practice

1. Write the next 3 terms in each pattern.

a) 7, 9, 11, 13, …
b) 1, 5, 25, 125, …
c) 4, 7, 10, 13, …
d) 1, 10, 100, 1000, …
e) 20, 19, 18, 17, …
f) 79, 77, 75, 73, …

2. Write the next 3 terms in each pattern.

a) 3, 4, 6, 9, …
b) 1, 4, 9, 16, …
c) 101, 111, 121, 131, …
d) 1, 12, 123, 1234, …
e) 1, 4, 16, 64, …
f) 256, 128, 64, 32, …

3. Describe each pattern in words.
Write the next 3 terms.

a) 200, 199, 201, 198, …
b) 4, 7, 12, 19, …
c) 100, 99, 97, 94, …
d) 2, 6, 12, 20, …
e) 50, 48, 44, 38, …
f) 2, 6, 18, 54, …

4. Create your own number pattern. Trade patterns with a classmate. Describe your classmate's pattern.
Write the next 3 terms.

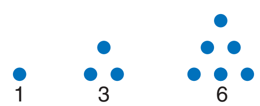
5. a) Copy this pattern. Find each product.

$99 \times 11 = \square$	$99 \times 111 = \square$	…
$99 \times 22 = \square$	$99 \times 222 = \square$	…
$99 \times 33 = \square$	$99 \times 333 = \square$	…
⋮	⋮	

b) Extend this pattern sideways and down.
Predict the next 6 terms in each row and column.

c) Check your predictions with a calculator.

6. This pattern shows the first 3 triangular numbers.



a) Draw the next 3 terms in the pattern.
b) List the first 6 triangular numbers.
c) Find the next 2 triangular numbers without drawing pictures.
Explain how you did this.

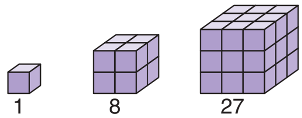
Number Strategies

One can of pop contains 355 mL. About how many 2-L pop bottles can be filled with one case of 24 cans of pop?

d) Add consecutive triangular numbers;
that is, Term 1 + Term 2; Term 2 + Term 3; and so on.
What pattern do you see?
Write the next 3 terms in this pattern.

e) Subtract consecutive triangular numbers;
that is, Term 2 − Term 1; Term 3 − Term 2; and so on.
What pattern do you see?
Write the next 3 terms in this pattern.

7. This pattern shows the first 3 cube numbers.



a) Sketch the next 3 cube numbers in the pattern.
Use interlocking cubes if they help.

b) Write the next 3 cube numbers without drawing pictures.
Explain how you did this.

8. Assessment Focus

a) Write the first 10 powers of 2; that is, 2^1 to 2^{10}, in standard form.

b) What pattern do you see in the units digits?

c) How can you use this pattern to find the units digit of 2^{40}?

d) Investigate powers of other numbers.
Look for patterns in the units digits.
Explain how you can use these patterns to find units digits for powers too large to display on the calculator.

Take It Further

9. Some sequences of numbers may represent different patterns.
Extend each pattern in as many different ways as you can.
Write the pattern rule for each pattern.

a) 1, 2, 4, … b) 1, 4, 9, … c) 5, 25, …

Reflect

Choose 3 different types of patterns from this section.
Describe each pattern.
Explain how you can use the pattern to predict the next term.

Using Different Strategies

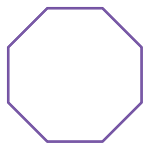
Problem

There are 8 people at a party.
Each person shakes hands with everyone else.
How many handshakes are there?

Think of a strategy

Strategy 1: **Draw a diagram and count.**

Draw 8 dots. Join every dot to every other dot.
Count the line segments.



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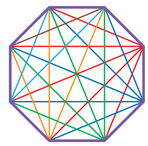
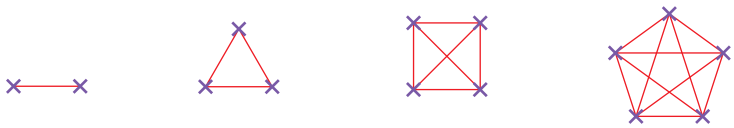


5 + 5 + 4 + 3 + 2 + 1

Number of line segments
$= 8 + 5 + 5 + 4 + 3 + 2 + 1$
$= 28$

There are 28 handshakes.

Strategy 2: **Solve simpler problems, then look for a pattern.**



2 people
1 handshake

3 people
3 handshakes

4 people
6 handshakes

5 people
10 handshakes

Make a table.
Each time you add a person,
you add one more
handshake than the time before.
So, 6 people: 10 + 5, or 15 handshakes
7 people: 15 + 6, or 21 handshakes
8 people: 21 + 7, or 28 handshakes

Number of People	Number of Handshakes	
2	1	
3	3	+2
4	6	+3
5	10	+4

Strategy 3: **Use reasoning.**

Each of 8 people shakes hands with
7 other people.
That is, 8×7, or 56 handshakes
But we have counted each handshake twice.
We have said that A shaking hands with B is different from B shaking hands with A.
So, we divide by 2: $\frac{56}{2} = 28$
There are 28 handshakes.

Look back

- Look at the numbers of handshakes for 2 to 8 people.
  Where have you seen this pattern before?
- What if there were 9 people at the party?
  How many handshakes would there be?
  How do you know?

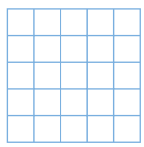
Problems

Solve each problem. Try to use more than one strategy.

1. A ball is dropped from a height of 16 m.
Each time it hits the ground, the ball bounces to one-half its previous height. The ball is caught when its greatest height for that bounce is 1 m. How far has the ball travelled?

2. A rectangular garden is 100 m long and 44 m wide.
A fence encloses the garden.
The fence posts are 2 m apart.
How many posts are needed?

3. Here is a 5 by 5 square.
How many squares of each different size can you find in this large square?



Reflect

Why might you want to solve a problem more than one way?

Unit Review

Review any lesson with etext online tutorial

What Do I Need to Know?

- ✓ A *factor* of a number divides into the number exactly; that is, there is no remainder.
  For example, $6 \div 2 = 3$, so 2 is a factor of 6.

- ✓ A *prime number* has only 2 factors, itself and 1.
  For example, the only factors of 17 are 17 and 1, so 17 is a prime number.

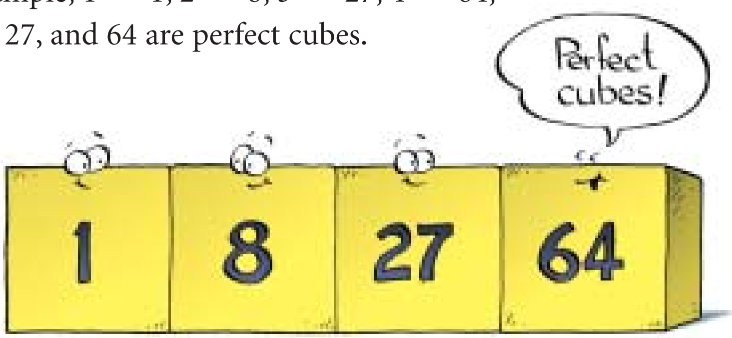
- ✓ A *composite number* has more than 2 factors.
  For example, 12 has factors 1, 2, 3, 4, 6, and 12, so 12 is a composite number.

- ✓ A *square number*, or *perfect square*, has an odd number of factors.
  It can also written as a power with exponent 2.
  For example, the factors of 9 are 1, 3, and 9, so 9 is a perfect square.
  We write $9 = 3^2$.

- ✓ A *square root* of a number is a factor that is squared to get the number.
  For example, 9 is a square root of 81 because $9^2 = 81$.
  We write $\sqrt{81} = 9$.

- ✓ When a number is written in *exponent form*, it is written as a *power*.
  For example, for the power 5^3:
  5 is the *base*.
  3 is the *exponent*.
  $5 \times 5 \times 5$ is the *expanded form*.
  125 is the *standard form*.

- ✓ A *cube number*, or *perfect cube*, is a power with exponent 3.
  For example, $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$,
  so 1, 8, 27, and 64 are perfect cubes.



What Should I Be Able to Do?

For extra practice, go to page 438.

LESSON

1.1

1. Find each answer.
Use pencil and paper.
a) 3621 + 8921
b) 5123 − 4123
c) 35 × 12
d) 125 × 27
e) 815 + 642 − 85
f) 1638 ÷ 21

2. This table shows the highest all-time scorers at the end of the 2000–2001 NBA season.

Kareem Abdul-Jabbar	38 387
Karl Malone	32 919
Wilt Chamberlain	31 419
Michael Jordan	29 277

a) What is the total number of points?
b) Write a problem about these data. Solve your problem. Justify the strategy you used.

3. **a)** Write the number 300 as the sum of 2 or more consecutive whole numbers. Find as many ways to do this as you can.
b) What patterns do you see in the numbers added?
c) Suppose you started with another 3-digit number. Will you see similar patterns? Investigate to find out.

4. Solve each problem. State any assumption you made.
a) Armin's house is 3 km from a mall. He walks 1 km in 15 min. How long does it take Armin to walk to the mall?
b) Tana makes $15, $21, and $19 for baby-sitting one weekend. How much will Tana make in a month?

5. The table shows the ticket prices and number of tickets sold for a popular movie.

	Ticket Price ($)	Number of Tickets Sold
Adults	12	125
Seniors	10	34
Youths	8	61

Calculate the total cost of the ticket sales.

1.2

6. Find all the factors of each number.
a) 36 **b)** 50
c) 75 **d)** 77

7. Find the first 10 multiples of each number.
a) 9 **b)** 7
c) 12 **d)** 15

8. For the numbers 18 and 60, find:
a) the GCF **b)** the LCM
Draw a Venn diagram to illustrate part a.

LESSON

9. How many prime numbers are even?
Justify your answer.

1.3 **10.** Find a square root of each number.
a) 121 **b)** 169 **c)** 225

11. Find each square root.
Draw a picture if it helps.
a) $\sqrt{25}$ **b)** $\sqrt{100}$ **c)** $\sqrt{81}$

12. Calculate the area of a square with each side length.
a) 7 cm **b)** 17 cm **c)** 93 m

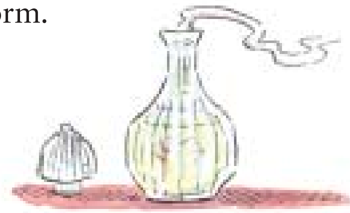
13. The area of a square is 81 m^2.
What is the perimeter of the square? How do you know?

14. Raquel cooks 8-cm square hamburgers on a grill.
The grill is a rectangle with dimensions 40 cm by 40 cm.
How many hamburgers can be grilled at one time?
Justify your answer.

1.4 **15.** Copy and complete this table.

	Exponent Form	Base	Exponent	Expanded Form	Standard Form
a)	3^4				
b)	2^5				
c)	10^7				
d)		5	4		
e)				$4 \times 4 \times 4 \times 4$	

1.5 **16.** A perfume formula requires 4 g of an essential oil per bottle.
a) How many grams are needed for 2500 bottles?
b) Write this number in exponent form.



17. Write these numbers in order from greatest to least.
3^4, 4^4, 5^3, 2^6

18. **a)** Write the next 3 terms in each pattern.
i) 3, 5, 6, 8, 9, …
ii) 1, 2, 4, 8, …
iii) 1, 4, 9, 16, …
iv) 3, 4, 6, 9, …
b) Describe each pattern in part a.

19. **a)** Copy and complete this pattern.
$1^2 + 2^2 = \square$
$2^2 + 3^2 = \square$
$3^2 + 4^2 = \square$
$4^2 + 5^2 = \square$
b) Write the next two rows in the pattern.
c) Describe the pattern.

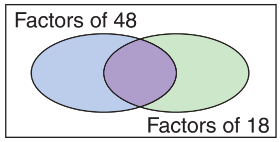
20. $1^2 = 1$
$1^2 + 2^2 = 5$
$1^2 + 2^2 + 3^2 = 14$
$1^2 + 2^2 + 3^2 + 4^2 = 30$
a) Write the next two lines in the pattern.
b) What pattern do you see?

Practice Test

1. Estimate. Describe your strategy.
   a) $624 + 1353$ b) $897 \div 23$ c) 752×36

2. Use mental math to evaluate $2 \times 395 \times 5$. Explain your strategy.

3. Use a Venn diagram to show the factors of 48 and 18. Circle the GCF.



4. Use patterns to find the first 6 common multiples of 15 and 6.

5. Use the clues below to find the mystery number. Explain your strategy and reasoning.
   Clue 1: I am a 2-digit number.
   Clue 2: I am less than 9^2.
   Clue 3: I have 26 and 6 as factors.

6. Sharma plays basketball every third day of the month. She baby-sits her little brother every seventh day of the month. How many times in a month will Sharma have a conflict between basketball and baby-sitting? Explain your thinking.

7. Write these numbers in order from least to greatest.
   a) $5^2, 2^5, \sqrt{25}, 10^2, 3^3$ b) $10 \times 10 \times 10, 2^3, \sqrt{400}, 3^2, 17$

8. The perimeter of a square is 32 cm. What is the area of the square? Explain your thinking. Include a diagram.

9. Write the next 3 terms in each pattern. Describe each pattern.
   a) 1, 3, 6, 10, … b) 23, 25, 27, … c) 100, 81, 64, 49, …

10. Write the number 35 as:
    a) the sum of 3 squares
    b) the difference between 2 squares
    c) the sum of a prime number and a square

Unit Problem

Fibonacci Numbers

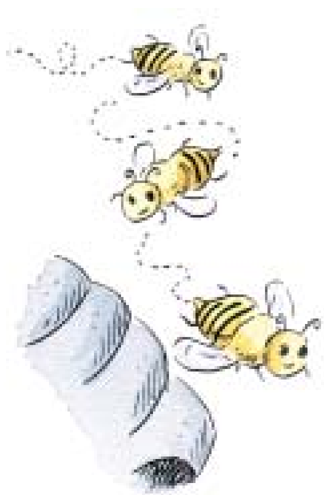
One of greatest mathematicians of the Middle Ages was an Italian, Leonardo Fibonacci.

Fibonacci is remembered for this problem:

A pair of rabbits is placed in a large pen.
When the rabbits are two months old, they produce another pair of rabbits.
Every month after that, they produce another pair of rabbits.
Each new pair of rabbits does the same.
None of the rabbits dies.
How many rabbits are there at the beginning of each month?

This table shows the rabbits at the beginning of the first 5 months.

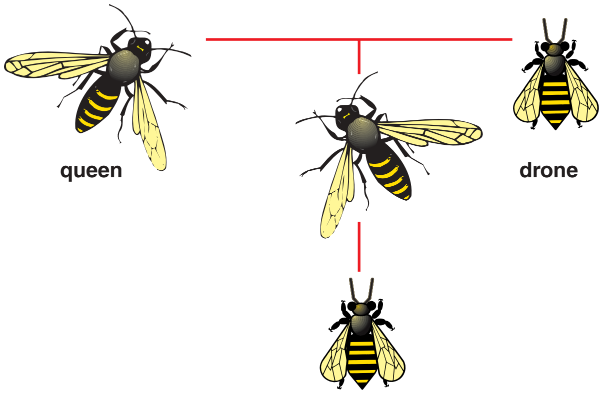
Beginning of Month	Number of Pairs	Number of Rabbits
1	1	
2	1	
3	2	
4	3	
5	5	



1. **a)** Continue the pattern for two more months.
Use different colours to show the new rabbits.

b) Write the number of pairs of rabbits at the beginning of each month, for the first 7 months.
These are the Fibonacci numbers.
What pattern do you see?
Explain how to find the next number in the pattern.

2. The Fibonacci sequence appears in the family tree of the drone, or male bee.
The drone has a mother, but no father.
Female bees are worker bees or queens.
They have a mother and a father.
The family tree of a male bee back to its grandparents is shown on the next page.



Check List

Your work should show:

- ✓ how you used patterns to find your answers
- ✓ all diagrams and charts, clearly presented
- ✓ a clear explanation of your results
- ✓ your understanding of Fibonacci numbers

a) Copy the diagram. Trace the male bee's ancestors back 5 more generations.

b) Explain how you can use this pattern to find the Fibonacci numbers.

There are many patterns you can find in the Fibonacci sequence.

3. Write the first 15 Fibonacci numbers.

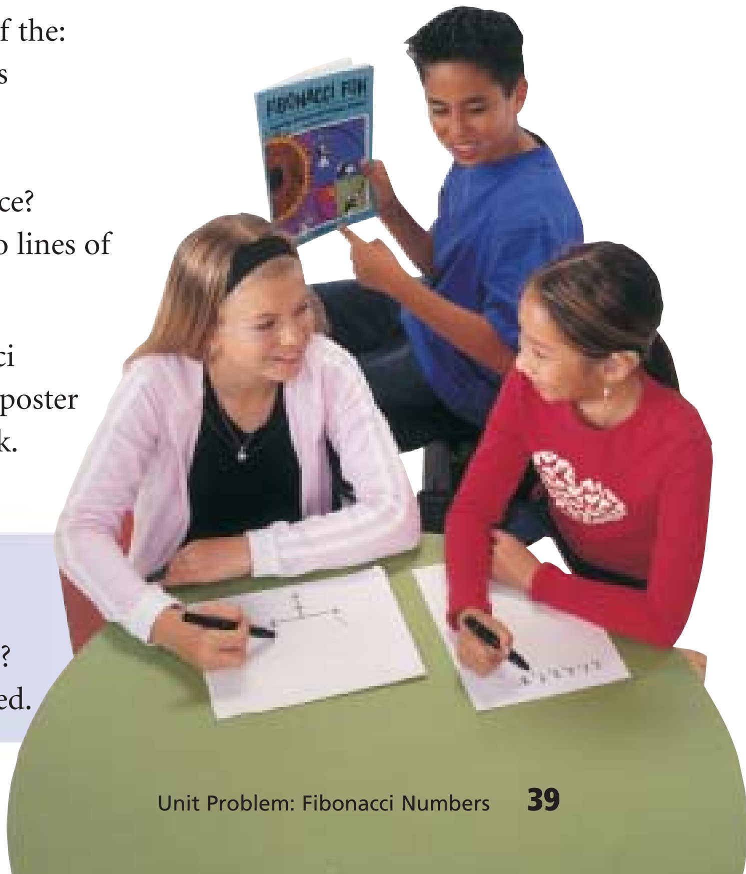
a) What type of number is every third number?

b) Which number is a factor of every fourth number?

c) Which number is a factor of every fifth number?

4. Add the squares of the:
2nd and 3rd terms
3rd and 4th terms
4th and 5th terms
What do you notice?
Write the next two lines of this pattern.

5. Research Fibonacci numbers. Make a poster to show your work.



Reflect on the Unit

What have you learned about whole numbers?
What have you learned about number patterns?
Write about some of the things you have learned.

UNIT 2

Ratio and Rate

The animal kingdom provides much interesting information. We use information to make comparisons. What comparisons can you make from these facts?

A sea otter eats about $\frac{1}{3}$ of its body mass a day.

A Great Dane can eat up to 4 kg of food a day.

A cheetah can reach a top speed of 110 km/h.

A human can run at 18 km/h.

The heart of a blue whale is the size of a small car.

One in 5000 North Atlantic lobsters is born bright blue.

What You'll Learn

- Understand what a ratio is.
- Find equivalent ratios.
- Compare ratios and use them to solve problems.
- Understand what a rate is.
- Find unit rates.
- Compare rates and use them to solve problems.

Why It's Important

You use ratios and rates to compare numbers and quantities; and to compare prices when you shop.

Key Words

- ratio
- part-to-whole ratio
- part-to-part ratio
- terms of a ratio
- equivalent ratios
- simplest form
- rate
- unit rate
- average speed

UNIT 2

Skills You'll Need

Greatest Common Factor

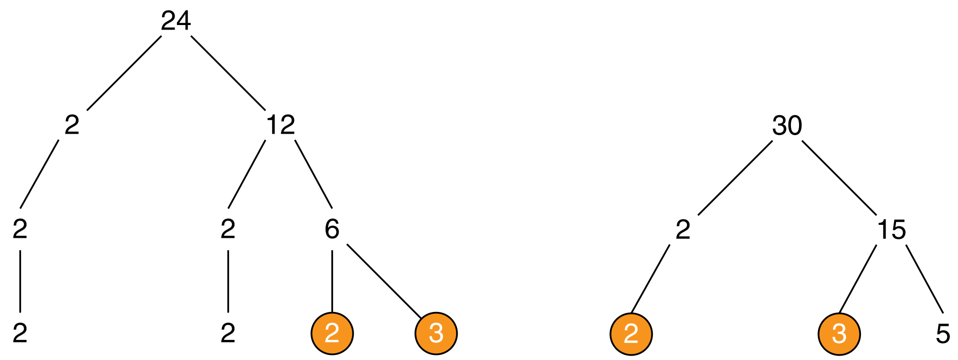
Recall that the greatest common factor (GCF) of a set of numbers is the greatest number that will divide exactly into the given numbers.
For example, 6 is the greatest common factor of 18 and 24.

Example 1

Find the GCF of 24 and 30.

Solution

Draw a factor tree for each number.



Circle the numbers that appear in the bottom row of both factor trees.
2 and 3 are common to both factor trees.
The GCF of 24 and 30 is $2 \times 3 = 6$.

✓ Check

1. Find the GCF of the numbers in each set.

a) 30, 75 **b)** 27, 63 **c)** 42, 56 **d)** 12, 18, 42

Lowest Common Multiple

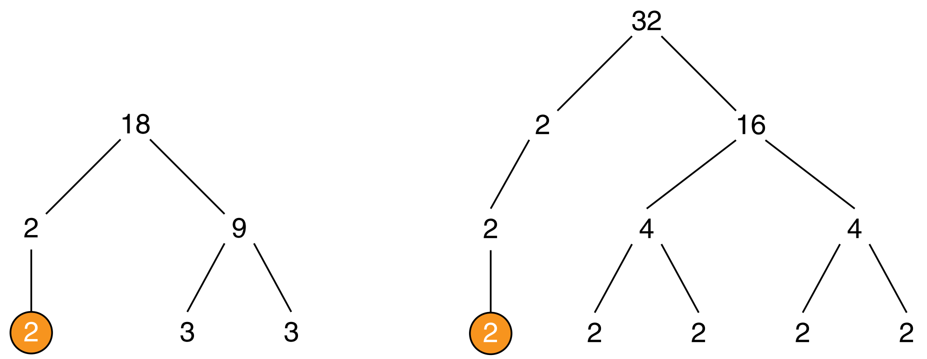
Recall that the lowest common multiple (LCM) of a set of numbers is the least number that is a multiple of each number in the set. This also means that each number in the set is a factor of the lowest common multiple.

Example 2

Find the LCM of 18 and 32.

Solution

Draw a factor tree for each number.



Circle the numbers that appear in the bottom row of both factor trees.
Multiply the numbers not circled and one each of the circled numbers:
$3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 = 288$
The LCM of 18 and 32 is 288.

✓ Check

2. Write the first 6 multiples of each number.

a) 4 **b)** 7 **c)** 9 **d)** 12

3. Find the LCM of the numbers in each pair.

a) 9, 12 **b)** 14, 35 **c)** 16, 40

4. Find the LCM of the numbers in each set.

a) 36, 45 **b)** 3, 4, 6 **c)** 12, 15, 20

Converting among Metric Units

100 cm = 1 m   1000 m = 1 km
1000 g = 1 kg   1000 mL = 1 L

- To convert centimetres to metres, divide by 100.
- To convert:
  metres to kilometres
  grams to kilograms
  millilitres to litres
  } Divide by 1000.

Divide to convert to a larger unit.

- To convert metres to centimetres, multiply by 100.
- To convert:
  kilometres to metres
  kilograms to grams
  litres to millilitres
  } Multiply by 1000.

Multiply to convert to a smaller unit.

Example 3

Convert.

a) 650 cm to metres   **b)** 82 km to metres
c) 2.4 kg to grams   **d)** 2840 mL to litres

Solution

a) 650 cm $= \frac{650}{100}$ m
$= 6.5$ m

b) 82 km $= 82 \times 1000$ m
$= 82\ 000$ m

c) 2.4 kg $= 2.4 \times 1000$ g
$= 2400$ g

d) 2840 mL $= \frac{2840}{1000}$ L
$= 2.84$ L

✓ Check

5. Convert.

a) 1280 cm to metres   **b)** 680 m to kilometres
c) 2454 g to kilograms   **d)** 1987 mL to litres
e) 8.2 m to centimetres   **f)** 1.25 km to metres
g) 0.45 kg to grams   **h)** 2.3 L to millilitres

2.1 What Is a Ratio?

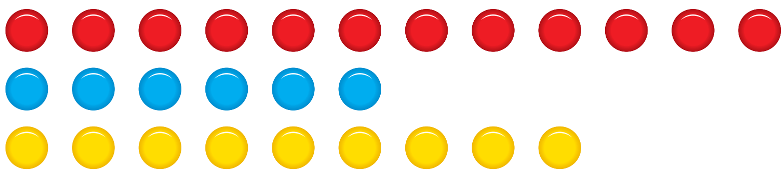
Focus Use models and diagrams to investigate ratios.

There are different ways to compare numbers.
Look at these advertisements.

How are the numbers in each advertisement compared?
Which advertisement is most effective? Explain.

Explore

Work with a partner.



How can you compare the number of blue counters to the number of yellow counters?
How many different ways can you compare the counters?
Write each way you find.

Reflect & Share

Share your list with another pair of classmates.
Add any new comparisons to your list.
Talk about the different ways you compared the counters.

Connect

Here is a collection of models.

This is a **part-to-whole** ratio.

➢ We can use a **ratio** to compare one part of the collection to the whole collection.
There are 9 cars compared to 13 models.
The ratio of cars to models is written as 9 to 13 or 9:13.

This is a **part-to-part** ratio.

➢ We can use a ratio to compare one part of the collection to another part.
There are 9 cars compared to 4 planes.
The ratio of cars to planes is written as 9 to 4 or 9:4.
9 and **4** are called the **terms** of the ratio.
9 is the first term and 4 is the second term.

Example

At a class party, there are 16 boys, 15 girls, and 4 adults.
What is each ratio?

a) boys to girls **b)** girls to adults
c) adults to total number of people at the party

Solution

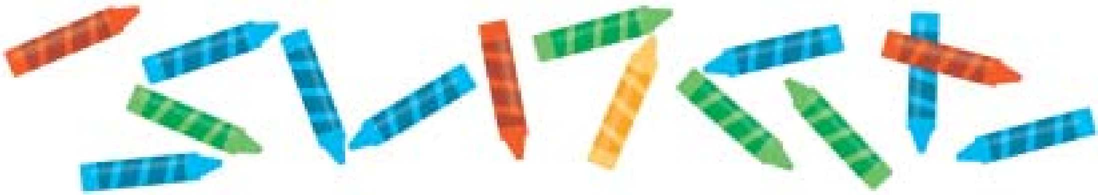
a) There are 16 boys and 15 girls.
So, the ratio of boys to girls is 16:15.

b) There are 15 girls and 4 adults.
So, the ratio of girls to adults is 15:4.

c) The total number of people is $16 + 15 + 4 = 35$.
So, the ratio of adults to total number of people is 4:35.

Practice

1. Look at the crayons below. Write each ratio.
   a) red crayons to the total number of crayons
   b) yellow to the total number of crayons
   c) blue crayons to green crayons

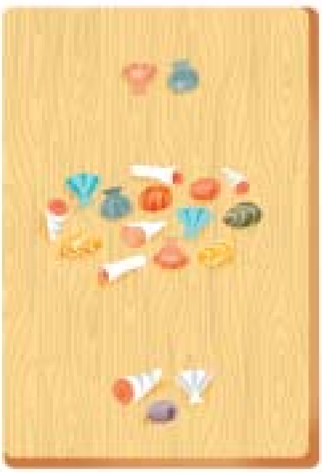


2. Use words, numbers, or pictures.
   Write a ratio to compare the items in each sentence.
   a) A student had 9 green counters and 7 red counters on his desk.
   b) In a dance team, there were 8 girls and 3 boys.
   c) The teacher had 2 fiction and 5 non-fiction books on her desk.

3. The ratio of T-shirts to shorts in Frank's closet is 5:2.
   Write the ratio of T-shirts to the total number of garments.

4. a) What is the ratio of boys to girls in your class?
   b) What is the ratio of girls to boys?
   c) What is the ratio of boys to the total number of students in your class?
   d) What if two boys leave the room?
   What is the ratio in part c now?

5. a) Draw two different diagrams to show the ratio 3:5.
   b) Draw a diagram to show the ratio 7:1.



6. Maria shares some seashells with Jeff.
   Maria says, "Two for you, three for me, two for you, three for me …"
   Tonya watches.
   At the end, she says, "So Jeff got $\frac{2}{3}$ of the shells."
   Do you agree with Tonya? Give reasons for your answer.

Number Strategies

What is the sum of all the prime numbers between 1 and 30?

7. A box contains 8 red, 5 green, 2 brown, 3 purple, 1 blue, and 6 yellow candles.

a) Write each ratio.

i) red:purple **ii)** green:blue

iii) purple:green **iv)** brown and yellow:total candles

b) What if 3 red, 2 green, and 4 yellow candles were burned? Write the new ratios for part a.

8. **Assessment Focus** Patrick plans to make salad. The recipe calls for 3 cups of cooked macaroni, 3 cups of sliced oranges, 2 cups of chopped apple, 1 cup of chopped celery, and 2 cups of mayonnaise.

a) What is the total amount of ingredients?

b) What is the ratio of oranges to apples? Mayonnaise to macaroni?

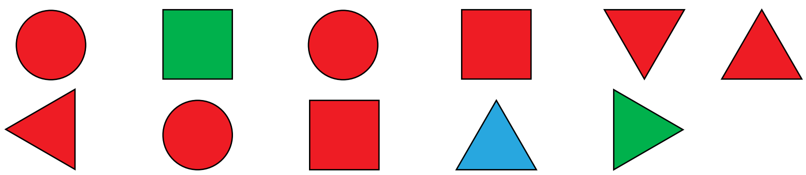
c) What is the ratio of apples and oranges to the total amount of ingredients?

d) Patrick makes a mistake. He uses 2 cups of oranges instead of 3. What are the new ratios in parts b and c?

e) Write your own ratio problem about this salad. Solve your problem.

Take It Further

9. **a)** Create four different ratios using these figures.



b) How can you change one figure to create ratios of 2:5 and 7:3? Explain.

Reflect

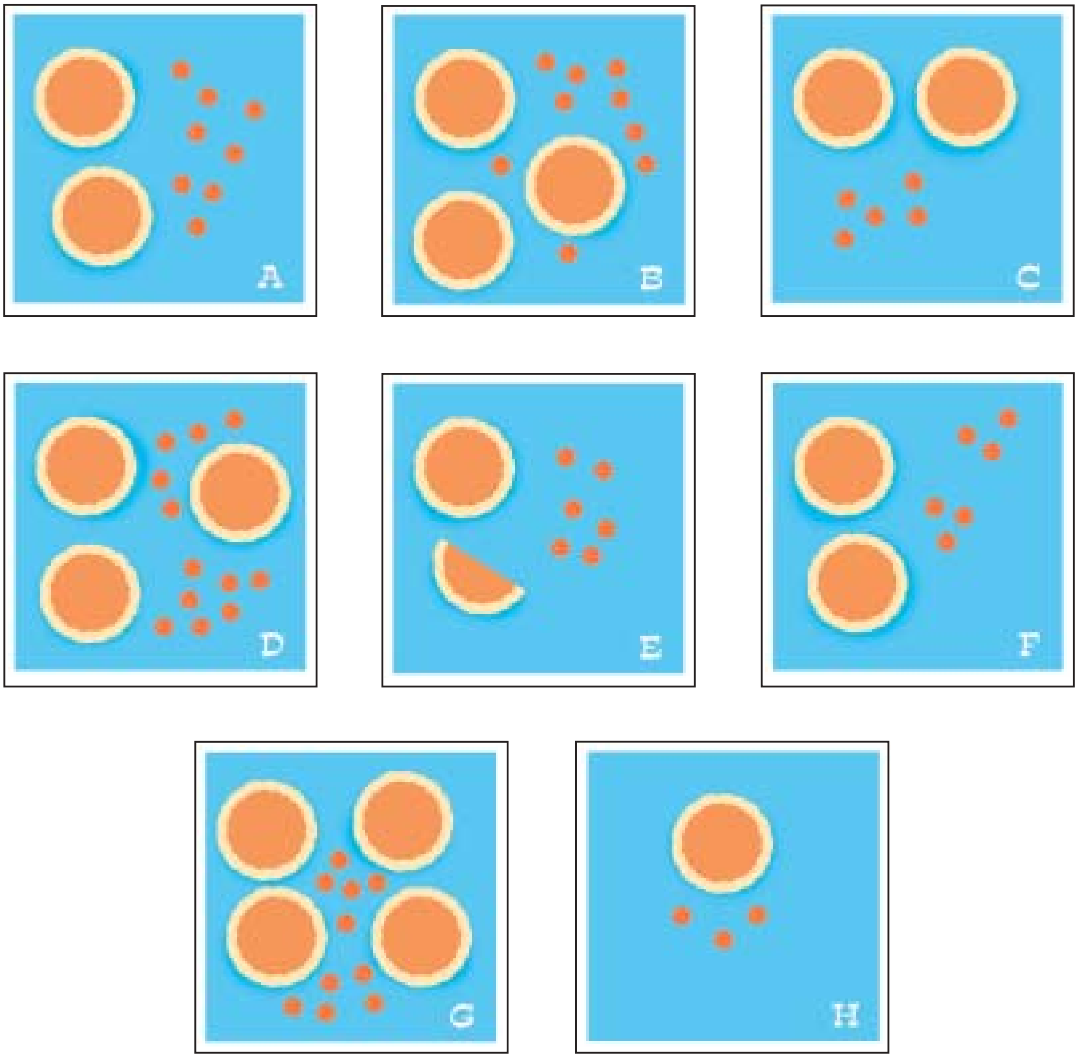
Look in newspapers and magazines for examples of ratios.
Cut out the examples. Paste them in your notebook.
Explain how the ratios are used.
What information can you get from them?

2.2 Equivalent Ratios

Focus Write equivalent ratios.

Explore

Work on your own.
Which cards have the same ratio of pepperoni pieces to pizzas?

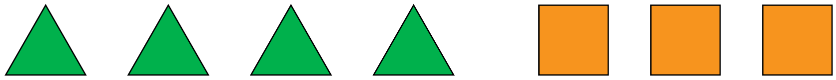


Reflect & Share

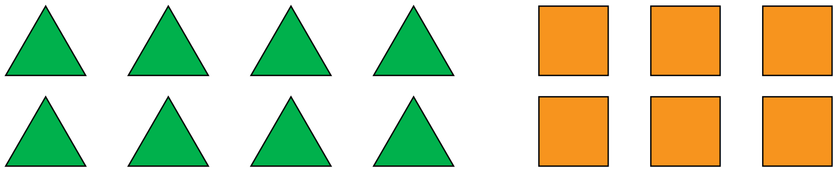
Share your answers with a classmate.
Why do you think your answers are correct?
What patterns do you see?

Connect

A ratio of 4:3 means that, for every 4 triangles, there are 3 squares.



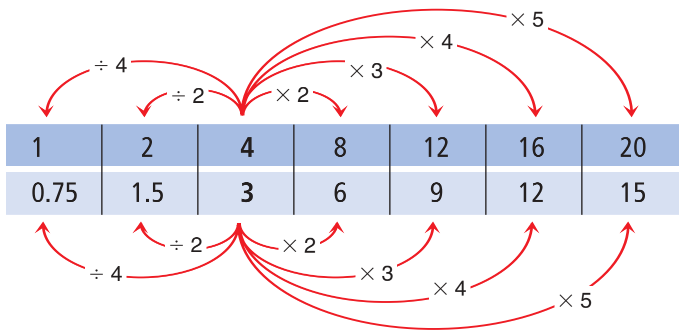
A ratio of 8:6 means that, for every 8 triangles, there are 6 squares.



The ratios 8:6 and 4:3 are called **equivalent ratios**.
Equivalent ratios are equal. 8:6 = 4:3

➢ An equivalent ratio can be formed by multiplying or dividing the terms of a ratio by the same number.

Note that
3 ÷ 2 is $\frac{3}{2}$ = 1.5
and
3 ÷ 4 is $\frac{3}{4}$ = 0.75



1	2	**4**	8	12	16	20
0.75	1.5	**3**	6	9	12	15

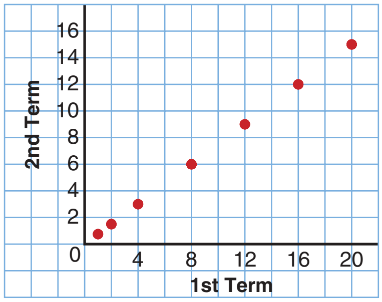
The equivalent ratios are:
1:0.75; 2:1.5; 4:3; 8:6; 12:9; 16:12; 20:15

➢ The equivalent ratios can be shown on a grid.

1st term	1	2	4	8	12	16	20
2nd term	0.75	1.5	3	6	9	12	15

The points representing the ratios lie on a straight line.

Graph of Equivalent Ratios



A ratio is in simplest form when its terms have no common factors.

➢ When we divide the terms in a ratio by their greatest common factor, we write the ratio in **simplest form**.

To write 24:16 in simplest form:

24:16 The GCF is 8. Divide by 8.

$= (24 \div 8):(16 \div 8)$

$=$ 3:2

So, 24:16 and 3:2 are equivalent ratios.

The ratio 3:2 is in simplest form.

Example

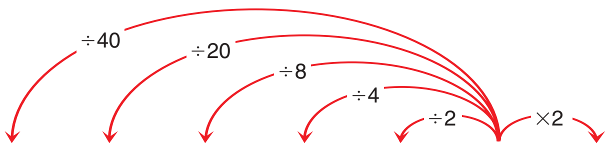
Construction kits come in different sizes.
The Regular Kit contains 120 long rods, 80 short rods, and 40 connectors.

a) What other kits could be created with the same ratio of rods and connectors?

b) One kit has 10 connectors. How many short and long rods does it have?

Solution

a) Use a table to find equivalent ratios.
Label each new kit.



Component	Kit A	Kit B	Kit C	Kit D	Kit E	Regular Kit	Kit F
Long Rods	3	6	15	30	60	120	240
Short Rods	2	4	10	20	40	80	160
Connectors	1	2	5	10	20	40	80

b) Use the table in part a.
The kit with 10 connectors is Kit D.
It has 20 short rods and 30 long rods.

Practice

Number Strategies

Find the greatest common factor of the numbers in each set.

- 16, 40, 24
- 33, 77, 88
- 45, 75, 30
- 150, 75, 225

1. Write three ratios equivalent to each ratio.
   Use tables to show your work.
   a) 3:4 **b)** 14:4

2. Rewrite each sentence as a ratio statement in simplest form.
   a) In a class, there are 15 girls and 12 boys.
   b) In a parking lot, there were 4 American cars and 12 Japanese cars.
   c) A paint mixture is made up of 6 L of blue paint and 2 L of white paint.
   d) A stamp collection contains 12 Canadian stamps and 24 American stamps.

3. Name the pairs of equivalent ratios:
   2:3, 9:12, 8:5, 1:2, 2:1, 16:10, 3:6, 6:9, 5:8, 3:4
   Tell how you know they are equivalent.

4. In a class library, 3 out of 4 books are non-fiction.
   The rest are fiction.
   a) How many non-fiction books could there be?
   How many fiction books?
   b) How many different answers can you find for part a?
   Which answers are reasonable? Explain.

5. The official Canadian flag has a length to width ratio of 2:1.
   Doreen has a sheet of paper that measures 30 cm by 20 cm.
   What are the length and width of the largest Canadian flag Doreen can draw? Sketch a picture of the flag.

6. **Assessment Focus** Use red, blue, and green counters.
   Make a set of counters with these two ratios:
   red:blue = 5:6 blue:green = 3:4
   How many different ways can you do this?
   Record each way you find.

Reflect

Choose a ratio. Use pictures, numbers, or words to show how to find two equivalent ratios.

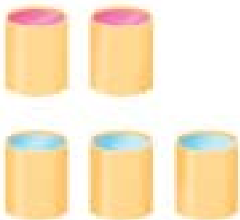
2.3 Comparing Ratios

Focus Use equivalent ratios to compare ratios.

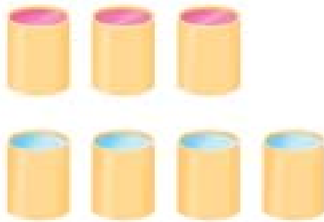
Explore

Work with a partner.

Recipe A for punch calls for 2 cans of concentrate and 3 cans of water.



Recipe B for punch calls for 3 cans of concentrate and 4 cans of water.



In which recipe is the punch stronger?
Or are the drinks the same?
Explain how you know.

Reflect & Share

Compare your answer with that of another pair of classmates.
Compare strategies.
If your answers are the same, which strategy do you prefer? Would there be a situation when the other strategy would be better? Explain.
If your answers are different, find out which is correct.

Connect

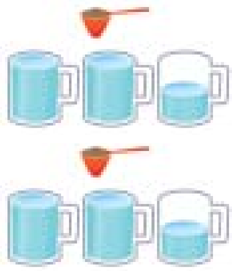
Erica makes her coffee with 2 scoops of coffee to 5 cups of water.

Jim makes his coffee with 3 scoops of coffee to 7 cups of water.

Here are two ways to find out which coffee is stronger.

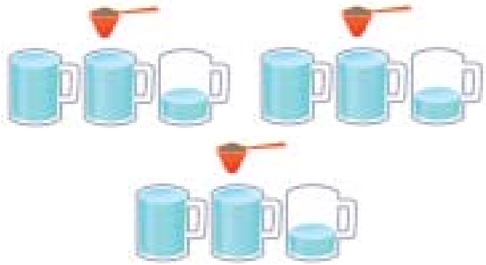
➢ Find how much water is used for 1 scoop of coffee.

Erica



1 scoop of coffee to
$2\frac{1}{2}$ cups of water.

Jim



1 scoop of coffee to
$2\frac{1}{3}$ cups of water.

Since $2\frac{1}{3}$ is less than $2\frac{1}{2}$,
Jim uses less water to 1 scoop of coffee.
So, Jim's coffee is stronger.

➢ Find how much coffee is used for the same amount of water.
Write each mixture as a ratio.
Write each ratio with the same second term,
then compare the first terms.
Use equivalent ratios.

Erica

2:5
4:10 ($\times$ 2)
6:15 ($\times$ 3)
8:20 ($\times$ 4)
10:25 ($\times$ 5)
12:30 ($\times$ 6)
14:35 ($\times$ 7)

Jim

3:7
6:14 ($\times$ 2)
9:21 ($\times$ 3)
12:28 ($\times$ 4)
15:35 ($\times$ 5)

Since 2:5 = 14:35,
Erica uses 14 scoops of
coffee to 35 cups of water.

Since 3:7 = 15:35,
Jim uses 15 scoops of
coffee to 35 cups of water.

Jim uses more coffee. So, Jim's coffee is stronger.
Notice that, as we multiply to get equivalent ratios,
we get multiples of the terms of the ratios.
That is, the first terms in Erica's equivalent ratios are multiples of 2;
the second terms are multiples of 5.

A quicker way to write each ratio with the same second term is to find the lowest common multiple of the second terms.
That is, the lowest common multiple of 5 and 7 is 35.

Example

At the outdoor centre, there were the same numbers of boys and girls. Five out of every 8 boys wanted to kayak. Two out of every 3 girls wanted to kayak. Do more boys than girls want to kayak? Explain.

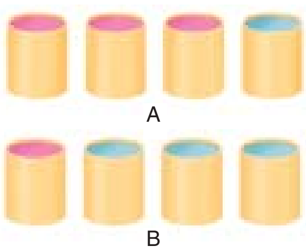
Solution

Five out of every 8 boys want to kayak.
This is a ratio of 5:8.
Two out of every 3 girls want to kayak.
This is a ratio of 2:3.
To compare the ratios, write them with the second terms the same.
The lowest common multiple of 8 and 3 is 24.
Multiply to make the second term of each ratio 24.

Boys	**Girls**
5:8	2:3
$= (5 \times 3):(8 \times 3)$	$= (2 \times 8):(3 \times 8)$
$=$ 15:24	$=$ 16:24
15 out of 24 boys want to kayak	16 out of 24 girls want to kayak

Since 16 is greater than 15, more girls want to kayak.

Practice



1. The concentrate and water in each picture are mixed.
Which mixture is stronger: A or B?
Draw a picture to show your answer.

2. Two boxes contain pictures of hockey and basketball players.
In one box, the ratio of hockey players to basketball players is 4:3.
In the other box, the ratio is 3:2.
The boxes contain the same number of pictures.
a) What could the total number of pictures be?
b) Which box contains more pictures of hockey players?
Draw a picture to show your answer.

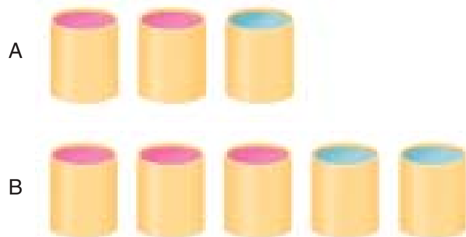
Number Strategies

There are 10 coins that total $0.60.
What are the coins?

3. In a basketball game, Alison made 6 of 13 free shots. Madhu made 5 of 9 free shots. Who played better? Explain.

4. The principal is deciding which shade of blue to have the classrooms painted. One shade of blue requires 3 cans of white paint mixed with 4 cans of blue paint. Another shade of blue requires 5 cans of white paint mixed with 7 cans of blue paint.
   a) Which mixture will give the darker shade of blue? Explain.
   b) Which mixture will require more white paint?

5. Look at the two mixtures.
   a) What is the ratio of concentrate to water in A and in B?



   b) Explain how you could add concentrate or water to make both ratios the same.
   Draw a picture to show your answer.

6. **Assessment Focus** The ratio of fiction to non-fiction books in Ms. Arbuckle's class library is 7:5.
   The ratio of fiction to non-fiction books in Mr. Albright's class library is 4:3.
   Each classroom has 30 non-fiction books.
   a) Which room has more fiction books? How many more?
   b) Mr. Albright added two non-fiction books to his class library. Does this make the ratio the same in both classes? Explain.

7. At Ria's party, there were 2 pizzas for every 3 people.
   At Amin's party, there were 5 pizzas for every 7 people.
   At which party did each person get more pizza? Explain.

Reflect

In one store, the ratio of DVDs to videos is 7:5.
In another store, the ratio of DVDs to videos is 4:3.
Explain why you cannot say which store has more DVDs.

Mid-Unit Review

LESSON

2.1 **1.** In the school parking lot, there are 4 Japanese cars, 7 American cars, 3 German cars, and 2 Korean cars. Write each ratio.

a) American cars to Japanese cars

b) German cars to Korean cars

c) American cars to total number of cars

d) German cars to Japanese cars and American cars

2. A box contains 5 cream chocolates, 2 chocolate-covered almonds, and 3 caramel chocolates.

a) What is each ratio? Sketch a picture for each ratio.

i) almond chocolates to caramel chocolates

ii) cream chocolates to caramel chocolates

iii) cream chocolates to all chocolates

b) Lesley ate one of each kind of chocolate. What is each new ratio for part a?

3. In Mary's closet, there are 7 T-shirts, 4 pairs of shorts, and 3 sweaters. Write each ratio.

a) T-shirts to shorts

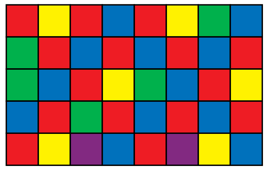
b) sweaters to shorts

c) sweaters to T-shirts and shorts

2.2 **4.** For each ratio, write three equivalent ratios.

a) 5:3 **b)** 6:24

5. a) Write each ratio below in simplest form.



i) green squares to red squares

ii) yellow squares to purple squares

iii) red squares to total number of squares

b) State the colours for each ratio.

i) 1:6 **ii)** 2:5

6. In Ms. Bell's class, the ratio of boys to girls is 5:4.

a) There are 15 boys in the class. How many girls are there?

b) Two girls leave. What is the new ratio of boys to girls?

2.3 **7.** A jug of orange juice requires 3 cans of orange concentrate and 5 cans of water.

a) Accidentally, 4 cans of concentrate were mixed with 5 cans of water. Is the mixture stronger or weaker than it should be? Explain.

b) Suppose 6 cans of water were mixed with 3 cans of concentrate. Is the mixture stronger or weaker than it should be? Explain.

2.4 Applications of Ratios

Focus Use ratios to solve problems.

Explore

Work on your own.
In the book *Gulliver's Travels*, Gulliver meets little people who are only 15 cm tall, and giants who are 18 m tall.
Gulliver is 1.80 m tall.
How many times as big as a little person is Gulliver?
How many times as big as Gulliver is a giant?
Write these comparisons as ratios.

Reflect & Share

Compare your answers with those of a classmate.
Work together to find the ratio of the height of the giants to the height of the little people.
How many times as big as a little person is a giant?

Connect

You can use diagrams and tables to model and solve ratio problems.

Example 1

Jolene makes a scale drawing of her home.
She uses a scale of 5 cm to represent 1.5 m.

a) What is the ratio of a length in the drawing to the actual length?
What does this ratio mean?

b) Jolene measures her bedroom on the drawing. It is 16 cm long.
What is the actual length of Jolene's bedroom?

c) The house measures 18 m by 12 m.
What are the dimensions of the scale drawing?

Solution

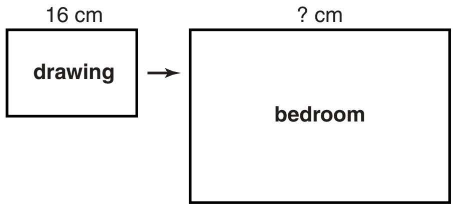
a) Length in the drawing:actual length

$$\begin{aligned} &= 5\text{ cm}:1.50\text{ m} \\ &= 5\text{ cm}:150\text{ cm} \\ &= (5\text{ cm} \div 5\text{ cm}):(150\text{ cm} \div 5\text{ cm}) \\ &= 1:30 \end{aligned}$$

When you write ratios of measurements, the units must be the same. Multiply 1.50 m by 100 to change metres to centimetres.

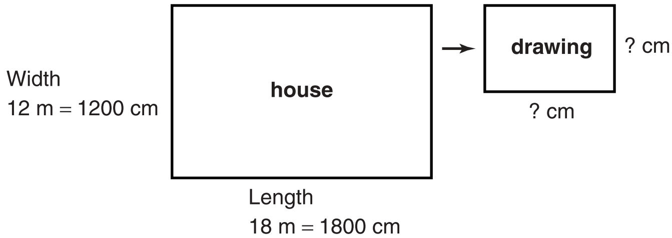
Each 1 cm in the drawing represents 30 cm in the home.

b) Draw a diagram.



The length in the drawing is 16 cm.
Each 1 cm in the drawing represents 30 cm in the bedroom.
So, the length of the bedroom is 16×30 cm $= 480$ cm
$= 4.8$ m

c) Draw a diagram.



1 cm in the drawing represents 30 cm in the house.
Divide each measurement in the house to find the measurement on the drawing.

Length in the drawing $= \frac{1800 \text{ cm}}{30}$
$= 60$ cm

Width in the drawing $= \frac{1200 \text{ cm}}{30}$
$= 40$ cm

The dimensions of the scale drawing are 60 cm by 40 cm.

Example 2

Wallpaper paste can be made by mixing flour and water.
The table shows the volume of water to be mixed with flour to make different quantities of paste.

Water (mL)	16	32	48
Flour (mL)	3	6	9

a) What is the ratio of the volume of water to the volume of flour?
b) How much water is required for 15 mL of flour?

Solution

a) Volume of water:volume of flour = 16:3

b) Use a pattern to continue the table in part a.
For the water, the pattern rule is: Start at 16, then add 16.
The next three terms are 64, 80, 96.
For the flour, the pattern rule is: Start at 3, then add 3.
The next three terms are 12, 15, 18.
Extend the table to include these numbers.

Water (mL)	16	32	48	64	80	96
Flour (mL)	3	6	9	12	15	18

From the table, 80 mL of water are needed for 15 mL of flour.

Practice

Mental Math

Find all the factors of each number.

7, 15, 27, 36, 48, 51

1. This table shows the quantities of concentrate and water needed to make punch.

Orange concentrate (cans)	2	4	6		
Cranberry concentrate (cans)	1	2	3		
Water (cans)	3	6	9		
Punch (L)	1.5	3	4.5		

a) What are the next two numbers in each row of the table?
b) What is the ratio of cranberry concentrate to water?
c) Suppose you use 10 cans of orange concentrate.
How much punch would you get?
d) Suppose you use 12 cans of concentrate in total.
How much punch would you get?
e) How much of each ingredient would you need to make 15 L of punch? Explain your reasoning.

Remember to convert the units so you work with all measurements in the same units.

2. Jenny makes models of Canadian Coast Guard ships.
She uses a scale of 3 cm to 1.5 m.
a) What is the ratio of a length on the model to a length on the ship?
b) The Terry Fox ship is 88 m long. How long is the model?
c) The model of the Otter Bay is 27 cm long.
What is the actual length of the Otter Bay?
Draw diagrams to show your answers.

3. A set designer builds a model of the stage and the different pieces of furniture on it. He uses a scale of 5 cm to 1 m.
   a) What is the ratio of a length on the model to a length on the stage?
   b) The length of a table is 1.4 m. What is the length of the model?
   c) The height of the model of a lamp is 3 cm. What is the actual height of the lamp?
   d) The height of the model of an ornament is 0.5 cm. What is the actual height of the ornament? How did you find out?

   Draw diagrams to show your answers.

Math Link

Social Studies

A scale on a provincial map is 1:1 500 000. This means that 1 cm on the map represents 1 500 000 cm on the ground, which is $\frac{1\,500\,000}{100}$ m, or 15 000 m on the ground.

And, 15 000 m is $\frac{15\,000}{1000}$ km, or 15 km.

So, a scale of 1:1 500 000 is 1 cm to 15 km.

4. Janice builds a model using a scale of 2 cm to represent 3 m. Her friend William says she is using a ratio of 2:3. Is William correct? Explain.

5. This table shows the quantities of salt and water needed to make salt solutions.

Water (L)	2	4	6	8
Salt (g)	18	36	54	72

   a) How much salt would be needed for 7 L of water?
   b) How much water would be needed for 90 g of salt?

6. **Assessment Focus** Aston challenges his father to a 100-m race. Aston runs 4 m for every 5 m his father runs.
   a) Who wins the race? Draw a diagram to show your answer.
   b) How far will Aston have run when his father crosses the finish line?
   c) Aston asks to race again but wants to be given a head start. How much of a start should Aston's father give him to make it a close race? Explain your answer.

Reflect

Make up your own ratio problem.
Solve your problem. Show your work.

2.5 What Is a Rate?

Focus Investigate rates.

Explore

Work with a partner.
You will need a stopwatch.
One person is the "blinker." The other person is the timekeeper.
The blinker blinks as many times as possible.
Count the number of times the blinker blinks in 20 s.
Reverse roles.
Count the number of times the blinker blinks in 30 s.

➢ Who was the faster blinker?
How do you know?
➢ Estimate how many times each person would blink in 1 h.
What assumptions do you make?
Are these assumptions reasonable?

Reflect & Share

Compare your results with those of another pair of classmates.
How can you decide who is the fastest blinker?

Connect

When we compare two different things, we have a **rate**.
Here are some rates.

- We need 5 sandwiches for every 2 people.
- Oranges are on sale at $1.49 for 12.
- Gina earns $4.75 per hour for baby-sitting.
- There are 500 sheets on one roll of paper towels.

The last two rates above are **unit rates**.
Each rate compares a quantity to 1 unit.

Jamal skipped rope 80 times in 1 min.
We say that Jamal's rate of skipping is 80 skips per minute.
We write this as 80 skips/min.

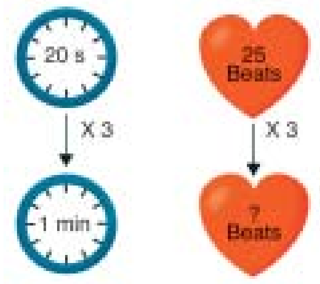
To find unit rates, we can use diagrams, tables, and graphs.

Example 1

The doctor took Marjorie's pulse. He counted 25 beats in 20 s.
What was Marjorie's heart rate in beats per minute?

Solution

Draw a diagram.
There are 25 beats in 20 s.
There are 60 s in 1 min.
We multiply 20 s by 3 to get 60 s.
So, multiply the number of beats in 20 s by 3 to get the number of beats per minute.



25 beats $\times$ 3 $=$ 75 beats

Marjorie's heart rate is 75 beats/min.

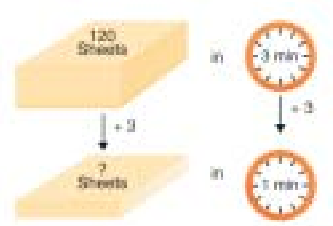
Example 2

A printing press prints 120 sheets in 3 min.

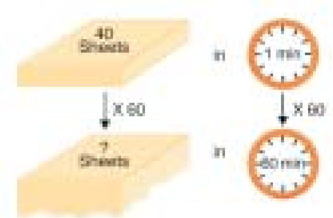
a) Express the printing as a rate.
b) How many sheets are printed in 1 h?
c) How long will it take to print 1000 sheets?

Solution

a) Draw a diagram.
The press prints 120 sheets in 3 min.
So, in 1 min, the press prints:
120 sheets $\div$ 3 $=$ 40 sheets
The rate of printing is 40 sheets/min.



b) In 1 min, the press prints 40 sheets.
One hour is 60 min.
So, in 60 min, the press prints:
60 $\times$ 40 sheets $=$ 2400 sheets
The press prints 2400 sheets in 1 h.



c) *Method 1*
In 1 min, the press prints 40 sheets.
So, in 5 min, the press prints: 5 $\times$ 40 $=$ 200 sheets
Make a table. Every 5 min, 200 more sheets are printed.
Extend the table until you get 1000 sheets.

Time (min)	5	10	15	20	25
Sheets printed	200	400	600	800	1000

Method 2

The press prints 40 sheets in 1 min.

Think: What do we multiply 40 by to get 1000?

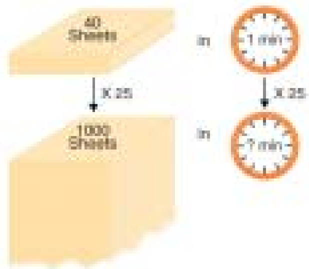
Use division: $1000 \div 40 = 25$

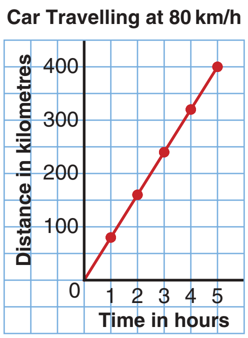
So, $40 \times 25 = 1000$

We multiply the time by the same number.

$1 \text{ min} \times 25 = 25 \text{ min}$

The press takes 25 min to print 1000 sheets.





The rate at which a car travels is its **average speed**.

When a car travels at an average speed of 80 km/h, it travels:

80 km in 1 h
160 km in 2 h
240 km in 3 h
320 km in 4 h
400 km in 5 h … and so on

We can show this motion on a graph.

An average speed of 80 km/h is a unit rate.

Practice

1. Express as a unit rate.

a) Morag typed 60 words in one minute.

b) Peter swam 25 m in one minute.

c) Abdu read 20 pages in one hour.

2. Express as a unit rate.

a) June cycled 30 km in 2 h.

b) An elephant travelled 18 km in 30 min.

c) A plane flew 150 km in 15 min.

3. Before running in a 100-m race, Gaalen's heart rate was 70 beats/min. Which do you think is more likely after the race: 60 beats/min or 120 beats/min? Explain.

Mental Math

Find a square root of each number.

49, 25, 81, 16, 64, 144, 4, 121

4. Ribbon costs $1.44 for 3 m.
   a) What is the cost per metre?
   b) How much would 5 m of ribbon cost?
   c) How much ribbon could you buy for $12?

5. The graph shows the distance travelled by a cyclist in 3 h.
   a) How far did the cyclist travel in 1 h?
   b) What is the average speed of the cyclist?
      How do you know?

In England, the currency is pennies and pounds. There are 100 pennies in one pound, £1.

6. James and Lucinda came to Canada on holiday from England. The rate of exchange for their money was $2.50 Can to £1.
   a) How many Canadian dollars would James get for £20?
   b) What is the value in English pounds of a gift Lucinda bought for $30 Can?

7. When a person runs a long-distance race, she thinks of the time she takes to run 1 km (min/km), rather than the distance run in 1 min (km/min).
   On a training run, Judy took 3 h 20 min to run 25 km.
   What was Judy's rate in minutes per kilometre?

8. **Assessment Focus** Scott trained for the marathon.
   On day 1, he took 70 min to run 10 km.
   On day 10, he took 2 h 40 min to run 20 km.
   On day 20, he took 4 h 15 min to run 30 km.
   a) What was Scott's running rate, in minutes per kilometre, for each day?
      i) Day 1 ii) Day 10 iii) Day 20
   b) What do you think Scott's running rate, in minutes per kilometre, might be for the 44 km of the marathon?
      How long do you think it will take him? Explain.

Reflect

Look through newspapers and magazines to find three different examples of rates. Explain how the rates are used.

Organizing a Math Notebook

1 You must include the date for each new note taken or activity performed.

2 You must include a title for each new note or activity.

3 All dates and titles must be underlined with a ruler.

4 All tables must be neat.
Use a ruler to draw a table.

For example:
Section 2.5, question 8.

April 13, 2005

Day	Time (min)	Distance (km)	Rate (min/km)
1			

5 Your daily work must be legible, complete, and well-organized.

6

You must make corrections where necessary.

7

To organize a math problem:

- Restate the problem in your own words.
  For example, "This problem is about…"
- Think about a strategy you will use and tell about it.
  For example, "The strategy I will use is…_________ because…"
- Solve the problem. Show all your work.
- State the answer to your problem, and explain how you know it is reasonable and correct.
  For example, "I found the answer to be…
  I know my answer is reasonable because…
  I know my answer is correct because…"
- Extend your thinking. Make up a similar problem by asking "What if?" questions.
  For example, "What if all the numbers in this question were doubled? How would the answer change?"

Race Engineer

The equivalent of the 100-m sprint in the world of cars is the $\frac{1}{4}$-mile drag race. One-quarter of one mile is about 400 m. The race engineers (men and women who help design, build, and tune the cars) are an important part of the racing team. Theirs is a world filled with gear ratios, cylinder compression ratios, fuel mixture ratios, acceleration, and speed. It's a constant cycle of theory, real-world application, testing, and evaluating. And, sometimes, all these take place during a few hours between race runs! Helping the dragster team driver get to the finish line $\frac{1}{1000}$ th of a second faster than a previous run could mean the difference between winning and losing the race.

In October, 2003, a top fuel dragster recorded a new "fastest time" of 4.441 s for the $\frac{1}{4}$-mile drag race. But another car and driver continued to hold the record for the fastest recorded speed achieved during a race—an incredible 536 km/h! Why do you suppose the second car doesn't hold the record for fastest time?

Unit Review

Review any lesson with etext online tutorial

What Do I Need to Know?

✓ A ratio is a comparison of quantities.
For example, 3 dogs to 7 cats is 3:7.

✓ An equivalent ratio can be formed by multiplying or dividing the terms of a ratio by the same number.

For example:

	5:8	and	36:30
=	(5 × 3):(8 × 3)	=	(36 ÷ 6):(30 ÷ 6)
=	15:24	=	6:5

5:8 and 15:24 are equivalent ratios.

36:30 and 6:5 are equivalent ratios.

✓ Two ratios can be compared when the second terms are the same.
For example, Scott's scoring record was 16:5.
Brittany's scoring record was 10:3.
To find who had the better record, use equivalent ratios to make the second term of each ratio 15.

	Scott		**Brittany**
	16:5		10:3
=	(16 × 3):(5 × 3)	=	(10 × 5):(3 × 5)
=	48:15	=	50:15

Brittany had the better record.

✓ A rate is a comparison of two quantities with different units.
For example:
Heart rate is measured in beats per minute (beats/min).
Average speed is measured in kilometres per hour (km/h).
Fuel consumption of an aircraft is measured in litres per hour (L/h).

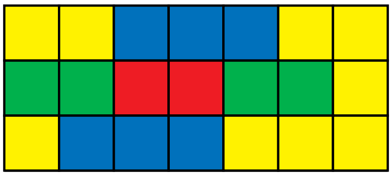
What Should I Be Able to Do?

For extra practice, go to page 439.

LESSON

2.1 **1.** On a school trip, there are 9 boys, 10 girls, and 4 adults.
Write each ratio.
a) girls to boys
b) boys and girls to adults
c) adults to boys and girls

2.2 **2.** Write four ratios to describe the coloured squares.



3. Explain two different ways to get ratios equivalent to 25:10.

4. Jake has to draw a flag with a length to width ratio of 3 to 2. What is the largest flag Jake can draw on a 25-cm by 20-cm sheet of paper?

2.3 **5.** The ratio of computers to students in Ms. Beveridge's class is 2:3. The ratio of computers to students in Mr. Walker's class is 3:5. Each class has the same number of students.
Which room has more computers? Explain.

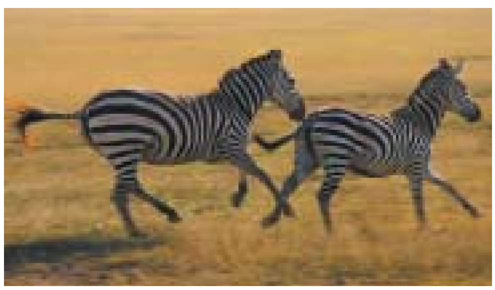
2.4 **6.** Ali builds model planes. He uses a scale of 8 cm to represent 1.8 m.
a) What is the ratio of a length on the model to the actual length on the plane?
b) A plane has a length of 72 m. What is the length of the model?
c) A model has a length of 60 cm. What is the actual length of the plane?

Draw pictures to show your answers.

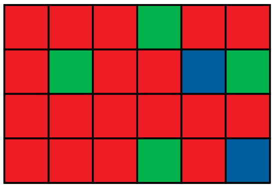
7. Red and white paint is mixed in the ratio of 3 to 2.
a) How many cans of red paint would be needed with 6 cans of white paint?
b) How many cans of each colour are needed to make 20 cans of mixture?

2.5 **8.** Express as a unit rate.
a) A bus travelled 120 km in 3 h.
b) An athlete ran 1500 m in 6 min.
c) A student earned $16 for 2 h work.

9. A lion can run 550 m in 25 s.
A zebra can run 270 m in 15 s.
a) Which animal is faster?
b) What is the ratio of their average speeds?



Practice Test

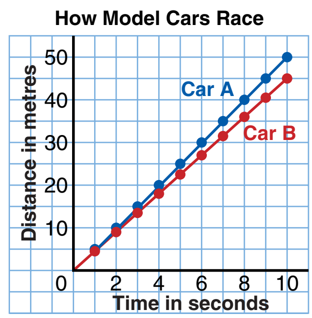


1. a) Write each ratio in simplest form.
   i) red squares to blue squares
   ii) blue squares to green squares
   iii) red squares and blue squares to total numbers of squares
   b) Suppose the grid is increased to a rectangle measuring 9 units by 4 units. The ratios of the colours remain the same. How many red squares will there be in the new rectangle?

2. In the league, the Leos play 8 games. Their win to loss ratio is 5:3. The Tigers play 11 games. Their win to loss ratio is 7:4.
   a) Which team has the better record? Explain.
   b) Suppose the Leos win their next game and the Tigers lose theirs. Which team would have the better record? Explain.

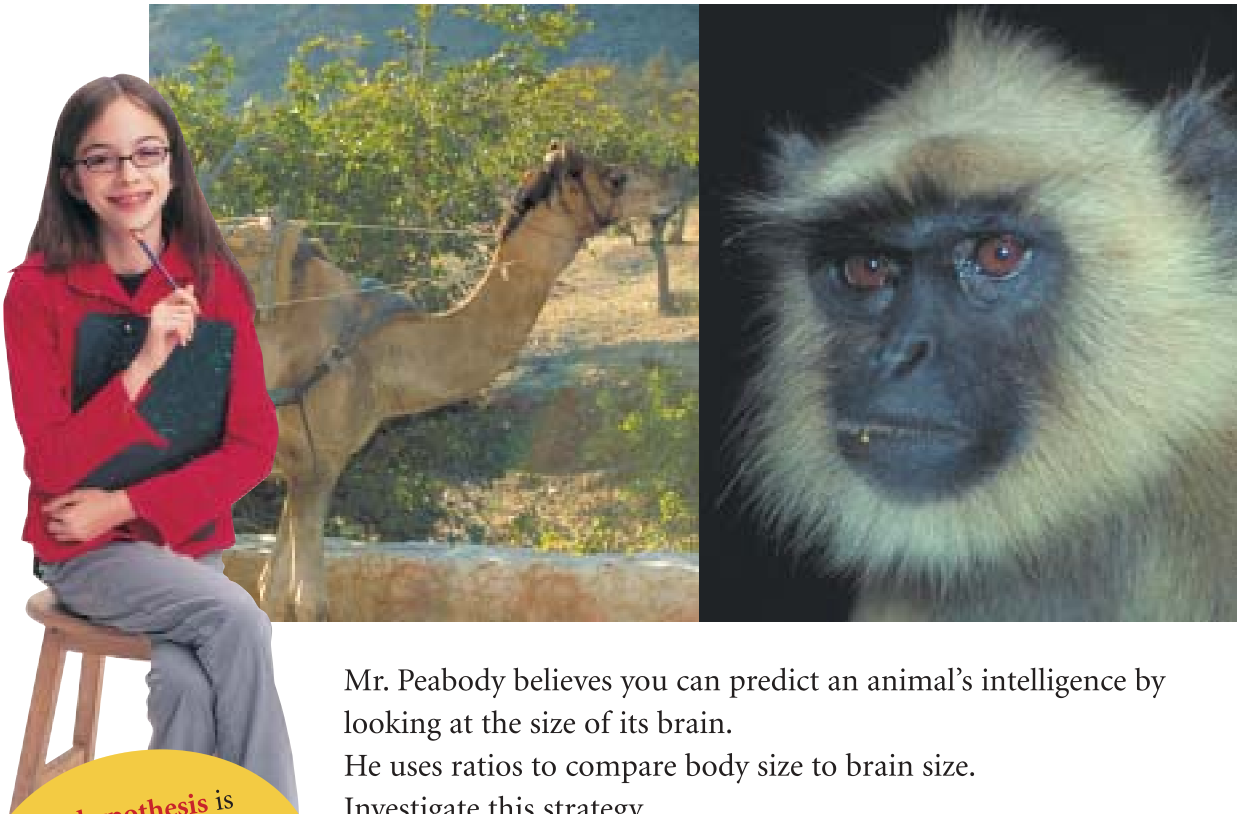
3. Hessa is building a scale model of a park. She uses a scale of 12 cm to represent 1.5 m.
   a) The actual length of the bridge is 20 m. What is the length of the model bridge?
   b) In the model, the height of the climbing frame is 10 cm. What is the actual height of the frame?

4. Look at this graph.
   a) What is the speed of each car?
   b) How far apart are the cars after 4 s?



5. Trevor's mark on a math test was 10 out of 15. Anne's mark on another test was 15 out of 20. Trevor said, "Each of us got 5 wrong. So, our marks are equal." Do you agree? Give reasons for your answer.

Unit Problem Who's the Smartest?



A **hypothesis** is something that seems likely to be true. It needs to be tested and proved or disproved.

The mass of a human brain is about the same mass as your math textbook.

Mr. Peabody believes you can predict an animal's intelligence by looking at the size of its brain.
He uses ratios to compare body size to brain size.
Investigate this strategy.

Analyse the data below to test Mr. Peabody's **hypothesis**.

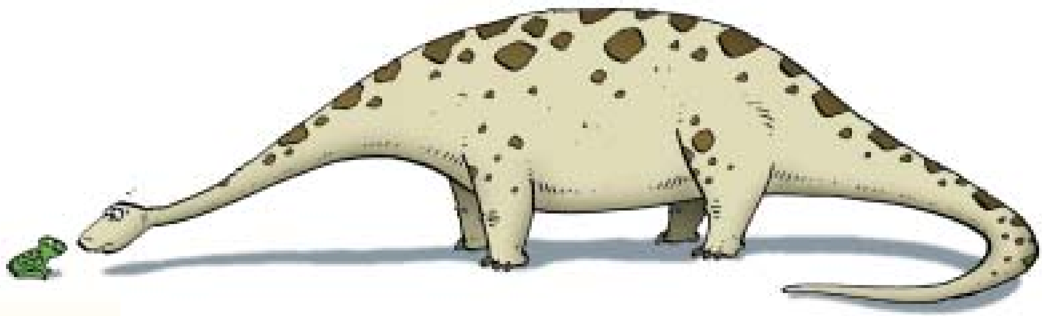
Species	Comparing Mass (g)		Comparing Length (cm)	
	Body	Brain	Body	Brain
Human	56 000	1400	150	15
Monkey	7 000	100	30	5
Camel	520 000	650	200	15

1. Compare masses: find the ratio of body mass to brain mass.
- How does a human compare to a monkey?
- How does a human compare to a camel?
- How does a camel compare to a monkey?

According to Mr. Peabody's hypothesis, which animal is smartest? Explain your reasoning.

2. Compare lengths: find the ratio of body length to brain length.
   - How does a human compare to a monkey?
   - How does a human compare to a camel?
   - How does a camel compare to a monkey?

   According to Mr. Peabody's hypothesis, which animal is smartest? Explain your reasoning.



Check List

Your work should show:

- ✓ how you calculated each ratio
- ✓ your reasoning for which animal is the smartest
- ✓ your conclusions about the hypothesis
- ✓ the correct use of mathematical language

3. The dinosaur, diplodocus, lived about 150 million years ago.
   The brain of a diplodocus was about 9 cm long.
   The body of a diplodocus was about 27 m long.
   - How long would a human's brain be if it had the same brain length to body length ratio as a diplodocus?

   A frog's brain is about 2 cm long.
   A frog's length is about 10 cm long
   - How long would a human's brain be if it had the same brain length to body length ratio as a frog?

4. Review your results.
   Write a short letter to Mr. Peabody telling him whether you agree with his hypothesis, and why.
   Use mathematical language to support your opinion.

Reflect on the Unit

How is a rate like a ratio?
How is it different?
Use examples in your explanation.

UNIT 3

Geometry and Measurement

Most products are packaged in boxes or cans.
How is a package made?
How do you think the manufacturer chooses the shape and style of package? What things need to be considered?

Look at the packages on this page.
Choose one package.
Why do you think the manufacturer chose that form of packaging?

What You'll Learn

- Recognize different views of an object.
- Sketch different views of an object.
- Sketch an object.
- Build an object from a net.
- Develop and use a formula for the surface area of a rectangular prism.
- Develop and use a formula for the volume of a rectangular prism.

Why It's Important

- Drawing a picture is one way to help solve a problem or explain a solution.
- Calculating the volume and surface area of a prism is an extension of the measuring you did in earlier grades.

Key Words

- polyhedron (polyhedra)
- prism
- pyramid
- regular polyhedron
- cube
- tetrahedron
- isometric
- pictorial diagram
- icosahedron
- octahedron
- dodecahedron
- frustum
- variable
- surface area

UNIT 3

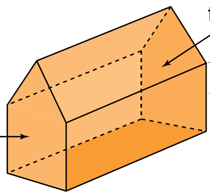
Skills You'll Need

Identifying Polyhedra

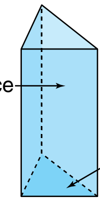
A **polyhedron** is a solid with faces that are polygons.
Two faces meet at an edge.
Three or more edges meet at a vertex.

A **prism** has 2 congruent bases, and is named for its bases.
Its other faces are rectangles.

A pentagonal prism

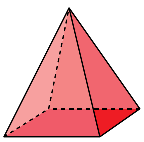


A triangular prism

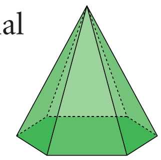


A **pyramid** has 1 base and is named for that base.
Its other faces are triangles.

A square pyramid

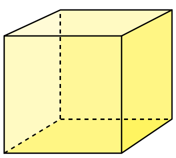


A hexagonal pyramid

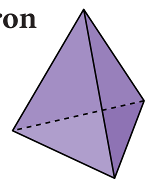


A **regular polyhedron** has all faces congruent.
The same number of faces meet at each vertex.
The same number of edges meet at each vertex.

A **cube** is a regular rectangular prism.



A **tetrahedron** is a regular triangular pyramid.



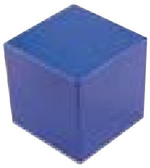
✓ Check

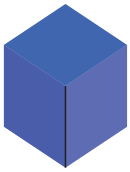
Use the pictures of the solids above. Use the solids if you have them.

1. **a)** How are the solids alike? How are they different?
b) Name a real-life object that has the shape of each solid.

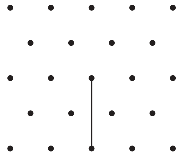
Using Isometric Dot Paper to Draw a Cube

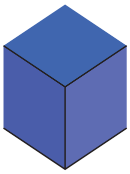
To draw this cube:



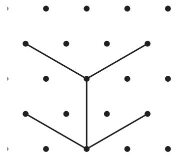


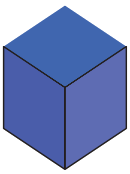
Join 2 dots for one vertical edge.



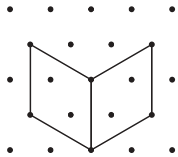


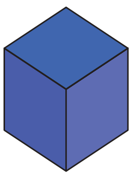
Join pairs of dots diagonally for
the front horizontal edges,
top and bottom.



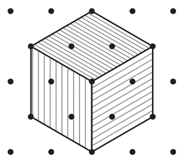


Join the dots for the
other 2 vertical edges.





Complete the cube.
Join dots diagonally for the back
horizontal edges at the top.



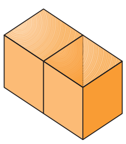
Shade visible faces differently to
get a three-dimensional (3-D) look.

Isometric means "equal measure." On isometric dot paper, the line segments joining 2 adjacent dots in any direction are equal.

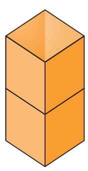
✓ Check

2. Use isometric dot paper. Draw each object. Use linking cubes when they help.

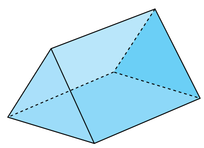
a)



b)



c)



3.1 Sketching Views of Solids

Focus Recognize and sketch different views of objects.

Which objects do you see in this picture?

Choose an object. What does it look like from the top? From the side? From the back?

Explore

Work on your own.
Choose a classroom object.

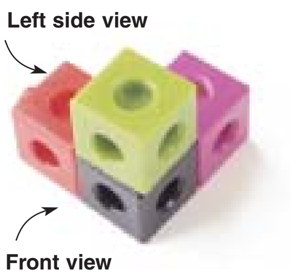
Sketch the object from every view possible. Label each view.
Use dot paper or grid paper if it helps.
Describe each sketch.

Reflect & Share

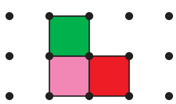
Trade sketches with a classmate.
Try to identify the object your classmate drew.

Connect

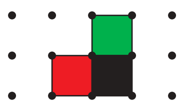
We can use square dot paper to draw each view of the object at the left.
We ignore the holes in each face.



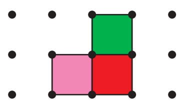
Back view



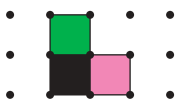
Front view



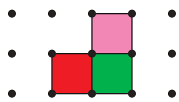
Left side view



Right side view

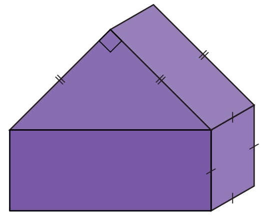


Top view



Example

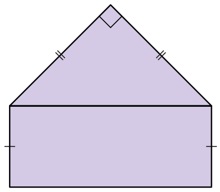
A triangular prism fits on top of a rectangular prism. The bases of the triangular prism are right isosceles triangles. The rectangular prism has two square faces. Sketch the front, back, side, and top views.



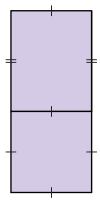
A right isosceles triangle has a 90° angle and two equal sides.

Solution

The front view and back view are the same. Sketch a right isosceles triangle on top of a rectangle.

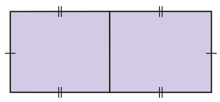


Front view



Side view

The two side views are the same. The side face of the triangular prism is a rectangle. Sketch a rectangle on top of a square.



Top view

The top view is two congruent rectangles.

Practice

Use linking cubes when they help.

1. Many signs are views of objects. Identify the view (front, back, side, or top) and the object on each sign.

a) Child care **b)** Airport transportation **c)** Fully accessible

2. Use linking cubes. Make each object A to E. Figures J to Q are views of objects A to E. Match each view (J to Q) to each object (A to E), in as many ways as you can.

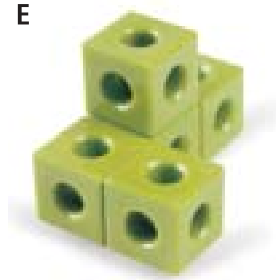
A

B

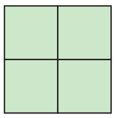
C

D

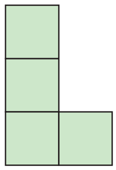
E



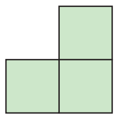
J



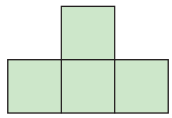
K



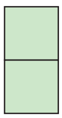
L



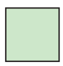
M



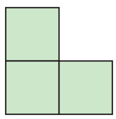
N

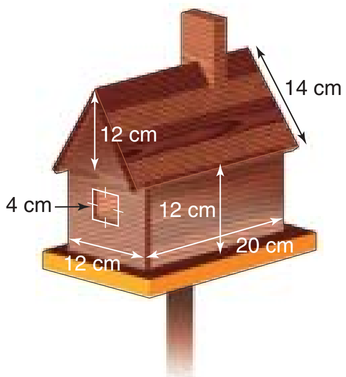


P



Q





3. Use linking cubes. Make each object. Use square dot paper. Draw the front, back, side, and top views of each object.

a)

b)

c)

4. Sketch the top, front, side, and back views of the birdhouse at the left. Label each view.

5. Design a road safety or information sign for each situation. Tell which view you used. Explain how each sign shows the information.
   a) playground ahead
   b) tennis court
   c) skateboarding allowed
   d) no flash cameras allowed

6. Find each object in the classroom.
   Sketch the front, back, top, and side views of each object.
   a) filing cabinet
   b) vase
   c) teacher's desk



7. **Assessment Focus** Use 4 linking cubes. Make as many different objects as possible. Draw the front, back, top, and side views for each object. Label each view. Use your sketches to explain how you know all the objects you made are different.

8. All 5 views of a cube are the same.
   Use the solids in the classroom.
   a) Are there any other prisms with all views the same? Explain.
   b) Which solid has 4 views the same?
   c) Which solid has only 3 views the same?
   d) Which solid has no views the same?
   If you cannot name a solid for parts b to d, use linking cubes to make a solid.

9. Here is one view of an object.
   Sketch a possible different view of the object.
   a) 
   b) 
   c) 

Mental Math

How many:
- days in a year?
- months in a year?
- days in a week?
- hours in a day?
- minutes in an hour?
- seconds in a minute?

Reflect

Choose an object. How many views do you need to sketch, so someone else can identify it? Explain. Sketch the views you describe.

Using a Computer to Draw Views of Solids

Focus Use technology to sketch views of objects.

Software, such as *The Geometer's Sketchpad*, can be used to draw different views of solids.

Follow these steps.

1. Open *The Geometer's Sketchpad*.

To make a "dot paper" screen:

2. From the **Edit** menu, choose **Preferences**.
Select the Units tab. Check the Distance units are cm.
Click **OK**.

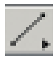
3. From the **Graph** menu, choose **Define Coordinate System**.
Click on a point where the grid lines intersect.
The grid is now highlighted in pink.

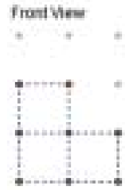
4. From the **Display** menu, choose **Line Width**, then **Dotted**.
There are dots at the grid intersections.
Click anywhere on the screen other than the dots.
The dots are no longer highlighted. Hold down the shift key.
Click on each numbered axis and the two red dots.
Release the shift key.
The axes and the dots are now highlighted.

5. From the **Display** menu, choose **Hide Objects**.
The axes and dots disappear.
The screen now appears like a piece of dot paper.
From the **Graph** menu, choose **Snap Points**.

Make this object with linking cubes.
Follow the steps below to create
views of this object.

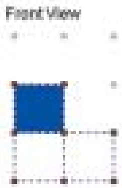
6. From the **Toolbox** menu, choose **A** (Text Tool).
Move the cursor to the screen and a finger appears.
Click and drag to make a box at the top left.
Inside the box, type: Front View

7. From the **Toolbox** menu, choose  (Straightedge Tool).



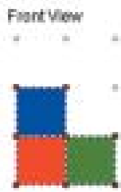
Move to a dot on the screen below the title.
Click and drag to draw a line segment.
Release the mouse button.
Continue to draw line segments to draw the front view.
Click to select each line segment.
From the **Display** menu, choose **Line Width**, then **Dashed**.
This draws the line segments as broken lines.

8. From the **Toolbox** menu, click on  (Selection Arrow Tool).



Click to select the four corners of the top square, in clockwise order.
From the **Construct** menu, choose **Quadrilateral Interior**.
From the **Display** menu, choose **Color**, then choose blue.

9. Repeat *Step 8* to make the bottom squares red and green.



10. Repeat *Step 6* to *9* to draw and label the left side view and the back view.

11. Draw the top view and right side view.

✓ Check

1. Open a new sketch. Draw different views of objects from *Section 3.1, Practice* question 3. Compare the hand-drawn views with the computer-drawn views.

3.2 Sketching Solids

Focus Sketch pictures of objects from models and drawings.

Explore

Work with a partner.
You will need isometric dot paper and 4 linking cubes.
Make an object so its front, top, and side views are all different.
Draw the object on isometric dot paper so all 4 cubes are visible.

Reflect & Share

Trade isometric drawings with another pair of classmates.
Make your classmates' object.
Was it easy to make the object? Explain.
Compare objects. If they are different, find out why.

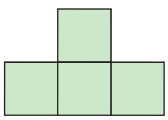
Connect

An object has 3 dimensions: length, height, and width or depth.
A drawing is a picture of an object on paper.
It has 2 dimensions: length and width.

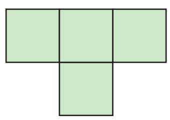
➢ We use isometric dot paper to show the 3 dimensions of an object. Parallel edges on an object are drawn as parallel line segments. An object made with 5 linking cubes is shown at the left.

When we draw the object from the front or the top, we see only 4 cubes.

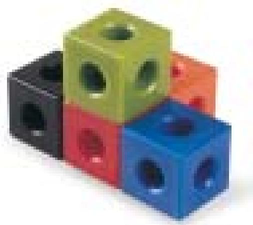
Front view



Top view



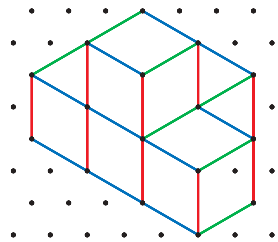
Place the object so that all 5 cubes are seen.

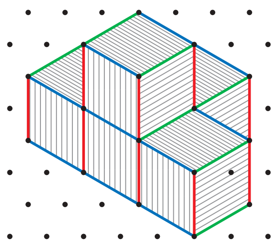


Draw the edges in this order:

- vertical edges (red)
- horizontal edges that appear to go down to the right (blue)
- horizontal edges that appear to go up to the right (green)

Shade faces to produce a 3-D effect.





Number Strategies

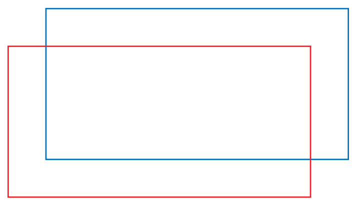
Simplify.
Use order of operations.

- $17 - 7 \times 2 + 3$
- $15 - 6 \div 2$
- $14 \div 2 \times 3$
- $8 \times 2 + 21 - 6$
- $15 + 3 \times 5$

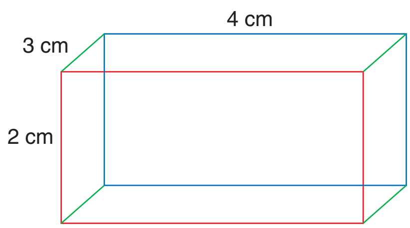
➢ We can use translations to draw a **pictorial diagram**.
A pictorial diagram shows the shape of an object in 2 dimensions.
It gives the impression of 3 dimensions.
To sketch the rectangular prism above right:

Draw a rectangle 4 cm by 2 cm.

Draw another rectangle that is the image of the first rectangle after a translation up and to the right.



Join corresponding vertices for a sketch of a rectangular prism.
Label the dimensions.



Note that the depth of the prism is 3 cm but, on the drawing, this distance is less than 3 cm. In a pictorial drawing, the depth of an object is drawn to a smaller scale than the length and width. This gives the appearance of 3 dimensions.

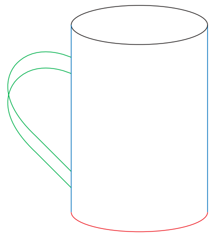
We can use similar ideas to sketch a 3-D picture of any object.

Example

Sketch this mug.

Solution

The mug is an open cylinder with a handle.
Draw an oval for the circular top.
Draw vertical lines for the curved surface (blue).
Draw half an oval for the circular base (red).
Draw 2 overlapping curves for the handle (green).



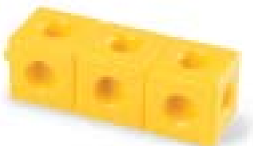
In a pictorial drawing, a circle is drawn as an oval.

Practice

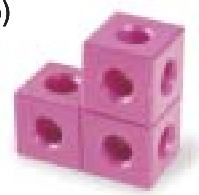
Use linking cubes when they help.

1. Make each object. Draw it on isometric dot paper.

   a)

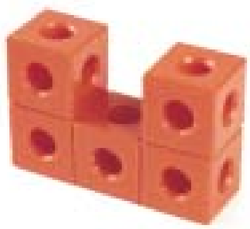
   b)

2. Turn each object in question 1.
   Draw it a different way on isometric dot paper.

3. Use linking cubes. Make each object.
   Draw it on plain paper or isometric dot paper.

   a)

   b)

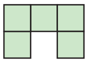
   

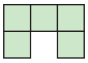
Math Link

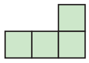
Art

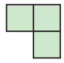
When you view an IMAX film, you "see" it in 3-D. The scenes are filmed from two slightly different angles. This imitates how our eyes view the world. Both films are projected onto the screen to give you the feeling of depth.

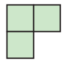
4. Here are different views of an object made with linking cubes.


Front


Back


Top


Left Side


Right Side

Make the object.
Draw it on isometric dot paper or plain paper.

5. Sketch a 3-D picture of each object on plain paper.

a) 

b) 

c) 

6. Square pieces of shelving snap together to make cube-shaped stacking shelves. All the faces are square and measure 30 cm by 30 cm. A side face costs $1.50. A top or bottom face costs $1.30. The back face costs $1.10. Kate built shelves using 9 side faces, 6 back faces, and 9 top/bottom faces.

a) Use linking cubes to build a possible design.
b) Draw a picture of your design on isometric dot paper.
c) How much did the shelving cost?
d) Is it possible to have the same number of cubes but use fewer pieces? Explain.
e) If your answer to part d is yes, what is the new cost of shelving? Explain.

7. **Assessment Focus** Use 5 linking cubes. Make a solid.

a) Sketch the solid on isometric dot paper.
b) Sketch a 3-D picture of the solid on plain paper.
c) How is sketching a solid on isometric paper different from sketching it on plain paper? How are the methods alike? Use your sketches to explain.

Reflect

How do you draw an object to show its 3 dimensions?
Use words and pictures to explain.

Forensic Graphics Specialist

A forensic investigator collects information from a crime scene to find out who was involved and exactly what happened. The forensic investigator also presents evidence in court as required by law.

The forensic graphics specialist visits the crime scene to take photographs and measurements. She may research to compose technical drawings to help with the investigation. When she prepares these drawings, the graphics specialist pays attention to shading and relative line thickness. It is important that she does not present optical illusions such as an object appearing to be "inside out."

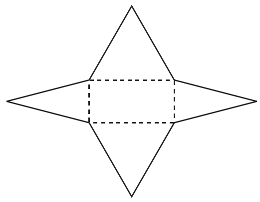
3.3 Building Objects from Nets

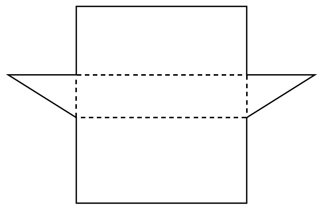
Focus Build objects from nets.

In earlier grades, you designed nets for prisms and pyramids.
Now, you will fold nets to make prisms, pyramids, and other polyhedra.

Explore

Work with a partner.
You will need scissors and tape.
Your teacher will give you large copies of the nets below.





➢ Identify the polyhedron for each net above.
➢ Use a copy of each net.
Cut it out. Fold, then tape it to make a polyhedron.
➢ Identify congruent faces on each polyhedron.

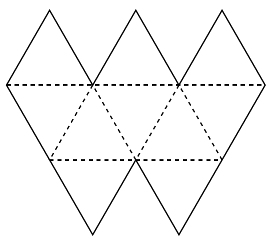
Reflect & Share

Look at each polyhedron from the top, front, back, and sides.
Which views are the same? Explain.

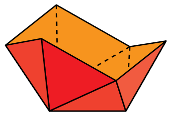
Connect

There are 5 regular polyhedra.
You reviewed 2 of them, the cube and the tetrahedron,
in *Skills You'll Need*, page 76.

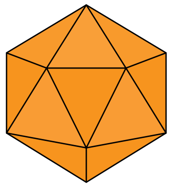
One regular polyhedron is
an **icosahedron**.
Here is a net for one-half
of an icosahedron.



This net can be cut out and folded.
The net has 10 congruent triangles.

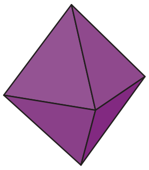


Two of these pieces are taped together to make an icosahedron.

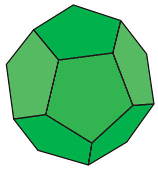


An icosahedron has 20 congruent faces.
Each face is an equilateral triangle.
In the *Practice* questions, you will make the other 2 regular polyhedra.

A regular **octahedron**



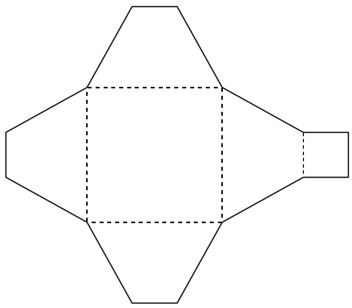
A regular **dodecahedron**



In previous grades, you designed and sketched nets of prisms and pyramids.
In the *Example* that follows, and in *Practice* questions, you will investigate nets for other objects.

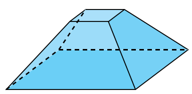
Example

Fold this net to make an object.
Describe the object.



Solution

The net is folded along the broken line segments. Each edge touches another edge. The edges are taped.



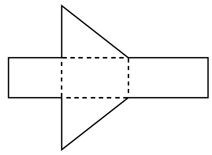
The object looks like the bottom of a pyramid;
that is, a pyramid with the top part removed.
The object is called a **frustum** of a pyramid.

The object has 2 non-congruent square faces, and 4 congruent trapezoid faces. The 2 square faces are parallel.

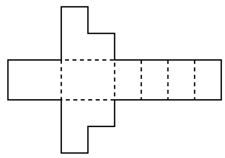
Practice

Your teacher will give you a large copy of each net.

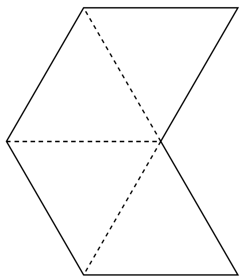
1. Fold this net to make a polyhedron.
   a) Identify the polyhedron.
   b) Describe the polyhedron.



2. Fold this net.
   a) Is the object a polyhedron? If so, what are the attributes that make it a polyhedron?
   b) Identify parallel faces and perpendicular faces.

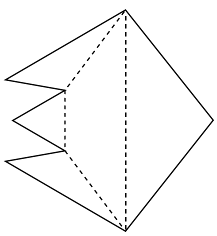


3. Fold two of these nets to make two square pyramids with no base.
Tape the two pyramids together at their missing bases.
You have made a regular octahedron.
   a) Why does it have this name?
   b) Describe the octahedron.
   How do you know it is regular?

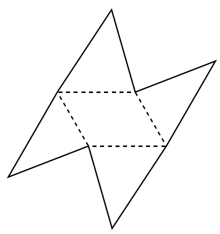


4. Fold each net. Describe each object.
How are the objects the same? How are they different?

a)



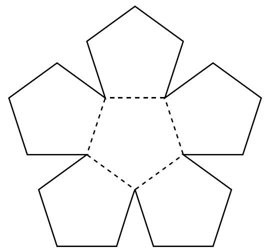
b)



Calculator Skills

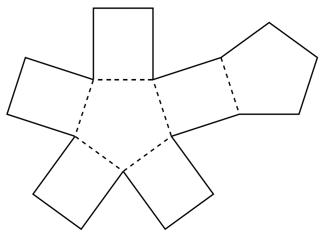
Find three prime numbers that have a sum of 43 and a product of 1085.

5. Fold two of these nets. Put the open parts together. Tape the pieces to make a regular dodecahedron. Describe the dodecahedron.



6. A soccer ball is not a sphere. It is a polyhedron.
Look at a soccer ball.
Explain which polygons are joined to make the ball.
How are the polygons joined?

7. **Assessment Focus** Fold the net shown.

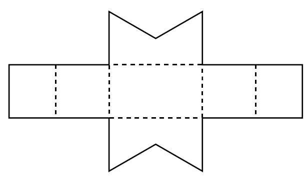


a) Describe the object formed.
b) Name the object. Justify your answer.

8. All the nets you have used fold to make objects.
Which features must be true for this to happen?

Take It Further

9. Use isometric dot paper or plain paper.
Draw the object that has this net. Explain your thinking.



Reflect

Choose a product that has a package in the shape of a polyhedron.
Sketch the package. Include appropriate dimensions.
Cut along the edges to make the net.
Why do you think the manufacturer used this shape for the package?
Can you find a better shape for the packaging? Explain.

Mid-Unit Review

LESSON

3.1 **1.** Sketch the front, back, top, and side views of each object. Label each view.

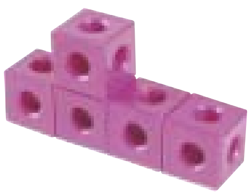
a) Television

b) Baseball mitt

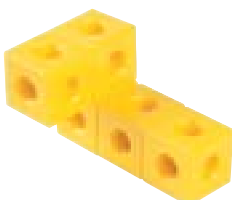
c) Chair

3.2 **2.** Make each object.
Sketch a 3-D view of the object on isometric dot paper.

a)



b)



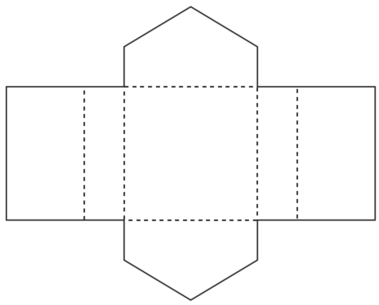
3. Sketch a 3-D view of each object on plain paper.

a)

b)

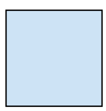
4. Suppose you had to construct an object from linking cubes. Which would you prefer to use: a drawing of the object on isometric dot paper, or 5 different views on plain paper? Justify your choice. Use diagrams to support your choice.

3.3 **5.** Use a large copy of this net.
Fold the net to make an object.
Name the object. List its attributes.

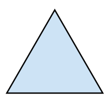


6. Which polyhedra could have each view shown below?
Find as many polyhedra as you can for each view.

a)



b)



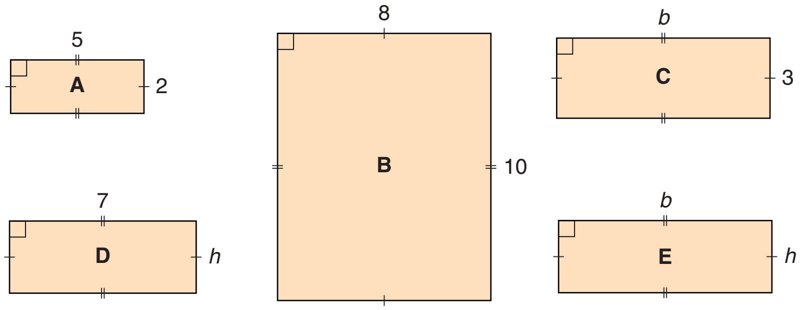
3.4 Using Variables in Measurement Formulas

Focus Use measurement formulas to introduce the concept of a variable.

Explore

Work with a partner.

➢ Find the area of each rectangle below, in square units.



➢ Find the perimeter of each rectangle above, in units.

Reflect & Share

Compare your results with those of another pair of classmates.
How did you find the area when you did not know:

- the base?
- the height?
- the base and height?

How did you find the perimeter in each case?

Connect

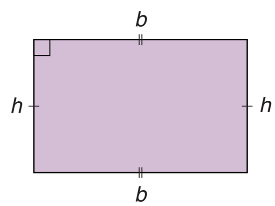
When we do not know the dimensions of a figure,
we use letters to represent them.
For a rectangle, we use b to represent the length of the base,
and h to represent the height.

One way to find the area of a rectangle is to use a formula.
The formula for the area of a rectangle is:

$A = \text{base} \times \text{height}$

Then $A = b \times h$

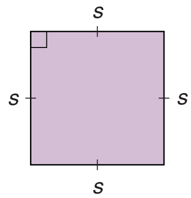
We write $A = b \times h$ as $A = bh$.



The formula for the perimeter of a rectangle is:
$P = 2 \times (\text{base} + \text{height})$
Then $P = 2 \times (b + h)$
We write $P = 2 \times (b + h)$ as $P = 2(b + h)$.

The letters we use to represent the base and height are called **variables**. Variables in a formula can represent different numbers. When we know the values of the variables, we **substitute** for the variables. That is, we replace each variable with a number.

A square is a rectangle with all sides equal.
The formula for the area of a square is:
$A = \text{side length} \times \text{side length}$
Use the variable s for the side length.
Then, $A = s \times s$
or $A = s^2$



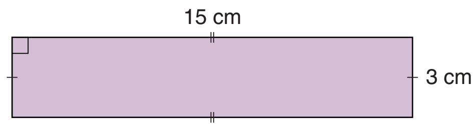
We use exponents to represent repeated multiplication.

The formula for the perimeter of a square is:
$P = 4 \times \text{side length}$
or $P = 4 \times s$

We write $P = 4 \times s$ as $P = 4s$.

Example

A rectangle has base 15 cm and height 3 cm. Use formulas to find its area and perimeter.



Solution

Area, $A = bh$
$b = 15$ and $h = 3$
Substitute for b and h in the formula.
$A = 15 \times 3$
$A = 45$
The area is 45 cm^2.

Perimeter, $P = 2(b + h)$
Substitute for b and h.
$P = 2(15 + 3)$
$P = 2(18)$
$P = 36$
The perimeter is 36 cm.

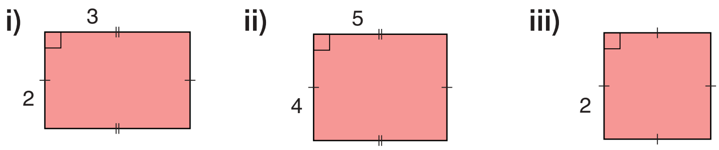
Use order of operations. Do the operation in brackets first. Then multiply.

Practice

1. Use the formula: $P = 4s$
Find the perimeter of the square with each side length.
a) 5 cm **b)** 9 cm **c)** 2 cm **d)** 8 cm

2. Use the formula for the area of a rectangle: $A = bh$
Find the area of each rectangle.
a) base: 6 cm; height: 3 cm **b)** base: 11 cm; height: 2 cm

3. a) For each rectangle, what is the value of b and the value of h?



b) Use your answers to part a to explain why we call b and h variables.
c) What is true about b and h in part a, iii?

Number Strategies

Find the LCM and GCF of 10, 15, and 25.

4. Find the area and perimeter of each rectangle.
a) base: 12 cm; height: 4 cm **b)** base: 10.5 cm; height: 3.0 cm

5. Find the perimeter and area of each square.
a) side length: 2.8 cm **b)** side length: 3.1 cm

6. Assessment Focus
Here is another formula for the perimeter of a rectangle:
$P = 2b + 2h$
Write this formula in words.
Explain why there are two formulas for the perimeter of a rectangle.

Take It Further

7. a) How could you use the formula for the perimeter of a rectangle to get the formula for the perimeter of a square?
b) How could you use the formula for the area of a rectangle to get the formula for the area of a square?

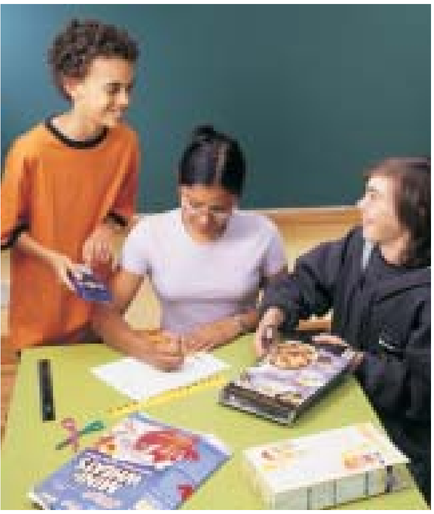
Reflect

What is a variable?
Why do we use variables to write measurement formulas?

3.5 Surface Area of a Rectangular Prism

Focus Use a formula to calculate the surface area of a rectangular prism.

Explore



Work in a group.
You will need several different empty cereal boxes, scissors, and a ruler.

➢ Cut along the edges of a box to make a net.
What is the area of the surface of the box?

➢ Repeat the activity above for 2 other boxes.

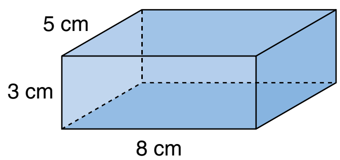
➢ Write a formula to find the area of the surface of a rectangular prism.

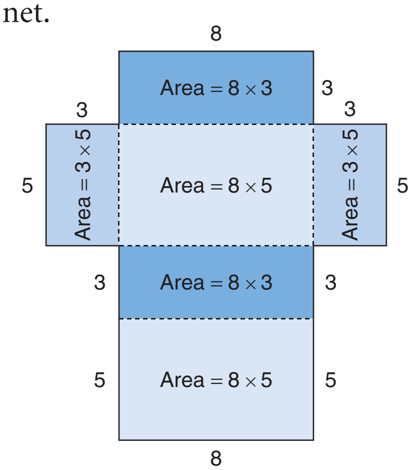
Reflect & Share

Compare your formula with that of another group.
Did you write the same formula?
If not, do both formulas work? Explain.

Connect

Here is a rectangular prism, and its net.





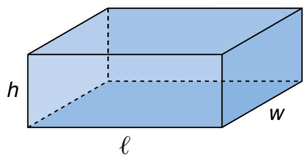
In the net, there are 3 pairs of congruent rectangles.
The area of the net is the sum of the areas of the rectangles.

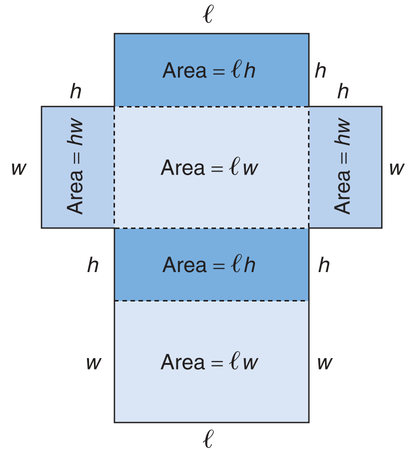
The area of the net $= 2(8 \times 3) + 2(8 \times 5) + 2(3 \times 5)$
$= 2(24) + 2(40) + 2(15)$ Multiply.
$= 48 + 80 + 30$ Add.
$= 158$

Use order of operations, with brackets first.

The area of the net is 158 cm^2.
We say that the **surface area** of the rectangular prism is 158 cm^2.

We can use the net of a rectangular prism to write a formula for its surface area.
We use a variable to label each dimension.
The prism has length l, width w, and height h.





In the net, there are 3 pairs of congruent rectangles.
The surface area of the prism is the sum of the areas of the rectangles.

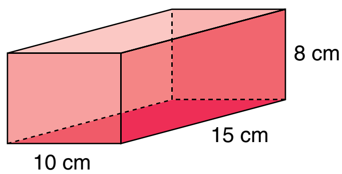
Remember that lw means $l \times w$.

The surface area of the prism $= 2lh + 2lw + 2hw$
We write: $SA = 2lh + 2lw + 2hw$

We can use this formula to find the surface area of a rectangular prism, without drawing a net first.

Example

Find the surface area of this rectangular prism.



Solution

Use the formula:
$SA = 2lh + 2lw + 2hw$
Substitute: $l = 15$, $h = 8$, and $w = 10$

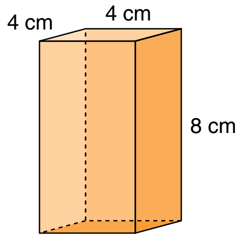
$$\begin{aligned} SA &= 2(15 \times 8) + 2(15 \times 10) + 2(8 \times 10) \\ &= 2(120) + 2(150) + 2(80) \\ &= 240 + 300 + 160 \\ &= 700 \end{aligned}$$

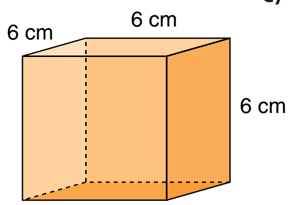
The surface area of the rectangular prism is 700 cm^2.

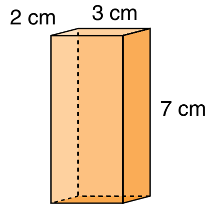
Practice

Visualize the net. How does that help to find the surface area?

1. Find the surface area of each rectangular prism.

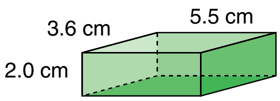
a)


b)


c)


2. Use isometric dot paper or plain paper.
Sketch a rectangular prism with these dimensions:
6 cm by 3 cm by 2 cm.
Find the surface area of the prism.

3. Find the surface area of each rectangular prism.

a)


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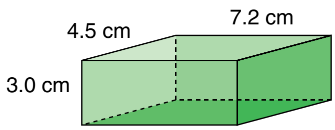
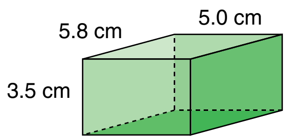


c)


Mental Math

Estimate each product.
What strategies did you use?

Order the products from least to greatest.

- 24.8×3.2
- 35.2×2.8
- 13.2×5.7
- 5.5×15.5

4. Use linking cubes.

a) Find the surface area of a cube with edge length 1 unit.

b) What if the edge length is doubled?
What happens to the surface area?
Make a new cube to find out.

c) What if the edge length is tripled?
What happens to the surface area?
Make a new cube to find out.

d) Predict the surface area of a cube with edge length 4 units.
Explain your prediction. Make a new cube to check.

5. a) Find a rectangular prism in the classroom.
Measure its length, width, and height.
Find its surface area.

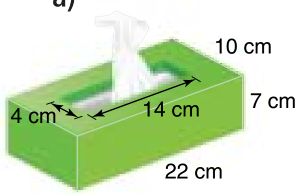
b) Suppose each dimension of the prism is halved.
What happens to the surface area? Explain.

6. Tanya paints the walls of her family room.
The room measures 3 m by 4 m by 7 m.
The walls need 2 coats of paint. A 4-L can of paint covers 40 m^2.
   a) How much paint should Tanya buy?
   b) What assumptions do you make? Explain.

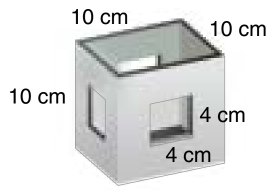
7. **Assessment Focus** Sketch a rectangular prism. Label its dimensions. What do you think happens to the surface area of a prism when its length is doubled? Its length is halved? Investigate to find out. Show your thinking.

8. Each object has the shape of a rectangular prism, but one face or parts of faces are missing. Find each surface area.

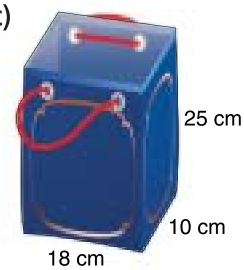
a)



b)



c)



9. The surface area of a cube is 54 cm^2.
   a) What is the area of one face of the cube?
   b) What is the length of one edge of the cube?

Take It Further

10. A 400-g cereal box measures 20 cm by 7 cm by 31 cm.
A 750-g cereal box measures 24 cm by 9 cm by 33 cm.
   a) Find the surface area of each box.
   b) What is an approximate ratio of surface areas?
   What is an approximate ratio of masses?
   c) Compare the ratios in part b.
   Do you expect the ratios to be equal? Explain.

11. A rectangular prism has a square base with area 4 m^2.
The surface area of the prism is 48 m^2.
What are the dimensions of the prism?

12. A rectangular prism has faces with these areas: 12 cm^2, 24 cm^2, and 18 cm^2. What are the dimensions of the prism? Explain.

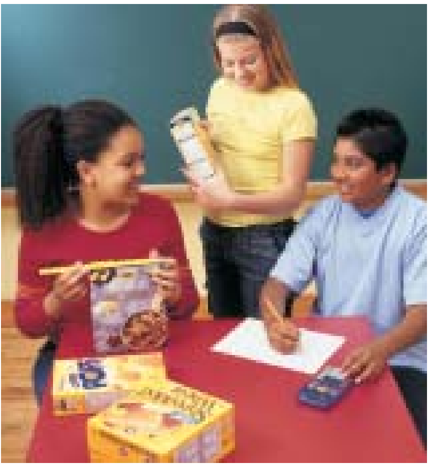
Reflect

Explain how to find the surface area of a rectangular prism using a formula. Include an example in your explanation.

3.6 Volume of a Rectangular Prism

Focus Use a formula to calculate the volume of a rectangular prism.

Explore



Work in a group.
You will need several empty cereal boxes and a ruler.

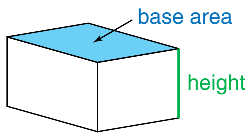
➢ Find the volumes of 3 cereal boxes.
➢ Write a formula you can use to find the volume of a rectangular prism.
➢ Measure a 4th cereal box. Substitute its dimensions in your formula to check that your formula is correct.

Reflect & Share

Compare the formulas for the volume of a rectangular prism and the area of a rectangle. What do you notice? Explain.

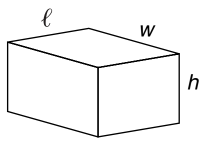
Connect

Recall that the volume of a rectangular prism is:
Volume = base area × height



The base of a rectangular prism can be any face of the prism.

The base of the prism is a rectangle.
Label the length l and the width w.
Then, the area of the base is $l \times w$, or lw.



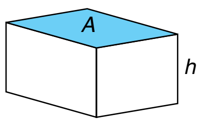
**The height is measured from the base to the opposite face.
The height is perpendicular to the base.**

Label the height h.
Volume = base area × height

$$V = lw \times h$$
$$V = lwh$$

The volume of a rectangular prism is: $V = lwh$

If we let A represent the area of the base, then $A = lw$.



Another way to write the volume is: $V = A \times h$, or $V = Ah$

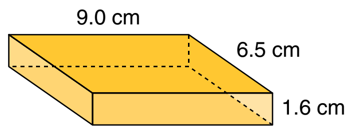
Example

A deck of 54 cards fits in a box with dimensions 6.5 cm by 9.0 cm by 1.6 cm. What is the volume of the box? Give the answer to the nearest cubic centimetre.

Solution

Recall that, when the dimensions are measured in centimetres (cm), the volume is measured in cubic centimetres (cm^3).

Draw a diagram.
The box is a rectangular prism.
Label each dimension.



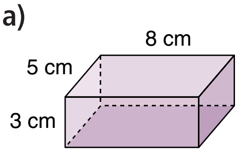
Use the formula: $V = lwh$
Substitute: $l = 9.0$, $w = 6.5$, and $h = 1.6$
$V = 9.0 \times 6.5 \times 1.6$ Use a calculator.
$\;\;\; = 93.6$
The volume is 93.6 cm^3, or about 94 cm^3.

Practice

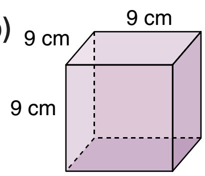
Use a calculator when it helps.

1. For each rectangular prism: find the area of its base, then find its volume.

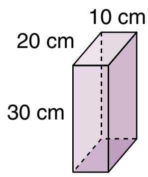
a)



b)

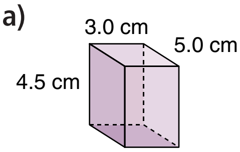


c)

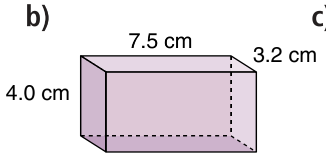


2. Find the volume of each rectangular prism.

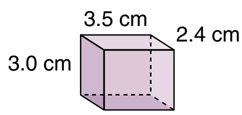
a)



b)



c)



3. Use linking cubes.
Make all possible rectangular prisms with volume 36 $units^3$.
Sketch each prism you make.
Label each prism with its dimensions in units.
How do you know you have found all possible prisms?

4. Philip made fudge that filled a 20-cm by 21-cm by 3-cm pan.
   a) What is the volume of the fudge?
   b) Philip shares the fudge with his classmates. There are 30 people in the class. How much fudge will each person get?
   c) How should Philip cut the fudge so each person gets the same size piece? Sketch the cuts he should make.
   d) What are the dimensions of each piece of fudge?

5. Sketch a rectangular prism. Label its dimensions.
   What do you think happens to the volume of the prism when:
   a) its length is doubled?
   b) its length and width are doubled?
   c) its length, width, and height are doubled?
   Investigate to find out. Show your work.
   Will the results be true for all rectangular prisms?
   How do you know?

6. How can you double the volume of a rectangular prism?
   Does its surface area double, too? Explain.

7. **Assessment Focus** Use linking cubes.
   a) How many rectangular prisms can you make with 2 cubes? 3 cubes? 4 cubes? 5 cubes? 6 cubes? and so on, up to 20 cubes?
   b) How many cubes do you need to make exactly 1 prism? Exactly 2 prisms? Exactly 3 prisms? Exactly 4 prisms?
   c) What patterns do you see in your answers to part b?

8. a) Sketch 3 different rectangular prisms with volume 24 cm^3.
   b) Which prism has the greatest surface area? The least surface area?
   c) Try to find a prism with a greater surface area. Describe the shape of this prism.
   d) Try to find a prism with a lesser surface area. Describe the shape of this prism.

Mental Math

Estimate each quotient. What strategies did you use?

Order the quotients from least to greatest.

- $3572 \div 71$
- $4675 \div 58$
- $5007 \div 43$
- $6729 \div 95$

Reflect

Suppose you know the volume of a rectangular prism.
How can you find its dimensions? Use words and pictures to explain.

Decoding Word Problems

1

READ the problem.

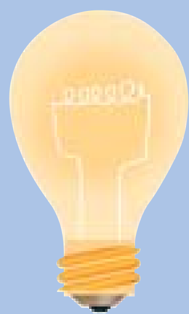
- Do I understand all the words?
- Highlight those words I don't understand. Whom can I ask?
- Are there key words that give me clues? Circle these words.

David, Jennifer, and Mark collect baseball cards. Jennifer has the most cards. She has 15 more cards than David. David has 4 times as many cards as Mark has today. Mark often loses some of his cards. This morning Mark has 18 cards. How many baseball cards does Jennifer have?

2

THINK about the problem.

- What is the problem about?
- What information am I given?
- Am I missing any information?
- Is this problem like another problem I have solved before?



STRATEGIES to consider . . .

Use a model.
Make an organized list.
Make a table.
Use logical reasoning.
Solve a simpler problem.
Draw a diagram.
Guess and check.
Use a pattern.
Work backward.
Draw a graph.

3 MAKE a plan.

- What strategy should I use?
- Would counters, geometric figures, other materials, calculators, and so on, help?

4 TRY out the plan.

- Is the plan working? Should I try something else?
- Have I shown *all* my work?

5 LOOK back.

- Does my answer make sense? How can I check?
- Is there another way to solve this problem?
- How do I know my answer is correct?
- Have I answered the problem?

6 EXTEND.

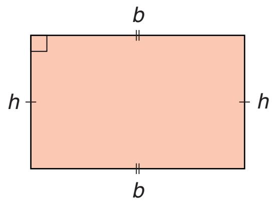
- Ask "What if ...?" questions. What if the numbers were different? What if there were more options?
- Make up similar problems of your own.
- STRETCH your thinking.

Unit Review

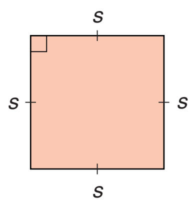
Review any lesson with etext online tutorial

What Do I Need to Know?

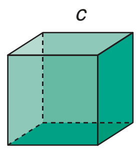
- ✓ Perimeter of a rectangle:
  $P = 2(b + h)$
- ✓ Area of a rectangle:
  $A = bh$



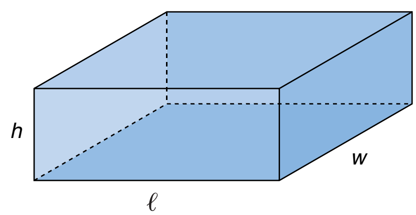
- ✓ Perimeter of a square:
  $P = 4s$
- ✓ Area of a square:
  $A = s^2$

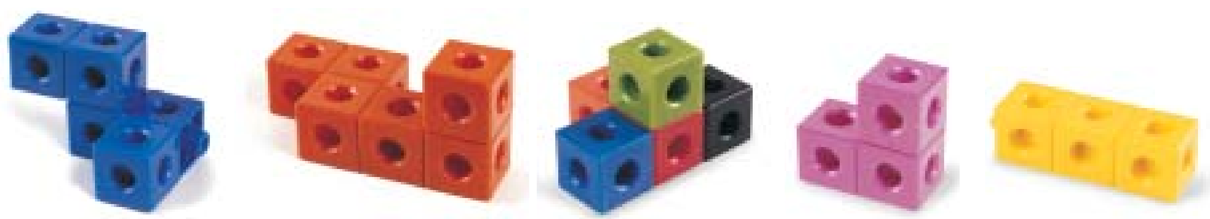


- ✓ Volume of a cube:
  $V = c^3$
- ✓ Surface area of a cube:
  $SA = 6c^2$



- ✓ Surface area of a rectangular prism:
  $SA = 2lh + 2lw + 2hw$
- ✓ Volume of a rectangular prism:
  $V = lwh$
  or $V = Ah$, where $A = lw$





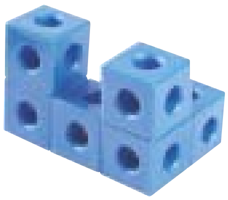
What Should I Be Able to Do?

For extra practice, go to page 440.

LESSON

3.1 **1.** Sketch the front, back, side, and top views of each object. Use square dot paper, plain paper, or *The Geometer's Sketchpad*.

a)

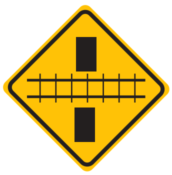


b)

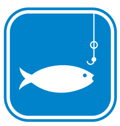
c)

3.2 **2.** **a)** Identify the view and the objects on each sign.

i) Railway Crossing Ahead

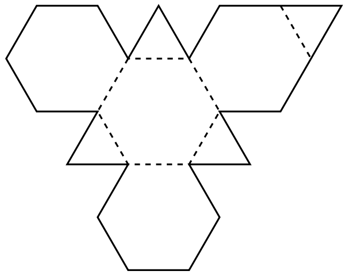


ii) Fishing Area



b) Design a new sign from a different view.
Sketch your design.

3.3 **3.** This is a net for an octahedron.



a) Fold a large copy of the net. Describe the object. Why do you think it is called an octahedron?

b) How is the object you built different from a regular octahedron?

3.4 **4.** Find the area of each figure.

a) a square with side length 7 cm

b) a rectangle with base 12 m and height 3 m

5. A cube has edge length c.

a) Write a formula for the surface area of the cube.

b) Use the formula to find the surface area when c is 4 cm.

6. A children's play area is 90 m long and 44 m wide. A fence will enclose the area.

a) How much fencing is needed?

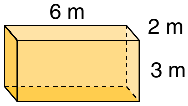
b) Fencing comes in 12-m bundles. Each bundle costs \$35. How much will the fence cost? Justify your answer.

LESSON

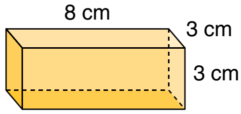
3.5
3.6

7. Find the surface area and volume of each rectangular prism.

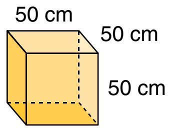
a)



b)



c)



8. Elizabeth pastes wallpaper on 3 walls of her bedroom. She paints the 4th wall. This is one of the smaller walls. The dimensions of the room are 3 m by 5 m by 6 m. A roll of wallpaper covers about 5 m^2. A 4-L can of paint covers about 40 m^2.

a) How much wallpaper and paint should Elizabeth buy?

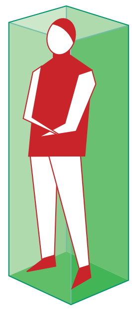
b) What assumptions do you make?

9. The surface area and volume of a cube have the same numerical value. Find the dimensions of this cube. How many answers can you find?

10. Sketch all possible rectangular prisms with a volume of 28 m^3. Each edge length is a whole number of metres. Label each prism with its dimensions. Calculate the surface area of each prism.

11. In *Gulliver's Travels* by Jonathan Swift, Gulliver visits a land where each of his dimensions is 12 times as large as that of the inhabitants.

a) Assume you can be modelled as a rectangular prism. Measure your height, width, and thickness.



b) Suppose you were one of the inhabitants. What would Gulliver's dimensions be?

c) Use your dimensions. Calculate your surface area and volume.

d) Calculate Gulliver's surface area and volume.

12. The volume, V, and base area, A, of a rectangular prism are given. Find the height of each prism. Sketch the prism. What are possible dimensions for the prism? Explain.

a) $V = 18\text{ m}^3$, $A = 6\text{ m}^2$

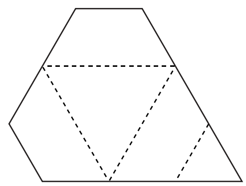
b) $V = 60\text{ cm}^3$, $A = 15\text{ cm}^2$

13. Samya is making candles. She uses a 1-L milk carton as a mould. Its base is a 7-cm square. Samya pours 500 mL of wax into the carton. Recall that 1 mL = 1 cm^3. Approximately how tall will the candle be? Justify your answer.

Practice Test

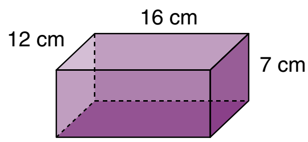
1. Build the letter H with linking cubes.
   Draw the front, back, side, and top views.

2. Fold a copy of this net.



   a) Describe the object.
      What name could you give it?
      Justify your answer.
   b) Sketch the object.

3. Find the surface area and volume of this rectangular prism.



4. Each edge length of a rectangular prism is 7 cm.
   a) Sketch the prism.
   b) Use formulas to find its volume and surface area.

5. Here are the dimensions of two designs for a sandbox for the primary playground. Each sandbox is a rectangular prism.
   One design is 3 m by 3 m by 30 cm.
   A second design is 3.25 m by 3.25 m by 25 cm.
   a) Sketch each sandbox. Include its dimensions.
   b) Compare the designs. For each design, calculate the area of material needed to build it and the volume of sand it will hold.
   c) Which sandbox should be built? Justify your answer.

6. Think about the nets you have folded and the solids you have made.
   Suppose you are a tent manufacturer.
   Which net would you use to make a tent?
   Explain why it would be a good tent design.

Unit Problem Making Food for a Bake Sale

Suppose you are planning to sell baked goods at the local charity bake sale.
You have to decide the selling price for your baked goods.
You want to make sure that the price covers the cost of the ingredients and the packaging.

Decide on the baked goods you will sell.
For example, here is a recipe for Rice Krispies Treats.

Rice Krispies Treats

50 mL margarine
250 g marshmallows
1.5 L Rice Krispies

Melt the margarine.
Add marshmallows and stir until melted.
Remove from heat.
Add Rice Krispies.
Stir until the Rice Krispies are coated.
Press mixture into greased 32-cm by 23-cm pan.
When mixture is cool, cut into squares.

Use this recipe or find your own recipe for treats.
How many batches of treats will you make?
Calculate how much of each ingredient you need.
Use grocery flyers to find how much the ingredients will cost.

1. Calculate the cost of 1 batch of treats.

2. How many treats will you sell in 1 package?
   Design a box as the package for your treats.
   Decide if the box is open and covered with plastic wrap, or if the box is closed.
   Draw a 3-D picture of your box.
   Sketch a net for your box.

3. Calculate the surface area and volume of your box.
   Suppose the cost of cardboard is 50¢/m^2.
   How many boxes can you make from 1 m^2 of cardboard?
   How much does each box cost?
   What other costs are incurred to make the boxes?
   What assumptions do you make?

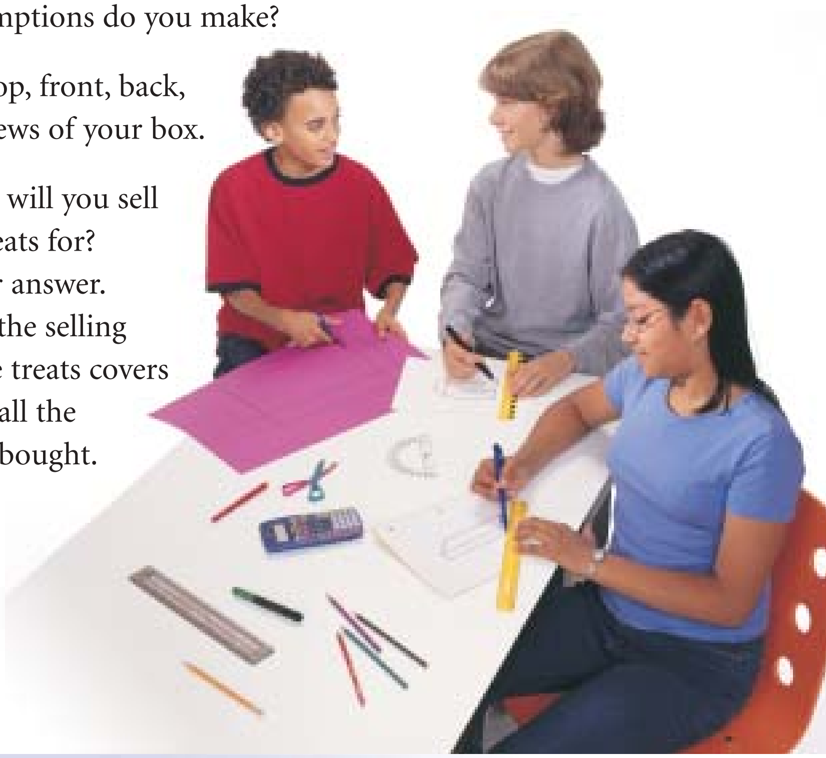
4. Draw the top, front, back, and side views of your box.

5. How much will you sell 1 box of treats for?
   Justify your answer.
   Show how the selling price of the treats covers the cost of all the things you bought.

Check List

Your work should show:

- ✓ all calculations in detail
- ✓ all sketches clearly labelled
- ✓ a clear explanation of your results
- ✓ how you decided the selling price of 1 box of treats



Reflect on the Unit

Write a paragraph to tell what you have learned about polyhedra in this unit. Try to include something from each lesson in the unit.

Cross Strand Investigation

Ratios in Scale Drawings

Materials:
- 1-cm grid paper
  2-cm grid paper
  0.5-cm grid paper
- a ruler

Work with a partner.

A **scale drawing** is larger or smaller than the original drawing or object, but has the same shape. The scale of the drawing is the ratio of a length on the scale drawing to the same length on the original drawing or object.

Polygons that have the same shape but different sizes are **similar**.

As you complete this *Investigation*, include all your work in a report that you will hand in.

Part 1

How are the ratios of corresponding sides in similar polygons related?

➢ Use grid paper.
Draw a polygon with vertices where the grid lines meet.
Trade drawings with your partner.

- Draw a similar polygon that is larger or smaller than the polygon drawn by your partner. You can use the same sheet of grid paper or a sheet with different grid dimensions. Your drawing is a scale drawing of the polygon.
- Measure and record the lengths of the sides of the two polygons.
- Compare the lengths of corresponding sides of the two polygons. What are the ratios? Are the ratios the same for each pair? Explain.
- Predict the ratio of the perimeters of the polygons. Check your prediction.
- Compare your findings with those of your partner. Explain any differences.

Part 2

How are the ratios of corresponding sides related to the ratios of the areas of the polygons?

- Draw a polygon with vertices where grid lines meet. Draw a similar polygon that is larger. What is the ratio of the corresponding sides?
- Predict the ratio of the area of the original polygon to the area of the second polygon. Explain your reasoning for making this prediction.
- Find the area of each polygon. What is the ratio of the two areas?
- How does your prediction compare to your findings? Explain any differences.
- How is the ratio of the corresponding sides of the polygons related to the ratio of the areas of the polygons?

Take It Further

- Suppose you have to enlarge a polygon so that its area is doubled. Should you double the length of each side? Explain. Use an example in your explanation.
- Suppose you have to enlarge a polygon so that its area is four times its original area. How would you do that?
- What patterns do you see? How could you use these patterns to enlarge a polygon even more?

UNIT 4

Fractions and Decimals

Many newspapers and magazines sell advertising space. Why do small companies run small advertisements?

Selling advertising space is a good way to raise funds. Students at Garden Avenue School plan to sell advertising space in their yearbook.

How can fractions and decimals be used in advertising space and advertising rates?

What You'll Learn

- Add and subtract fractions.
- Multiply a fraction by a whole number.
- Compare and order decimals.
- Multiply and divide decimals.
- Use order of operations with decimals.
- Solve problems using fractions and decimals.

Why It's Important

- You use fractions when you share or divide.
- You use decimals when you shop and when you measure.

Key Words

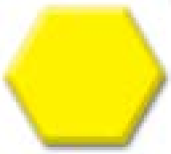
- fraction strips
- equivalent fractions
- related denominators
- unrelated denominators
- common denominator
- lowest common denominator
- unit fraction
- terminating decimal
- repeating decimal

UNIT 4

Skills You'll Need

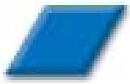
Adding and Subtracting Fractions with Pattern Blocks

Let the yellow hexagon represent 1:

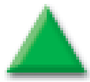


Then the red trapezoid represents $\frac{1}{2}$:

the blue rhombus represents $\frac{1}{3}$:



and the green triangle represents $\frac{1}{6}$:



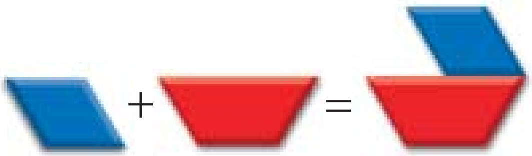
Example 1

Use Pattern Blocks to add and subtract.

a) $\frac{1}{3} + \frac{1}{2}$ b) $\frac{1}{2} - \frac{1}{3}$

Solution

a) $\frac{1}{3} + \frac{1}{2}$



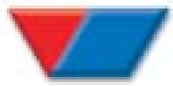
$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

b) $\frac{1}{2} - \frac{1}{3}$

Show $\frac{1}{2}$.

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Cover $\frac{1}{3}$.



$\frac{1}{6}$ is not covered.

✓ Check

1. Use Pattern Blocks to add.

a) $\frac{1}{3} + \frac{2}{3}$ b) $\frac{4}{6} + \frac{1}{6}$ c) $\frac{7}{6} + \frac{1}{2}$ d) $\frac{5}{6} + \frac{2}{3}$

2. Use Pattern Blocks to subtract.

a) $\frac{5}{6} - \frac{2}{6}$ b) $\frac{2}{3} - \frac{1}{6}$ c) $\frac{3}{2} - \frac{2}{3}$ d) $\frac{5}{6} - \frac{2}{3}$

Multiplying by 0.1, 0.01, and 0.001

We can use patterns to multiply by 0.1, 0.01, and 0.001.
We can show products on a place-value chart.

	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
24×100	2	4	0	0 •			
24×10		2	4	0 •			
24×1			2	4 •			
24×0.1				2 •	4		
24×0.01				0 •	2	4	
24×0.001				0 •	0	2	4

On the chart:

- To multiply by 0.1, move each digit 1 place to the right.
- To multiply by 0.01, move each digit 2 places to the right.
- To multiply by 0.001, move each digit 3 places to the right.

To get the product:

- Move the decimal point 1 place to the left.
- Move the decimal point 2 places to the left.
- Move the decimal point 3 places to the left.

Example 2

Multiply.

a) 372×0.1 **b)** 56×0.01 **c)** 41×0.001

Solution

Mark the decimal point in the whole number.

a) $372. \times 0.1 = 37.2$ The decimal point moves 1 place to the left.

b) $56. \times 0.01 = 0.56$ The decimal point moves 2 places to the left.

c) $41. \times 0.001 = 0.041$ Write zeros as placeholders. Then move the decimal point 3 places to the left.

✓ Check

3. Multiply.

a) 5×0.1 **b)** 98×0.1 **c)** 124×0.1

d) 326×0.01 **e)** 72×0.01 **f)** 6×0.01

g) 56×0.001 **h)** 276×0.001 **i)** 8×0.001

Operations with Decimals

Example 3

Evaluate.

a) $82.34 + 4.7$ **b)** $79.1 - 43.8$ **c)** 426.31×2 **d)** $9.47 \div 2$

Solution

a) $82.34 + 4.7$

Add the hundredths.
Add the tenths: 10 tenths = 1 whole
Add the ones. Add the tens.

Estimate: $80 + 5 = 85$

$$\begin{array}{r} \scriptstyle 1 \\ 82.34 \\ +\ 4.7 \\ \hline 87.04 \end{array}$$

b) $79.1 - 43.8$

To subtract the tenths, trade 1 whole for 10 tenths.
Subtract the ones. Subtract the tens.

Estimate: $80 - 40 = 40$

$$\begin{array}{r} \scriptstyle 8\ 11 \\ 7\cancel{9}.\cancel{1} \\ -\ 43.8 \\ \hline 35.3 \end{array}$$

c) 426.31×2

Ignore the decimal point.
Multiply as you would with whole numbers.
Place the decimal point in the answer by estimation:
426.31×2 is about $400 \times 2 = 800$
So, $426.31 \times 2 = 852.62$

Estimate: $400 \times 2 = 800$

$$\begin{array}{r} 426.31 \\ \times \qquad 2 \\ \hline 852.62 \end{array}$$

d) $9.47 \div 2$

Use short division.
Divide 9 ones by 2. There is 1 whole left.
Trade 1 whole for 10 tenths.
Divide 14 tenths by 2.
Divide 7 hundredths by 2. There is 1 hundredth left.
Trade 1 hundredth for 10 thousandths. Divide 10 thousandths by 2.
So, $9.47 \div 2 = 4.735$

Estimate: $10 \div 2 = 5$

$$\begin{array}{r|l} 2 & 9.{}^{1}47{}^{1}0 \\ \hline & 4.\ 73\ 5 \end{array}$$

✓ Check

4. Evaluate.

a) $12.3 + 3.5$ **b)** $21.41 - 13.8$ **c)** 31.47×4 **d)** $7.44 \div 2$

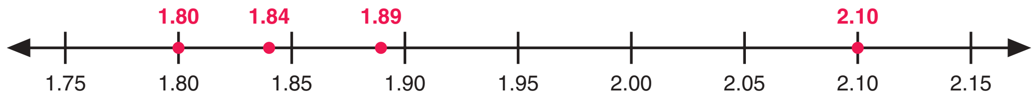
e) 182.34×7 **f)** $52.103 + 71.81$ **g)** $49.35 \div 3$ **h)** 138.97×6

Comparing and Ordering Decimals

Here are two ways to order these decimals from least to greatest: 1.84, 2.10, 1.80, 1.89

➢ Use a number line.

Mark each decimal on a number line in hundredths from 1.75 to 2.15.
1.84 is between 1.80 and 1.85, but closer to 1.85.
1.89 is between 1.85 and 1.90, but closer to 1.90.



➢ Use place value.

1.84, 2.10, **1**.80, **1**.89
In the ones place, the least digit is 1.
Three decimals have 1 as a ones digit.
Compare these 3 decimals: 1.8**4**, 1.8**0**, 1.8**9**
In the tenths place, each decimal has the digit 8.
So, look in the hundredths place:
The least hundredths digit is 0; so, 1.80 is the least decimal.
The next hundredths digit is 4; so, 1.84 is the next largest decimal.
The next hundredths digit is 9; so, 1.89 is the next largest decimal.

So, from least to greatest: 1.80, 1.84, 1.89, 2.10

Example 4

Compare each pair of decimals. Place > or < between each pair.

a) 0.5 and 0.08

b) 47.305 and 47.5

Solution

a) 0.**5** and 0.**0**8
Both decimals have 0 ones.
In the tenths place, 5 > 0
So, 0.5 > 0.08

b) 47.**3**05 and 47.**5**
Both decimals have 4 tens and 7 ones.
In the tenths place, 3 < 5
So, 47.305 < 47.5

✓ Check

5. Order the numbers in each set from least to greatest.

a) 7.32, 4.116, 3.79, 4.12, 3.1

b) 4.4, 0.62, 2.591, 0.65, 4.15

c) 1.25, 3.62, 1.43, 2.81, 2.55

d) 3.669, 1.752, 3.68, 2.67, 1.8

4.1 Combining Fractions

Focus Find equivalent fractions and add fractions.

We can use an area model to show fractions of one whole.

Explore

Work with a partner.
Your teacher will give you a copy of the map.
The map shows a section of land owned by 8 families.

- What fraction of the land did each family own?
  What strategies did you use?

Four families sold land to the other 4 families.

- Use the clues below to draw the new map.
- Write addition equations, such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, to keep track of the land sales.

CLUES

1: When all the sales were finished, four families owned all the land – Smith, Perry, Haynes, and Chan.

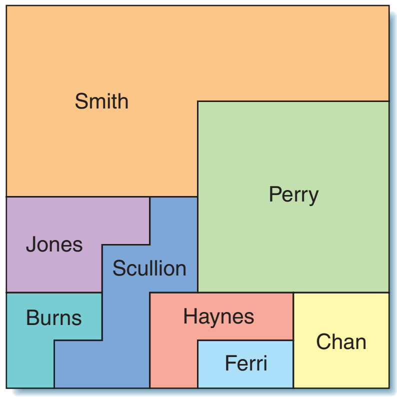
2: Each owner can walk on her or his land without having to cross someone else's property.

3: Smith now owns $\frac{1}{2}$ of the land.

4: Perry kept $\frac{1}{2}$ of her land, and sold the other half to Chan.

5: Haynes bought land from two other people.
He now owns $\frac{3}{16}$ of the land.

6 : Chan now owns the same amount of land as Haynes.



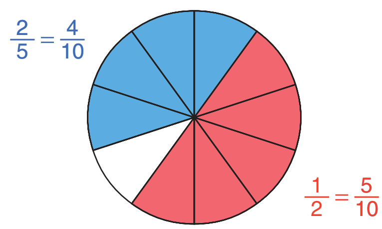
Reflect & Share

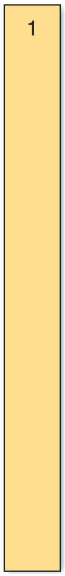
Did you find any equivalent fractions? How do you know?
Which clues helped you most to draw the new map?
Explain how they helped.

Connect

This circle shows equivalent fractions.

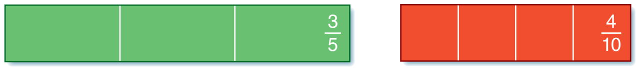
The circle also shows:
$\frac{1}{2} + \frac{2}{5} + \frac{1}{10} = 1$



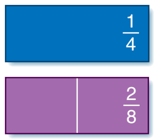


This strip represents 1 whole.

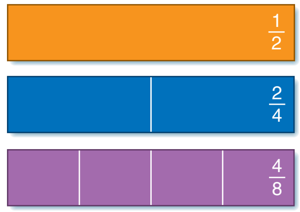
You can model fractions with strips of paper called **fraction strips**.



Here are more fraction strips and some equivalent fractions they show.

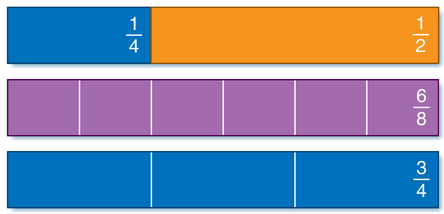


$\frac{1}{4} = \frac{2}{8}$



$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

To add $\frac{1}{4} + \frac{1}{2}$, estimate first.
You know that $\frac{1}{2} + \frac{1}{2} = 1$. Since $\frac{1}{4} < \frac{1}{2}$, then $\frac{1}{4} + \frac{1}{2} < 1$
Use fraction strips to add. Align the strips for $\frac{1}{4}$ and $\frac{1}{2}$.
Find a single strip that has the same length as the two strips.
There are 2 single strips: $\frac{6}{8}$ and $\frac{3}{4}$.
So, $\frac{1}{4} + \frac{1}{2} = \frac{6}{8}$
And, $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
$\frac{3}{4}$ and $\frac{6}{8}$ are **equivalent fractions.**
They represent the same amount.



Example

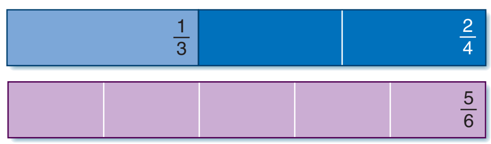
Use fraction strips to add.
$\frac{1}{3} + \frac{2}{4}$

Solution

Estimate: $\frac{1}{3} < \frac{1}{2}$; $\frac{2}{4} = \frac{1}{2}$
So, $\frac{1}{3} + \frac{2}{4} < 1$

$\frac{1}{3} + \frac{2}{4}$

Use the $\frac{1}{3}$- and $\frac{2}{4}$-fraction strips.
These strips align with the $\frac{5}{6}$-strip.



$\frac{1}{3} + \frac{2}{4} = \frac{5}{6}$

Practice

Use models.

1. Which fraction is greater? How do you know?

a) $\frac{2}{3}, \frac{2}{5}$ **b)** $\frac{3}{4}, \frac{2}{3}$ **c)** $\frac{5}{8}, \frac{3}{5}$ **d)** $\frac{3}{4}, \frac{5}{6}$

2. **a)** Add.

i) $\frac{1}{5} + \frac{1}{5}$ **ii)** $\frac{2}{3} + \frac{1}{3}$ **iii)** $\frac{4}{10} + \frac{3}{10}$ **iv)** $\frac{1}{4} + \frac{2}{4}$

b) Look at your work in part a.
How else could you add fractions with the same denominator?

3. Add. Estimate first.

a) $\frac{1}{5} + \frac{1}{10}$ **b)** $\frac{1}{2} + \frac{1}{3}$ **c)** $\frac{1}{6} + \frac{1}{3}$ **d)** $\frac{1}{4} + \frac{1}{8}$

Number Strategies

A loonie has a diameter of about 25 mm. About how many loonies, laid side by side, would there be in 1 km?

4. Add. Estimate first.

a) $\frac{2}{4} + \frac{3}{8}$ **b)** $\frac{2}{3} + \frac{1}{6}$ **c)** $\frac{2}{5} + \frac{2}{10}$ **d)** $\frac{3}{6} + \frac{4}{8}$

5. Find 2 fractions that have a sum of 1.
Try to do this as many ways as you can.

6. Meena's family had a pizza for dinner. The pizza was cut into 8 equal pieces. Meena ate 1 piece, her brother ate 2 pieces, and her mother ate 3 pieces.

a) What fraction of the pizza did Meena eat? Her brother eat? Her mother eat?

b) Which person's fraction can you write in more than one way? Explain.

c) What fraction of the pizza was eaten? What fraction was left?

7. Find the missing number that makes both sides equal.

a) $\frac{1}{5} + \frac{1}{2} = \frac{\square}{10}$ **b)** $\frac{\square}{10} + \frac{2}{5} = \frac{6}{10}$ **c)** $\frac{1}{2} + \frac{3}{\square} = \frac{7}{8}$

8. **Assessment Focus** Boris added 2 fractions. Their sum was $\frac{5}{6}$.
Which 2 fractions might Boris have added?
Find as many pairs of fractions as you can.

Reflect

Write 4 equivalent fractions.
How are these fractions the same? How are they different? Explain.

Join the Dots

YOU WILL NEED

Square dot paper

NUMBER OF PLAYERS

2 or more

GOAL OF THE GAME

To write the greatest number of equivalent fractions

HOW TO PLAY THE GAME:

1. Use dot paper. Mark an array of 49 dots.

2. Take turns to join 2 dots.

3. The player who completes a square claims that square by writing her initials in it.

4. Continue playing until all squares are claimed.

5. Each player writes the squares he has as a fraction of the whole.

6. The winner is the player who can write the greatest number of equivalent fractions for her share.

What if you used a square array of 81 dots. How would this affect the game?

4.2 Adding Fractions Using Models

Focus Add fractions using circles or fraction strips.

Explore

Work on your own.

Baljit trains for cross-country one hour a day.
One day, she ran for $\frac{1}{3}$ of the time, walked for 25 minutes, then got a second wind and ran for the rest of the time.

How long did Baljit run altogether?
What fraction of the hour is this?

- Use fractions to write an addition equation to show how Baljit spent her hour. Baljit never runs for the whole hour.
- Write another possible training schedule for Baljit. Share it with a classmate.
- Write an additon equation for your classmate's training schedule.

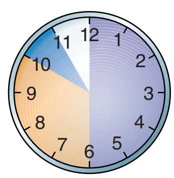
Reflect & Share

Compare your equations with your classmate's equation.
Were the equations the same? Explain.
When might a clock be a good model for thinking about adding fractions?
When is a clock not a good model?

Connect

There are many models that help us add fractions.

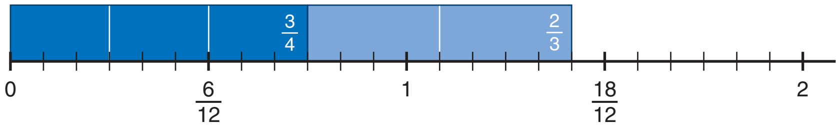
- We could use clocks to model halves, thirds, fourths, sixths, and twelfths.



$$\frac{1}{2} + \frac{1}{3} + \frac{1}{12} = \frac{11}{12}$$

Circle models are useful when the sum is less than 1.

- When the sum is greater than 1, we could use fraction strips and a number line.



$\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$

Example

Add. $\frac{1}{2} + \frac{4}{5}$

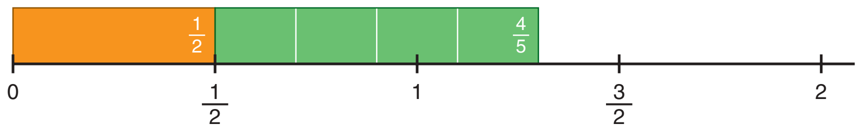
Solution

$\frac{1}{2} + \frac{4}{5}$

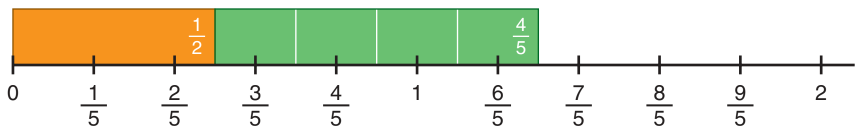
Estimate:
$\frac{4}{5}$ is close to 1; so, $\frac{1}{2} + \frac{4}{5} > 1$

Place both strips end-to-end on the halves line.

The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the halves line.

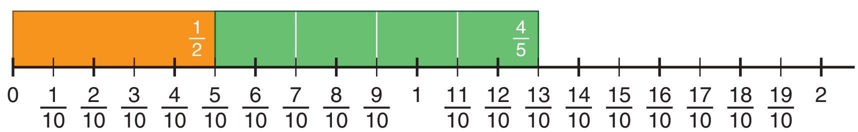


Place both strips on the fifths line.



The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the fifths line.

Find a line on which to place both strips so the end of the $\frac{4}{5}$-strip lines up with a fraction.



The end of the $\frac{4}{5}$-strip lines up with a fraction on the tenths line.

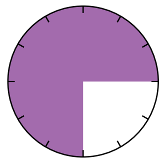
The strips end at $\frac{13}{10}$.

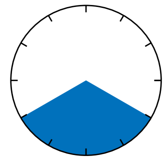
So, $\frac{1}{2} + \frac{4}{5} = \frac{13}{10}$

Practice

Use models.

1. Write 2 equivalent fractions for each fraction shown.

a) 

b) 

2. Add.

a) $\frac{2}{4} + \frac{3}{4}$ b) $\frac{8}{10} + \frac{9}{10}$ c) $\frac{3}{5} + \frac{4}{5}$ d) $\frac{7}{8} + \frac{7}{8}$

3. Find 2 fractions with a sum of $\frac{3}{2}$.
Try to do this as many ways as you can.
Record each way you find.

4. Add. Estimate first.

a) $\frac{7}{8} + \frac{1}{2}$ b) $\frac{7}{10} + \frac{3}{5}$ c) $\frac{1}{2} + \frac{3}{4}$ d) $\frac{5}{6} + \frac{2}{3}$

5. Add. Estimate first.

a) $\frac{3}{8} + \frac{3}{4}$ b) $\frac{4}{4} + \frac{1}{2}$ c) $\frac{2}{2} + \frac{4}{6}$ d) $\frac{1}{2} + \frac{9}{10}$

6. Use your answers to questions 4 and 5.

a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?

b) The denominators in each part of questions 4 and 5 are **related denominators**.
Why do you think they have this name?

7. Add.

a) $\frac{1}{2} + \frac{2}{3}$ b) $\frac{1}{2} + \frac{2}{5}$ c) $\frac{1}{3} + \frac{3}{4}$ d) $\frac{2}{2} + \frac{3}{5}$

8. Look at your answers to question 7.

a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?

b) The denominators in each part of question 7 are called **unrelated denominators**.
Why do you think they have this name?

c) When you add 2 fractions with unrelated denominators, how do you decide which number line to use?

Calculator Skills

How many days are there in one million seconds?

9. One day Ryan ran for 30 min, rested for 20 min, and then ran for another 45 min. Use fractions of one hour. Write an addition equation that represents his training session.

10. A jug holds 2 cups of liquid. A recipe for punch is $\frac{1}{2}$ cup of orange juice, $\frac{1}{4}$ cup of raspberry juice, $\frac{3}{8}$ cup of grapefruit juice, and $\frac{5}{8}$ cup of lemonade. Is the jug big enough for the punch? Explain.

11. **Assessment Focus** Use any of the digits 1, 2, 3, 4, 5, 6 only once. Copy and complete. Replace each □ with a number.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

a) Find as many sums as you can that are between 1 and 2.
b) Find the least sum that is greater than 1.
Show your work.

12. Abey and Anoki are eating chocolate bars. The bars are the same size. Abey has $\frac{3}{4}$ left. Anoki has $\frac{7}{8}$ left. How much chocolate is left altogether?

Take It Further

13. A pitcher of juice is half empty. After $\frac{1}{2}$ cup of juice is added, the pitcher is $\frac{3}{4}$ full. How much juice does the pitcher hold when it is full? Show your thinking.

14. Which number line would you need to add each pair of fractions? Explain.

a) $\frac{1}{3} + \frac{1}{8}$ **b)** $\frac{1}{3} + \frac{1}{5}$ **c)** $\frac{1}{4} + \frac{1}{5}$ **d)** $\frac{1}{4} + \frac{1}{6}$

Reflect

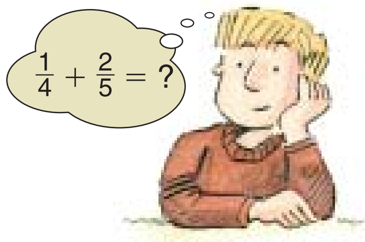
Which strategies do you use to add 2 fractions?
Include 3 different examples in your answer.

4.3 Adding Fractions

Focus Add fractions using symbols.

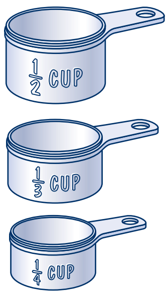
In *Section 4.2*, you used models to add fractions.
A clock model only works with certain fractions.
You may not always have suitable fraction strips.

We need a new strategy we can use to add fractions without using a model.



Explore

Work with a partner.



A cookie recipe calls for $\frac{3}{8}$ cup of brown sugar and $\frac{1}{3}$ cup of white sugar.
How much sugar is needed altogether?
How can you find out?
Show your work.

Reflect & Share

Describe your strategy.
Will your strategy work with all fractions?

Test it with $\frac{4}{5} + \frac{2}{3}$.

Use models to justify your strategy.

Connect

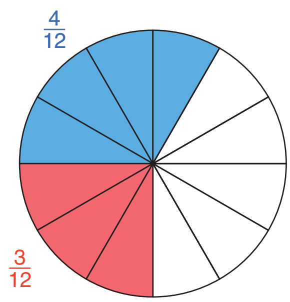
We can use equivalent fractions to add $\frac{1}{4} + \frac{1}{3}$.

Use equivalent fractions that have the same denominators.
12 is a multiple of 3 and 4.
12 is a **common denominator**.

$\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{3} = \frac{4}{12}$

$$\text{So, } \frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12}$$
$$= \frac{7}{12}$$

Both fractions are written with the same denominator.



A fraction is in **simplest form** when the numerator and denominator have no common factors.

Look at the pattern in the equivalent fractions below.

$$\frac{1}{4} \overset{\times 3}{\underset{\times 3}{=}} \frac{3}{12} \qquad \frac{1}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{4}{12}$$

So, to get an equivalent fraction, multiply numerator and denominator by the same number.

We may also get equivalent fractions by dividing.

For example, $\frac{8}{10}$ can be written $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

This fraction is in simplest form.

Example 1

Add. $\frac{5}{6} + \frac{2}{9}$

Solution

Estimate:
$\frac{5}{6}$ is close to 1; $\frac{2}{9} < \frac{1}{2}$
So, $\frac{5}{6} + \frac{2}{9}$ is less than $1\frac{1}{2}$.

$\frac{5}{6} + \frac{2}{9}$

The common denominator is a multiple of 6 and 9.

List the multiples of 6: 6, 12, **18**, 24, …

List the multiples of 9: 9, **18**, 27, 36, …

18 is the lowest common multiple of 6 and 9.

So, choose 18 as the common denominator.

$$\frac{5}{6} \overset{\times 3}{\underset{\times 3}{=}} \frac{15}{18} \qquad \frac{2}{9} \overset{\times 2}{\underset{\times 2}{=}} \frac{4}{18}$$

We say that 18 is the **lowest common denominator**.

$$\frac{5}{6} + \frac{2}{9} = \frac{15}{18} + \frac{4}{18}$$
$$= \frac{19}{18}$$

We can write $\frac{19}{18}$ as a mixed number: $1\frac{1}{18}$

Example 2

Add. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

Solution

Estimate:
$\frac{2}{3} > \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$
So, $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ is greater than $1\frac{1}{2}$.

$\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

2 is a multiple of 4.

So, the common denominator is a multiple of 3 and 4.

List the multiples of 3: 3, 6, 9, **12**, 15, 18, …

List the multiples of 4: 4, 8, **12**, 16, 20, 24, …

12 is the lowest common multiple of 3 and 4.

So, 12 is the lowest common denominator.

$$\frac{1}{2} \xrightarrow{\times 6} \frac{6}{12} \qquad \frac{2}{3} \xrightarrow{\times 4} \frac{8}{12} \qquad \frac{3}{4} \xrightarrow{\times 3} \frac{9}{12}$$

$$\frac{1}{2} = \frac{6}{12} \qquad \frac{2}{3} = \frac{8}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{6}{12} + \frac{8}{12} + \frac{9}{12}$$

$$= \frac{23}{12}$$

We can write $\frac{23}{12}$ as a mixed number: $1\frac{11}{12}$

Practice

A fraction with numerator 1 is a **unit fraction**.

1. Add. Use grid paper. Draw a picture to show each sum.

a) $\frac{1}{2} + \frac{1}{3}$ **b)** $\frac{1}{3} + \frac{1}{5}$ **c)** $\frac{1}{4} + \frac{1}{5}$ **d)** $\frac{1}{5} + \frac{1}{6}$

2. Copy and complete. Replace each $\square$ with a number to make a true sentence.

a) $\frac{3}{12} = \frac{\square}{4}$ **b)** $\frac{3}{4} = \frac{6}{\square}$ **c)** $\frac{3}{6} = \frac{\square}{4}$ **d)** $\frac{6}{8} = \frac{15}{\square}$

3. Add. Estimate first.

a) $\frac{4}{5} + \frac{1}{2}$ **b)** $\frac{3}{4} + \frac{1}{3}$ **c)** $\frac{2}{3} + \frac{4}{5}$ **d)** $\frac{2}{3} + \frac{3}{4}$

4. Add.

a) $\frac{2}{3} + \frac{2}{9}$ **b)** $\frac{1}{6} + \frac{5}{12}$ **c)** $\frac{3}{8} + \frac{1}{2}$ **d)** $\frac{3}{4} + \frac{7}{8}$

5. Add.

a) $\frac{5}{6} + \frac{1}{4}$ **b)** $\frac{1}{6} + \frac{4}{9}$ **c)** $\frac{7}{10} + \frac{4}{6}$ **d)** $\frac{3}{4} + \frac{3}{10}$

6. One page of a magazine had 2 advertisements.
One was $\frac{1}{8}$ of the page, the other $\frac{1}{16}$.
What fraction of the page did these occupy?

7. Which sum is greater? How do you know?

$\frac{2}{3} + \frac{5}{6}$ or $\frac{3}{4} + \frac{4}{5}$

8. Add.

a) $\frac{3}{8} + \frac{1}{2} + \frac{3}{4}$ **b)** $\frac{1}{4} + \frac{3}{2} + \frac{2}{5}$ **c)** $\frac{2}{3} + \frac{5}{6} + \frac{4}{9}$

Mental Math

Multiply.
- $5 \times 27 \times 2$
- $10 \times 3 \times 31$
- $4 \times 9 \times 25$
- $6 \times 20 \times 5$

Which mental math strategies did you use?

To add 2 mixed numbers:
Add the whole numbers.
Add the fractions.
Write the sum as a mixed number.

9. Add.

a) $1\frac{1}{6} + 2\frac{1}{2}$ **b)** $3\frac{1}{3} + 1\frac{1}{2}$ **c)** $4\frac{1}{6} + 2\frac{3}{8}$

10. A recipe for punch calls for $2\frac{2}{3}$ cups of fruit concentrate and $6\frac{3}{4}$ cups of water.
How many cups of punch will the recipe make?

11. **Assessment Focus** Three people shared a pie.
Which statement is true? Can both statements be true?
Use diagrams to show your thinking.

a) Edna ate $\frac{1}{10}$, Farrah ate $\frac{3}{5}$, and Fran ate $\frac{1}{2}$.

b) Edna ate $\frac{3}{10}$, Farrah ate $\frac{1}{5}$, and Fran ate $\frac{1}{2}$.

Take It Further

12. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

Find 3 other fractions with different denominators that add to 1.
Explain your strategy.

13. Copy this sum. Replace □ with the correct digit.

$2\frac{1}{4} + 1\frac{\square}{3} = 3\frac{7}{12}$

Explain your strategy.

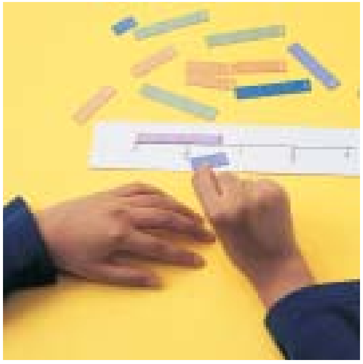
Reflect

When you add fractions, and the denominators are different, how do you add?
Give 2 different examples. Use pictures to show your thinking.

4.4 Subtracting Fractions Using Models

Focus Subtract fractions using fraction strips and number lines.

Explore



Work with a partner.
You will need fraction strips and number lines.

Find 2 fractions with a difference of $\frac{1}{2}$.

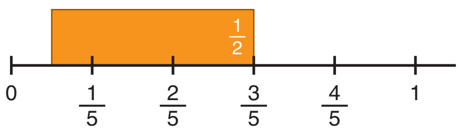
How many different pairs of fractions can you find?
Record each pair.

Reflect & Share

Discuss with your partner.
How are your strategies for subtracting fractions the same as your strategies for adding fractions? How are they different?

Connect

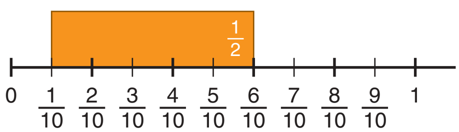
To subtract $\frac{3}{5} - \frac{1}{2}$, think addition: What do we add to $\frac{1}{2}$ to get $\frac{3}{5}$?
Try the fifths number line.
Place the $\frac{1}{2}$-strip with its right end at $\frac{3}{5}$.



Estimate:
$\frac{3}{5} < 1$; so, $\frac{3}{5} - \frac{1}{2} < \frac{1}{2}$

Equivalent fractions:
$\frac{3}{5} = \frac{6}{10}$
$\frac{1}{2} = \frac{5}{10}$

The left end of the strip does not line up with a fraction on the line.
Use a number line that has equivalent fractions for halves and fifths.
Put the $\frac{1}{2}$-strip on the tenths number line, with its right end at $\frac{6}{10}$.



The left end of the strip is at $\frac{1}{10}$.

So, $\frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

Example

Use fraction strips and number lines to subtract.

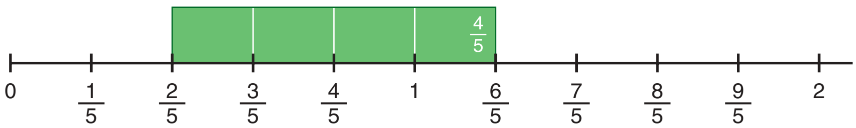
a) $\frac{6}{5} - \frac{4}{5}$ **b)** $\frac{5}{8} - \frac{1}{4}$

Solution

a) $\frac{6}{5} - \frac{4}{5}$

Estimate:
$\frac{6}{5} > 1, \frac{4}{5} < 1;$
so, $\frac{6}{5} - \frac{4}{5} < 1$

Think addition.
What do we add to $\frac{4}{5}$ to get $\frac{6}{5}$?
Use the fifths number line because both denominators are 5.
Place the $\frac{4}{5}$-strip on the fifths number line with its right end at $\frac{6}{5}$.



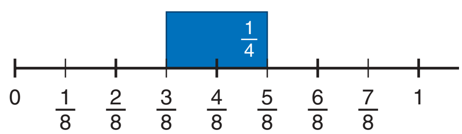
The left end of the strip is at $\frac{2}{5}$.
So, $\frac{6}{5} - \frac{4}{5} = \frac{2}{5}$

b) $\frac{5}{8} - \frac{1}{4}$

Estimate:
$\frac{5}{8}$ is between $\frac{1}{2}$ and 1,
$\frac{1}{4} < \frac{1}{2}$; so , $\frac{5}{8} - \frac{1}{4} < \frac{1}{2}$

Think addition.
What do we add to $\frac{1}{4}$ to get $\frac{5}{8}$?

Use a number line that shows equivalent fractions for eighths and fourths.
That is, use the eighths number line.
Place the $\frac{1}{4}$-strip on the eighths number line with its right end at $\frac{5}{8}$.



The left end of the strip is at $\frac{3}{8}$.
So, $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

Practice

Use models.

1. Subtract.

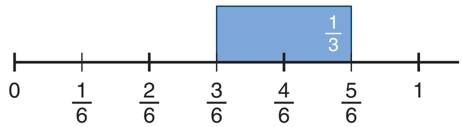
a) $\frac{3}{4} - \frac{2}{4}$ **b)** $\frac{4}{5} - \frac{1}{5}$ **c)** $\frac{2}{3} - \frac{1}{3}$ **d)** $\frac{5}{8} - \frac{3}{8}$

2. **a)** Write a rule you could use to subtract fractions with like denominators without using number lines or fraction strips.

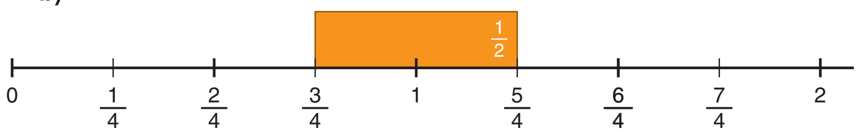
b) Write 3 subtraction questions with like denominators. Use your rule to subtract the fractions. Use fraction strips and number lines to check your answers.

3. Write the subtraction equation that each number line and fraction strip represent.

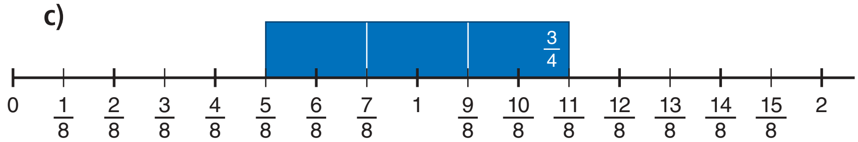
a)



b)



c)



4. Subtract. Estimate first.

a) $\frac{3}{8} - \frac{1}{4}$ **b)** $\frac{5}{6} - \frac{2}{3}$ **c)** $\frac{5}{4} - \frac{1}{2}$ **d)** $\frac{7}{10} - \frac{3}{5}$

5. Subtract. Estimate first.

a) $\frac{3}{4} - \frac{1}{2}$ **b)** $\frac{3}{4} - \frac{3}{8}$ **c)** $\frac{2}{3} - \frac{1}{6}$ **d)** $\frac{5}{6} - \frac{1}{2}$

6. Aaron has $\frac{2}{3}$ cup of raisins. He gives Raj $\frac{1}{2}$ cup. How much does Aaron have left?

7. Subtract.

a) $\frac{2}{3} - \frac{1}{4}$ **b)** $\frac{5}{3} - \frac{1}{2}$ **c)** $\frac{1}{2} - \frac{1}{5}$ **d)** $\frac{3}{2} - \frac{1}{3}$

Number Strategies

Find:

- 8 tenths less than 22.23
- 9 hundredths more than 94.43
- 36 hundredths more than 48.425
- 36 thousandths more than 48.425

8. Subtract.

a) $\frac{11}{8} - \frac{3}{4}$ **b)** $\frac{3}{2} - \frac{2}{3}$ **c)** $\frac{9}{5} - \frac{3}{2}$ **d)** $\frac{5}{3} - \frac{5}{6}$

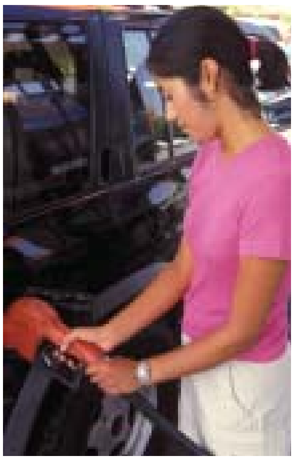
9. Subtract.

a) $2 - \frac{1}{2}$ **b)** $1 - \frac{3}{5}$ **c)** $2 - \frac{5}{4}$ **d)** $1 - \frac{2}{3}$

10. A cookie recipe calls for $\frac{3}{4}$ cup of chocolate chips.
Spencer has $\frac{2}{3}$ cup. Does he have enough?
If your answer is yes, explain.
If your answer is no, how much more does Spencer need?

11. Copy and replace each $\square$ with a number, to make each statement correct.
Try to do this more than one way.

a) $\frac{3}{4} - \frac{\square}{\square} = \frac{1}{4}$ **b)** $\frac{\square}{\square} - \frac{1}{5} = \frac{3}{5}$ **c)** $\frac{\square}{6} - \frac{2}{\square} = \frac{1}{6}$



12. **Assessment Focus** Kelly had $\frac{3}{4}$ of a tank of gas at the beginning of the week.
At the end of the week, Kelly had $\frac{1}{8}$ of a tank left.

a) Did Kelly use more or less than $\frac{1}{2}$ of a tank? Explain.

b) How much more or less than $\frac{1}{2}$ of a tank did Kelly use?

13. **a)** Which of these differences is greater than $\frac{1}{2}$?
How do you know?

i) $\frac{5}{6} - \frac{2}{3}$ **ii)** $\frac{5}{6} - \frac{1}{2}$ **iii)** $\frac{5}{6} - \frac{1}{6}$

b) Explain how you found your answers to part a.
Which other way can you find the fractions with a difference greater than $\frac{1}{2}$? Explain.

Reflect

Which pairs of fractions can you *not* subtract using the number lines and fraction strips you have? Give 3 examples.

4.5 Subtracting Fractions

Focus Subtract fractions using symbols.

Addition and subtraction are related operations.
You can use what you know about adding fractions to subtract them.

Explore

Work with a partner.
Use any of the digits 1, 2, 3, 4, 5, 6 only once.
Copy and complete. Replace each □ with a number.

$$\frac{\square}{\square} - \frac{\square}{\square}$$

Find the least difference greater than 0.
Which strategies did you use?

Reflect & Share

Compare your least difference with that of another pair of classmates.
Are the differences the same?
If your answer is no, what is the least difference?

Connect

To subtract $\frac{3}{4} - \frac{1}{6}$, estimate first.
$\frac{3}{4}$ is between $\frac{1}{2}$ and 1, and $\frac{1}{6} < \frac{1}{4}$.
So, $\frac{3}{4} - \frac{1}{6}$ is about $\frac{1}{2}$.
Use equivalent fractions to subtract.
Express $\frac{3}{4}$ and $\frac{1}{6}$ with a common denominator.
Find the lowest common denominator.
List the multiples of 4: 4, 8, **12**, 16, 20, …
List the multiples of 6: 6, **12**, 18, 24, 30, …
The lowest common denominator is 12.

$$\overset{\times 3}{\underset{\times 3}{\frac{3}{4} = \frac{9}{12}}} \qquad \overset{\times 2}{\underset{\times 2}{\frac{1}{6} = \frac{2}{12}}}$$

$$\begin{aligned}\frac{3}{4} - \frac{1}{6} &= \frac{9}{12} - \frac{2}{12} \\ &= \frac{7}{12}\end{aligned}$$

Think: 9 twelfths minus 2 twelfths is 7 twelfths.

Example

Subtract.

a) $\frac{3}{4} - \frac{5}{8}$

b) $\frac{3}{2} - \frac{2}{5}$

Solution

a) $\frac{3}{4} - \frac{5}{8}$

Since 8 is a multiple of 4, use 8 as the lowest common denominator.

> Estimate:
> Both $\frac{3}{4}$ and $\frac{5}{8}$ are between $\frac{1}{2}$ and 1; so, $\frac{3}{4} - \frac{5}{8} < \frac{1}{2}$

$$\frac{3}{4} \overset{\times 2}{\underset{\times 2}{=}} \frac{6}{8}$$

$$\frac{3}{4} - \frac{5}{8} = \frac{6}{8} - \frac{5}{8}$$
$$= \frac{1}{8}$$

b) $\frac{3}{2} - \frac{2}{5}$

> Estimate:
> $\frac{2}{5} < \frac{1}{2}$; so, $\frac{3}{2} - \frac{2}{5} > 1$

List the multiples of 2:
2, 4, 6, 8, **10**, 12, …
List the multiples of 5:
5, **10**, 15, 20, …
The lowest common denominator is 10.
Write each fraction with a denominator of 10.

$$\frac{3}{2} \overset{\times 5}{\underset{\times 5}{=}} \frac{15}{10} \qquad \frac{2}{5} \overset{\times 2}{\underset{\times 2}{=}} \frac{4}{10}$$

$$\frac{3}{2} - \frac{2}{5} = \frac{15}{10} - \frac{4}{10}$$
$$= \frac{11}{10}$$

We can write $\frac{11}{10}$ as the mixed number $1\frac{1}{10}$.

Practice

1. Subtract.

a) $\frac{4}{5} - \frac{2}{5}$ b) $\frac{2}{3} - \frac{1}{3}$ c) $\frac{7}{9} - \frac{4}{9}$ d) $\frac{5}{7} - \frac{3}{7}$

2. Subtract. Estimate first.

a) $\frac{5}{8} - \frac{1}{2}$ b) $\frac{4}{9} - \frac{1}{3}$ c) $\frac{3}{2} - \frac{4}{10}$ d) $\frac{5}{3} - \frac{5}{6}$

3. Subtract.

a) $\frac{5}{6} - \frac{2}{9}$ b) $\frac{5}{6} - \frac{2}{4}$ c) $\frac{5}{8} - \frac{3}{12}$ d) $\frac{7}{10} - \frac{4}{15}$

4. Subtract. Estimate first.

a) $\frac{3}{4} - \frac{2}{3}$ b) $\frac{5}{2} - \frac{3}{4}$ c) $\frac{4}{5} - \frac{2}{3}$ d) $\frac{5}{4} - \frac{4}{5}$

5. Subtract.

a) $\frac{4}{6} - \frac{1}{2}$ b) $\frac{5}{3} - \frac{3}{4}$ c) $\frac{7}{2} - \frac{3}{2}$ d) $\frac{5}{6} - \frac{3}{4}$

To subtract 2 mixed numbers: Subtract the whole numbers. Subtract the fractions. Write the difference as a mixed number.

6. Subtract.

a) $5\frac{5}{7} - 1\frac{2}{7}$ b) $3\frac{4}{9} - 2\frac{1}{6}$ c) $4\frac{3}{10} - 2\frac{1}{5}$ d) $4\frac{3}{5} - 2\frac{1}{2}$

7. **Assessment Focus** Terri biked $2\frac{1}{4}$ h on Sunday.
Terri increased the time she biked by $\frac{1}{4}$ h every day.
Sam biked $\frac{1}{2}$ h on Sunday.
Sam increased the time he biked by $\frac{1}{2}$ h every day.

a) Who will bike longer the next Saturday? Explain.
b) For how much longer will this person bike?
c) What did you need to know about fractions to answer these questions?

8. A recipe calls for $\frac{3}{4}$ cup of walnuts and $\frac{2}{3}$ cup of pecans.
Which type of nut is used more in the recipe? How much more?

9. Write as many different subtraction questions as you can where the answer is $\frac{3}{4}$.

10. The difference of 2 fractions is $\frac{1}{2}$.
The lesser fraction is between 0 and $\frac{1}{4}$.
What do you know about the other fraction?

Calculator Skills

Suppose the $\boxed{8}$ key on your calculator is broken. How would you use your calculator to find each answer?

- 18 + 27
- 118 – 85
- 18 × 27
- 225 ÷ 8

Reflect

When you subtract fractions, and the denominators are different, how do you subtract? Give 2 different examples.
Explain your steps.

Shade One

YOU WILL NEED

One gameboard;
coloured markers;
1 set of 42 fraction cards

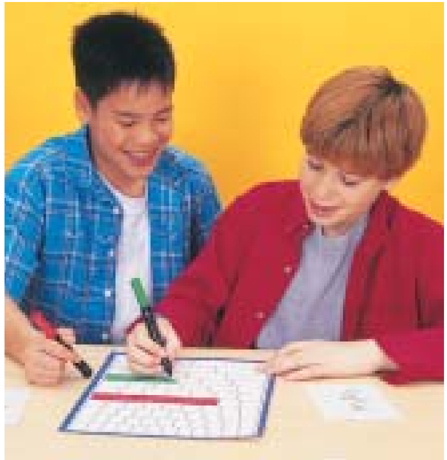
NUMBER OF PLAYERS

2 or more

GOAL OF THE GAME

To shade 1 whole on each fraction strip

HOW TO PLAY THE GAME:



1. Each player chooses a different colour marker.
2. Place the set of fraction cards face down.
3. Player A turns over a card. This is his *target fraction*.
4. Player A could shade the target fraction on one fraction strip, or share the fraction among several fraction strips. However, the total fraction shaded must equal the target fraction.

For example: If your target fraction is $\frac{3}{4}$, you could shade from 0 to $\frac{3}{4}$ on the fourths strip, or 0 to $\frac{6}{8}$ on the eighths strip. Or you could shade several fractions that add up to $\frac{3}{4}$. For example, you could shade $\frac{3}{12}$, $\frac{2}{8}$, and $\frac{1}{4}$; or you could shade $\frac{2}{5}$, $\frac{1}{10}$, and $\frac{1}{4}$.

5. Player B turns over the next fraction card. He repeats *Step 4* for his fraction.
6. Play continues with players taking turns to shade the target fraction. If there is not enough of a fraction strip left, and it is impossible to shade the target fraction, that player forfeits his turn. When a player is not able to shade any more fractions, the game is over.
7. The player who shades to complete 1 whole gets 1 point.
8. The person with the most points wins.

Mid-Unit Review

LESSON

4.1 **1.** Add. Use fraction strips to help you.

a) $\frac{2}{3} + \frac{1}{6}$ b) $\frac{2}{4} + \frac{1}{3}$

c) $\frac{2}{8} + \frac{2}{4}$ d) $\frac{2}{10} + \frac{3}{5}$

4.2 **2.** We know that $\frac{1}{2} + \frac{4}{5} = \frac{13}{10}$.
Use this result to find each sum.

a) $1\frac{1}{2} + \frac{4}{5}$ b) $5\frac{1}{2} + 2\frac{4}{5}$

3. Use models to add.

a) $\frac{4}{5} + \frac{3}{5}$ b) $\frac{2}{3} + \frac{1}{2}$

c) $\frac{1}{2} + \frac{5}{6}$ d) $\frac{3}{2} + \frac{1}{3}$

4.3 **4.** Add. Estimate first.

a) $\frac{3}{4} + \frac{5}{6}$ b) $\frac{3}{2} + \frac{2}{3}$

c) $\frac{7}{10} + \frac{2}{5}$ d) $\frac{5}{9} + \frac{5}{6}$

5. Add.

a) $\frac{2}{5} + \frac{3}{2} + \frac{3}{10}$

b) $\frac{3}{8} + \frac{3}{4} + \frac{1}{2}$

c) $\frac{1}{3} + \frac{5}{6} + \frac{2}{9}$

6. Add.

a) $2\frac{1}{3} + 3\frac{1}{3}$

b) $2\frac{1}{4} + 1\frac{1}{3}$

c) $1\frac{3}{4} + 3\frac{3}{8}$

4.4 **7.** Use models to subtract.

a) $\frac{7}{8} - \frac{3}{4}$ b) $\frac{3}{2} - \frac{3}{5}$

c) $\frac{4}{3} - \frac{5}{6}$ d) $\frac{11}{6} - \frac{2}{3}$

8. Peter put $\frac{4}{5}$ cup of chocolate chips into his cookie batter. Samantha put $\frac{7}{8}$ cup into hers. Whose batter contains more chips? How do you know?

9. Alex delivered flyers to $\frac{1}{4}$ of her route on Saturday morning. She delivered to $\frac{2}{3}$ of the route on Saturday afternoon, and the remainder on Sunday.

a) What fraction of her route did Alex deliver on Saturday?

b) What fraction did she deliver on Sunday?

4.5 **10.** Subtract. Estimate first.

a) $\frac{3}{2} - \frac{1}{5}$ b) $\frac{7}{6} - \frac{2}{3}$

c) $\frac{11}{6} - \frac{3}{4}$ d) $\frac{13}{10} - \frac{3}{5}$

11. We know that $\frac{9}{8} - \frac{1}{4} = \frac{7}{8}$.
Use this result to find each difference.
Explain how you did this.

a) $\frac{12}{8} - \frac{1}{4}$ b) $\frac{5}{8} - \frac{1}{4}$

12. Subtract.

a) $3\frac{5}{8} - 2\frac{3}{8}$ b) $2\frac{1}{2} - 1\frac{1}{4}$

4.6 Exploring Repeated Addition

Focus Multiply a fraction by a whole number.

How many ways can you find this sum?
$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$
You can use the same strategies in *Explore*.

Explore

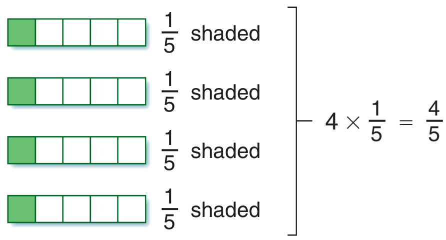
Work with a partner.
Jan takes $\frac{3}{4}$ h to walk to her music lesson.
Jan has a music lesson once a week, for 9 weeks.
How much time does Jan spend walking to her music lessons?

Reflect & Share

Compare your strategy for solving the problem
with that of another pair of classmates.
Did you get the same answers?
If not, who is correct? Explain.

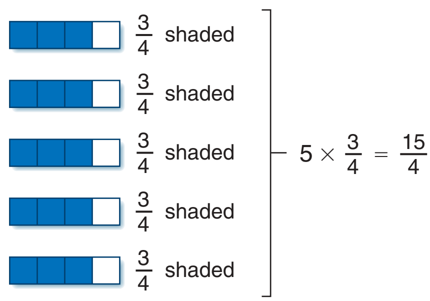
Connect

$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$
All the fractions added are $\frac{1}{5}$.
Repeated addition can be
written as multiplication.
$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 4 \times \frac{1}{5} = \frac{4}{5}$
We can show this as a picture.



Similarly: $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$
$= 5 \times \frac{3}{4}$
$= \frac{15}{4}$

We can show this as a picture.



Example 1

Use multiplication to find this sum.

$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

Solution

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = 9 \times \frac{3}{8}$$
$$= \frac{27}{8}$$

Think: 9 times 3 eighths is 27 eighths.

Example 2

Multiply.

a) $\frac{3}{5} \times 5$

b) $7 \times \frac{5}{6}$

Solution

a) $\frac{3}{5} \times 5 = \frac{15}{5}$
$= 3$

Think: $\frac{15}{5}$ means $15 \div 5$, which is 3.

b) $7 \times \frac{5}{6} = \frac{35}{6}$

Think: 7 times 5 sixths is 35 sixths.

Practice

1. Write each repeated addition as a multiplication question.

a) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

b) $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$

c) $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} + \frac{3}{10}$

2. Write each multiplication question as repeated addition. Draw a picture to show each answer.

a) $5 \times \frac{1}{8}$ b) $\frac{2}{5} \times 3$ c) $4 \times \frac{5}{12}$

3. Multiply. Draw a picture to show each answer.

a) $3 \times \frac{4}{7}$ b) $5 \times \frac{1}{12}$ c) $\frac{2}{15} \times 10$

d) $4 \times \frac{9}{4}$ e) $\frac{2}{5} \times 7$ f) $9 \times \frac{1}{2}$

4. Multiply.

a) $3 \times \frac{4}{5}$ b) $5 \times \frac{7}{10}$ c) $\frac{5}{6} \times 6$

d) $\frac{1}{2} \times 5$ e) $12 \times \frac{7}{12}$ f) $\frac{2}{3} \times 9$

5. It takes $\frac{2}{3}$ h to pick all the apples on one tree at Springwater Farms. There are 24 trees.
How long will it take to pick all the apples?
Show your work.

Number Strategies

Calculate each answer.

$6000 \div 10$	6000×0.1
$600 \div 10$	600×0.1
$60 \div 10$	60×0.1
$6 \div 10$	6×0.1

What patterns do you see in the questions and answers?

6. a) Draw a picture to show each product. What is each answer?

i) $4 \times \frac{3}{10}$ ii) $3 \times \frac{4}{10}$

b) How are the questions in part a related?
Write 2 more questions like these.
Find each product. What do you notice?

7. A cookie recipe calls for $\frac{3}{4}$ cup of oatmeal.
How much oatmeal is needed to make 3 batches of cookies?

8. **Assessment Focus**

a) Draw a picture to show $5 \times \frac{1}{2}$.

b) What meaning can you give to $\frac{1}{2} \times 5$?
Draw a picture to show your thinking.

Take It Further

9. Jacob takes $\frac{3}{4}$ h to fill one shelf at the supermarket.
Henry can fill the shelves in half Jacob's time.
There are 15 shelves. Henry and Jacob work together.
How long will it take to fill the shelves? Justify your answer.

Reflect

Draw a picture to show why $4 \times \frac{2}{5}$ is the same as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$.
Explain your picture.
Draw a different picture to show the same answer.

Fractions to Decimals

Technology

Recall that $\frac{1}{10}$ means $1 \div 10$, which is 0.1 as a decimal.

Similarly, $\frac{1}{2}$ means $1 \div 2$, which is 0.5 as a decimal.

0.1 and 0.5 are **terminating decimals**, because each has a definite number of decimal places.

On a calculator, press: $\boxed{1}$ $\boxed{\div}$ $\boxed{11}$ $\boxed{=}$ to display 0.090909091

The calculator rounds up the last digit.
The fraction $\frac{1}{11}$ is a **repeating decimal**.

The decimal for $\frac{1}{11}$ is $0.\overline{09}$, with a bar above the 0 and 9 to show they repeat.

A period above an equal sign shows the answer is approximate.

We can round $0.\overline{09}$ to an approximate decimal.

- $0.\overline{09} \doteq 0.1$ to 1 decimal place
- $0.\overline{09} \doteq 0.09$ to 2 decimal places
- $0.\overline{09} \doteq 0.091$ to 3 decimal places

✓ Check

1. Investigate other unit fractions; that is, fractions with numerator 1.

a) Which unit fractions from $\frac{1}{3}$ to $\frac{1}{20}$ produce terminating decimals?
Which produce repeating decimals? Explain why.

b) Investigate other fractions.
For example, how are the decimals for $\frac{1}{6}, \frac{2}{6}, \ldots, \frac{5}{6}$ related?

2. Investigate fractions with greater denominators.
For example, how are the decimals for $\frac{1}{9}, \frac{1}{99}, \frac{1}{999}, \ldots$ related?

3. Which is greater in each case? Justify your answer.

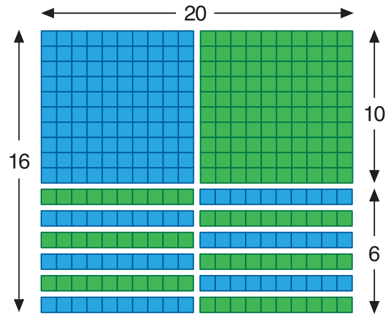
a) $0.\overline{3}$ or 0.3

b) $\frac{1}{9}$ or 0.11

4.7 Multiplying Decimals

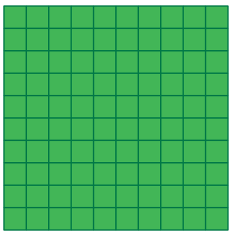
Focus Multiply decimals with tenths.

To multiply 2 whole numbers,
we can use Base Ten Blocks.
This picture shows the product
$20 \times 16 = 100 + 100 + 60 + 60$
$= 320$

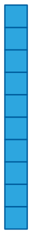


We can also use Base Ten Blocks to multiply a decimal and a whole number.

The flat represents 1.

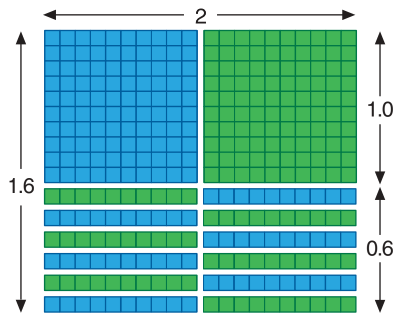


The rod represents 0.1.



The small cube represents 0.01.

This picture shows the product
$2 \times 1.6 = 1 + 1 + 0.6 + 0.6$
$= 3.2$



In *Explore*, you will use Base Ten Blocks to multiply 2 decimals.

Explore

Work with a partner.
A rectangular tabletop measures 2.4 m by 1.8 m.
Use Base Ten Blocks to find the area of the tabletop.
Record your work on grid paper.

Reflect & Share

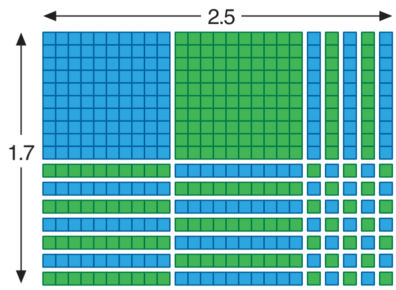
Compare your answer with that of another pair of classmates.
Did you draw the same picture? Explain.
Did you get the same area? Explain.

Connect

A rectangular park measures 1.7 km by 2.5 km.

Here are 2 ways to find the area of the park.

➢ Use Base Ten Blocks.



There are 2 flats: $2 \times 1 = 2$

There are 19 rods: $19 \times 0.1 = 1.9$

There are 35 small cubes: $35 \times 0.01 = 0.35$

The total area is $2 + 1.9 + 0.35 = 4.25$

The area of the park is 4.25 km^2.

➢ Use the method for multiplying 2 whole numbers.

The area, in square kilometres, is 1.7×2.5.

Multiply: 17×25

$$\begin{array}{r} 17 \\ \times\ 25 \\ \hline 85 \\ 340 \\ \hline 425 \end{array}$$

Estimate to place the decimal point in the answer.

1.7×2.5 is about $2 \times 3 = 6$

So, the product is 4.25.

The area of the park is 4.25 km^2.

Example 1

Multiply. 5.8×9.7

Solution

5.8×9.7

Multiply: 58×97

$$\begin{array}{r} 58 \\ \times\ 97 \\ \hline 406 \\ 5220 \\ \hline 5626 \end{array}$$

Estimate to place the decimal point.

5.8×9.7 is about $6 \times 10 = 60$.

So, the product is 56.26.

Example 2

Multiply.

a) 0.9×6.8 b) 0.5×0.4

Solution

a) 0.9×6.8

Multiply: 9×68

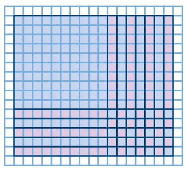
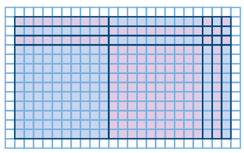
$$\begin{array}{r} 68 \\ \times\ 9 \\ \hline 612 \end{array}$$

Estimate to place the decimal point.
0.9×6.8 is about $1 \times 7 = 7$.
So, the product is 6.12.

b) 0.5×0.4
Multiply: $5 \times 4 = 20$
Estimate to place the decimal point.
0.5×0.4 is about $1 \times 0.4 = 0.4$.
So, the product is 0.20.

Practice

1. Write a multiplication equation for each picture.
Each small square represents 0.01.

a)  b) 

2. Use Base Ten Blocks to find each product.
a) 2.6×1.5 b) 1.4×2.8 c) 2.7×1.6

3. Use Base Ten Blocks to find each product.
Record your work on grid paper.
a) 2.3×0.4 b) 0.6×1.9 c) 0.8×0.7

4. Multiply: 36×24
Use this to find each product. Explain your work.
a) 36×2.4 b) 3.6×24 c) 3.6×2.4

5. Multiply.
a) 4.2×3.7 b) 8.9×0.3 c) 0.6×0.9

6. Carla drives 7.6 km to work. She drives the same distance home. How many kilometres does Carla drive in a 5-day workweek?

7. A rectangular plot of land measures 30.5 m by 5.3 m. What is the area of the plot?

8. **a)** Multiply.
  i) 6.3×1.8 **ii)** 4.2×0.7 **iii)** 0.8×0.5
 b) Look at the questions and products in part a. What patterns do you see in the numbers of decimal places in the question and the product? How could you use this pattern to place the decimal point in a product without estimating?

Calculator Skills

Which is greater in each pair?

0.3 or $\frac{1}{3}$ 0.4 or $\frac{1}{4}$

0.7 or $\frac{2}{3}$ 0.09 or $\frac{1}{11}$

Justify your answer.

9. The product of 2 decimals is 0.36. What might the decimals be? Find as many answers as possible.

10. Recall that dividing by 10 is the same as multiplying by 0.1. Multiply to find:
 a) $\frac{1}{10}$ of 25.2 **b)** $\frac{2}{10}$ of 37.3 **c)** $\frac{6}{10}$ of 58.7

11. **Assessment Focus** An area rug is rectangular. Its dimensions are 3.4 m by 2.7 m. Show different strategies you can use to find the area of the rug. Which strategy is best? Justify your answer.

12. **a)** Find each product.
  i) 4.8×5.3 **ii)** 4.8×0.6 **iii)** 0.4×0.6
 b) When you multiply 2 decimals, how does the product compare with the numbers you multiplied? Explain your reasoning.

Take It Further

13. Explain why the sum of the number of digits to the right of the decimal point in the factors of a product equals the number of digits to the right of the decimal point in the product.

Reflect

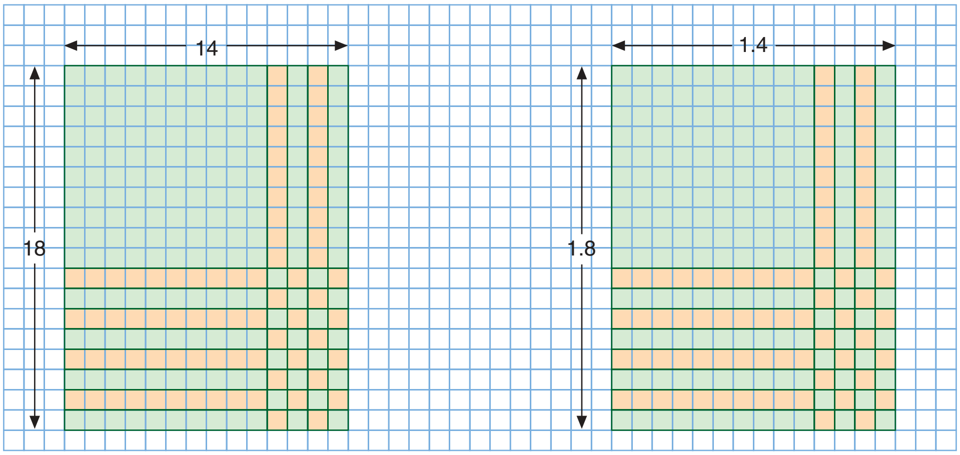
When you multiply 2 decimals, how do you know where to place the decimal point in the product? Use examples to explain.

4.8 Dividing Decimals

Focus Divide decimals with tenths.

Recall that we can use the same Base Ten Blocks to multiply:

$14 \times 18 = 252$ and $1.4 \times 1.8 = 2.52$



The only difference is the values we assign to the flat, the rod, and the small cube.

In a similar way, we can divide 2 decimals by thinking about whole numbers instead.

So, $252 \div 18 = 14$ and $2.52 \div 1.8 = 1.4$

To divide 2 decimals, ignore the decimal points.
Divide as you would whole numbers, then estimate to place the decimal point in the answer.

Explore

Work with a partner.
Mark bought 19.5 m of fabric to make costumes for a play.
Each costume needs 2.6 m of fabric.
How many costumes can Mark make?
How much material is left over?
Show your work.

Reflect & Share

How did you use division of whole numbers to find your answer?

Connect

Jan bought 18.9 m of framing to make picture frames.
Each picture needs 1.8 m of frame.
How many frames can Jan make?
How much framing material is left over?

When you divide 2 numbers, the answer is the **quotient**.

To divide: 18.9 ÷ 1.8, ignore the decimal points.
Find 189 ÷ 18, then estimate.

$$\begin{array}{r}
10.5 \\
18\overline{)189.0} \\
\underline{180} \\
9\,0 \\
\underline{9\,0} \\
0
\end{array}$$

189 ÷ 18 = 10.5
Estimate to place the decimal point correctly.
18.9 ÷ 1.8 is about 20 ÷ 2, which is 10.
So, 18.9 ÷ 1.8 is 10.5.

Jan can make 10 frames.
10 frames use 10 × 1.8 m = 18 m.
So, the framing material left is 18.9 m – 18 m = 0.9 m.

Sometimes when we divide 2 decimals, the quotient is not a terminating decimal.

Example

Divide.

a) 12.5 ÷ 0.6 **b)** 25.2 ÷ 4.7

Solution

a) 12.5 ÷ 0.6

$$\begin{array}{r|l}
6 & 125.{}^{5}0{}^{2}0{}^{2}0 \\
\hline
 & 20.\;8\;3\;3
\end{array}$$

Think: Use short division to find 125 ÷ 6.

If we continue to divide, the 3 in the quotient repeats.
So, 125 ÷ 6 = $20.8\overline{3}$
To estimate: 12.5 ÷ 0.6 is about 13 ÷ 1 = 13.
So, the quotient is $20.8\overline{3}$.

b) $25.2 \div 4.7$

Think: Use long division to find $252 \div 47$.

$$\begin{array}{r} 5.36 \\ 47\overline{)252.00} \\ \underline{235} \\ 17\,0 \\ \underline{14\,1} \\ 2\,90 \\ \underline{2\,82} \\ 8 \end{array}$$

There is 1 decimal place in each number in the question. Continue to divide until there are 2 decimal places in the quotient. That is, calculate the quotient to 1 more decimal place than in the question. Then, round the quotient to 1 decimal place.

Round the quotient to 5.4.
To estimate: $25.2 \div 4.7$ is about $25 \div 5 = 5$.
So, the quotient is approximately 5.4.

Practice

1. Look at each division equation on the left. Estimate each quotient on the right.

a) $234 \div 13 = 18$ — $23.4 \div 1.3$
b) $133 \div 7 = 19$ — $13.3 \div 7$
c) $714 \div 34 = 21$ — $71.4 \div 3.4$
d) $450 \div 18 = 25$ — $4.5 \div 1.8$
e) $51 \div 17 = 3$ — $5.1 \div 1.7$

2. Choose the correct quotient for each division question. Explain how you know.

Question	Possible Quotients		
a) $5.95 \div 3.5$	17	1.7	0.17
b) $195.3 \div 6.2$	315	31.5	3.15
c) $31.32 \div 1.8$	174	17.4	1.74
d) $1.44 \div 0.12$	12	1.2	0.12

Number Strategies

A rectangular prism has dimensions 6 cm by 4 cm by 8 cm. Find the surface area and volume of this rectangular prism.

3. Divide.

a) $8.7 \div 0.3$ b) $2.24 \div 0.7$ c) $10.3 \div 0.6$

4. Divide.

a) $10.92 \div 0.6$ b) $30.42 \div 1.3$ c) $18.56 \div 5.8$

Math Link

Measurement

The prefixes in units of length less than 1 m show fractions or decimals of 1 m.

One decimetre is $\frac{1}{10}$ or 0.1 m.

One centimetre is $\frac{1}{100}$ or 0.01 m.

One millimetre is $\frac{1}{1000}$ or 0.001 m.

One micrometre is $\frac{1}{1\,000\,000}$ or 0.000 001 m.

5. Divide. Round the quotient to the nearest tenth.
a) 172.5 ÷ 2.6 **b)** 21.68 ÷ 3.4 **c)** 92.8 ÷ 8.2

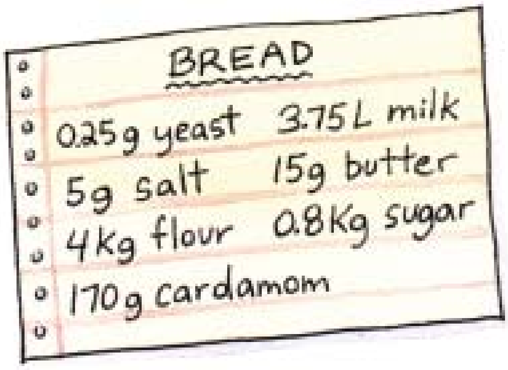
6. Divide. Round to the nearest tenth where necessary.
a) 7.3 ÷ 0.4 **b)** 1.98 ÷ 1.3 **c)** 426.8 ÷ 3.7

7. The quotient of 2 decimals is 0.12. What might the decimals be? Write as many different possible decimal pairs as you can.

8. **Assessment Focus** Alex finds a remnant of landscaping fabric at a garden store.
The fabric is the standard width, with length 9.7 m.
Alex needs twelve 0.85-m pieces for a garden patio.
a) Will Alex have more fabric than she needs? If so, how much more?
b) Will Alex need more fabric? If so, how much more?

9. The area of a rectangular lawn is 120.4 m^2.
The width is 5.6 m. What is the length?

10. The question 237 ÷ 7 does not have an exact quotient.
The first five digits of the quotient are 33857. The decimal point has been omitted. Use only this information and estimation.
Write an approximate quotient for each question.
Justify each answer.
a) 237 ÷ 0.7 **b)** 2.37 ÷ 7 **c)** 23.7 ÷ 7 **d)** 2370 ÷ 70



11. Here is a recipe for bread.
Sam wants to make one-half the amount.
Find how much of each ingredient Sam needs.

12. Jack has 2.5 L of juice.
Each day, he drinks 0.4 L.
How many days will it take Jack to drink the juice?
Justify your answer.

Reflect

Explain how you decide where to place the decimal point when you divide 2 decimals. Use an example in your explanation.

Advertising Sales Representative

Magazines and newspapers make money by selling advertising space.

The advertising sales representative contacts companies whose products might be of interest to readers. She offers to sell them various sizes of advertisement space at different rates. When talking about ads smaller than a full page, the sales rep uses fractions to describe them. It's much simpler to talk about a $\frac{2}{3}$-page ad instead of a 0.666 667 page ad!

The sales rep tries to sell combinations of ads that can fill pages, with no space left over. A sales rep has sold two $\frac{1}{4}$-page ads and one $\frac{1}{6}$-page ad. She wants to know the possible combinations of ad sizes she can sell to fill the page. What might they be?

4.9 Order of Operations with Decimals

Focus Use the order of operations to evaluate expressions.

We use a base ten, or decimal, number system.
Every whole number is also a decimal.
So, we use the same order of operations for decimals as for whole numbers.

Explore

Work on your own.
Evaluate this expression: $6 \times (15.9 + 36.4) \div 4$
Explain each step.

Reflect & Share

Compare your solution and answer with that of a classmate.
If your answers are different, who has the correct answer?

Connect

Here is the order of operations.

- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

Example

Evaluate. $12.4 - (4.7 + 1) + 2.4 \times 3 - 4.8 \div 2$

Solution

$12.4 - (4.7 + 1) + 2.4 \times 3 - 4.8 \div 2$ **Calculate in brackets.**

$= 12.4 - 5.7 + 2.4 \times 3 - 4.8 \div 2$ **Multiply and divide from left to right.**

$= 12.4 - 5.7 + 7.2 - 2.4$ **Add and subtract from left to right.**

$= 6.7 + 7.2 - 2.4$

$= 13.9 - 2.4$

$= 11.5$

Practice

1. Evaluate.
   a) $3.4 + 4 \times 7$ b) $14 - 2.2 \times 5$ c) $8 - 3.6 \div 2$

2. Evaluate.
   a) $7.4 - 3 + 2.3 \times 4$ b) $4.6 + 5.1 - 3.2 \div 2$
   c) $16.4 - (10.8 - 3.1)$ d) $23 \times 6.2 + 4$
   e) $81.2 - (35.8 + 2.1)$ f) $85.7 \div 0.4 \times 7$

3. Evaluate.
   a) $46.78 - 6.1 \times 2.3$ b) $75.06 \times (3.45 - 1.2)$
   c) $(98.5 + 7) \div 2.5$ d) $9.0023 \times 5.1 - 4.32 \times 6$
   e) $8.3 + 46.2 \div 1.4$ f) $70.56 - 32.8 \div 4.1$

Number Sense

A number cube has faces labelled 1 to 6.

The cube is rolled 75 times.

About how many times will the number 3 show?

4. Evaluate.
   a) $3.2 + 5.6 \times 7.2 \div 2.4 - 9.3$
   b) $8.5 \times 7 - 6.3 \div 9 + 10.6$
   c) $1.35 + (5 \times 4.9 \div 0.7) - 2.7 \times 2.1$
   d) $(4.7 - 3.1) \times 5 - 7.5 \div 2.5$

5. Evaluate.
   a) $164.5 \div 7 \times 10 + 7.2$ b) $73.8 \times (3.2 + 6.8) - 14.1 \div 0.2$

6. The cross-country team members ran timed circuits. Here are their times: 15.8 min, 12.5 min, 18.0 min, 14.2 min, 13.9 min, 16.0 min, 16.2 min, 17.5 min, 16.3 min, 15.6 min
   Find the mean time.

The mean time is the sum of the times divided by the number of times.

7. **Assessment Focus**
   Evaluate. Show all steps.
   $0.38 + 16.2 \times (2.1 + 4) + 21 \div 3.5$

Reflect

Explain why the brackets are unnecessary in this expression:
$(4.2 \times 3.8) - (15.25 \div 6.1)$
Find the answer. Show each step.

Writing a Math Journal Entry

1 THINK about what you did in math.

- Which materials did you use?
- Which problem did you solve?
  How did you solve it?
- What did you learn?
- What are you confused about?
- What vocabulary did you use?
- What was challenging? What was easy?

2 TALK about what you did in math.

- Work with a partner or in a group.
- Share your ideas. Listen to the ideas of others.
- What information did your partner or group share that was new?
- Ask your partner or group members about any issue that was confusing … what did they say?

3 BRAINSTORM all your ideas.

- Make dot jots of the things you discussed and thought about.
  Don't leave anything out … you can edit it later.
- Think about how you can support your written work.
  Use graphs, tables, numbers, symbols, diagrams, and so on.

4 WRITE your ideas.

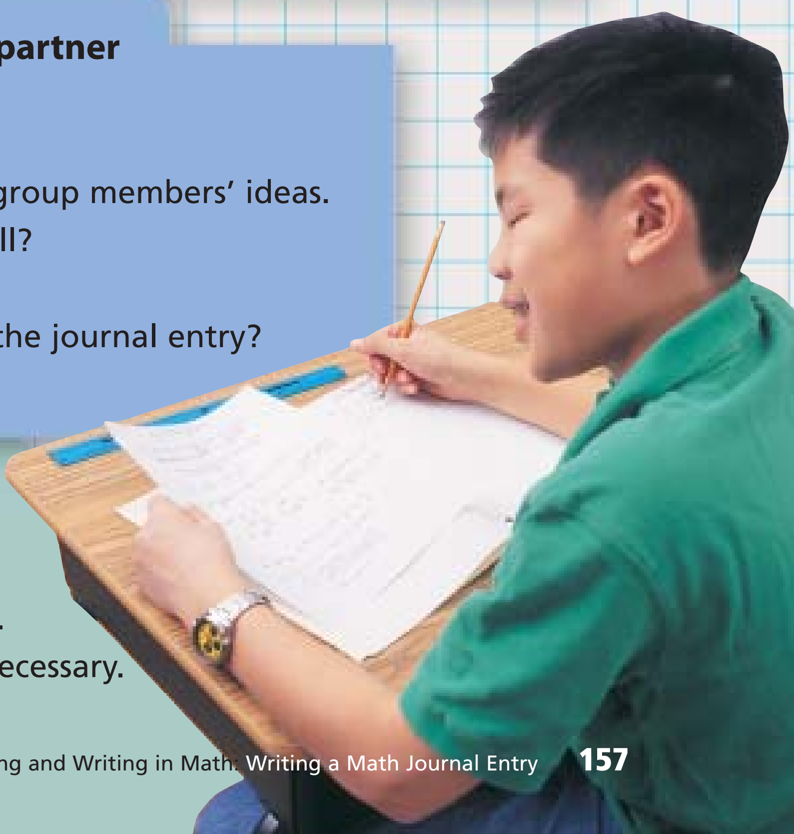
- Put your thoughts into words.
  Be sure you follow a logical order.
- Think about what you write. Are you using math language? Underline this language.

5 READ your ideas out loud.

- Listen for missing words.
- Be sure your thoughts are in a logical order.
- Add anything that is missing.
- What is unclear? Why? What would help?
- Add graphs, tables, and so on, that will help explain your written work.

6 READ your work to your partner or group.

- Listen to your partner's or group members' ideas.
- Discuss: What was done well?
  What part was not clear?
  What would help improve the journal entry?



7 REVISE and REWRITE.

- Make the changes that are necessary.
- Rewrite on a new piece of paper if necessary.

Unit Review

Review any lesson with etext online tutorial

What Do I Need to Know?

Adding and Subtracting Fractions

There are 4 types of fraction questions. Look at the denominators.

Same denominators

For example, $\frac{5}{6}$ and $\frac{2}{6}$

$\frac{5}{6} + \frac{2}{6} = \frac{7}{6}$ $\qquad$ $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

Related denominators

For example, $\frac{1}{6}$ and $\frac{1}{12}$

12 is a multiple of 6, so the lowest common denominator is 12.

$\frac{1}{6} = \frac{2}{12}$

$\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12}$ $\qquad$ $\frac{1}{6} - \frac{1}{12} = \frac{2}{12} - \frac{1}{12} = \frac{1}{12}$

Partially related denominators

For example, $\frac{1}{6}$ and $\frac{1}{9}$

6 and 9 have the common factor, 3.

So, list multiples to find the lowest common denominator.

Multiples of 6: 6, 12, **18**, 24, 30, …

Multiples of 9: 9, **18**, 27, 36, …

The lowest common denominator is 18.

$\frac{1}{6} = \frac{3}{18}$; $\frac{1}{9} = \frac{2}{18}$; $\frac{1}{6} + \frac{1}{9} = \frac{3}{18} + \frac{2}{18} = \frac{5}{18}$; $\frac{1}{6} - \frac{1}{9} = \frac{3}{18} - \frac{2}{18} = \frac{1}{18}$

Unrelated denominators

For example, $\frac{1}{5}$ and $\frac{1}{6}$

5 and 6 have no common factors, so the lowest common denominator is their product: $5 \times 6 = 30$

$\frac{1}{5} = \frac{6}{30}$; $\frac{1}{6} = \frac{5}{30}$; $\frac{1}{5} + \frac{1}{6} = \frac{6}{30} + \frac{5}{30} = \frac{11}{30}$; $\frac{1}{5} - \frac{1}{6} = \frac{6}{30} - \frac{5}{30} = \frac{1}{30}$

Multiplying a Fraction by a Whole Number

This is the same as repeatedly adding the same fraction.

$4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{12}{5}$

What Should I Be Able to Do?

For extra practice, go to page 441.

LESSON

4.1 **1.** Which fraction is greater? How do you know?

a) $\frac{2}{5}, \frac{2}{6}$ **b)** $\frac{3}{10}, \frac{1}{2}$

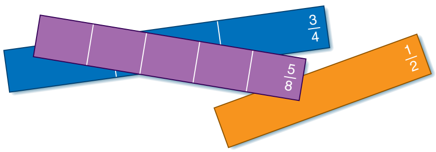
c) $\frac{4}{8}, \frac{3}{6}$ **d)** $\frac{7}{2}, \frac{7}{3}$

2. Add.

a) $\frac{2}{3} + \frac{1}{6}$ **b)** $\frac{3}{4} + \frac{2}{8}$

c) $\frac{1}{4} + \frac{3}{6}$ **d)** $\frac{1}{10} + \frac{3}{5}$

3. Find 2 fractions that add to $\frac{5}{8}$. Find as many pairs of fractions as you can.



4.2 **4.** Add.

a) $\frac{2}{3} + \frac{5}{6}$ **b)** $\frac{1}{2} + \frac{3}{4}$

c) $\frac{1}{2} + \frac{9}{10}$ **d)** $\frac{8}{8} + \frac{1}{4}$

5. Adam eats $\frac{3}{10}$ of a pizza. Julie eats $\frac{2}{5}$ of the pizza.

a) How much of the pizza is eaten?

b) How much is left?

4.3 **6.** Add. Draw a picture to show each sum.

a) $\frac{1}{3} + \frac{2}{5}$ **b)** $\frac{1}{2} + \frac{3}{8}$

c) $\frac{2}{3} + \frac{3}{10}$ **d)** $\frac{3}{5} + \frac{1}{4}$

7. Add.

a) $\frac{2}{5} + \frac{5}{6}$ **b)** $\frac{1}{4} + \frac{2}{5}$

c) $\frac{3}{10} + \frac{5}{6}$ **d)** $\frac{3}{8} + \frac{2}{3}$

8. Add.

a) $6\frac{1}{3} + 2\frac{1}{3}$ **b)** $1\frac{5}{12} + 2\frac{1}{6}$

c) $2\frac{3}{10} + 3\frac{1}{5}$ **d)** $5\frac{1}{4} + 2\frac{2}{5}$

9. Add.

a) $\frac{2}{3} + \frac{3}{4} + \frac{3}{8}$

b) $\frac{5}{2} + \frac{1}{3} + \frac{2}{5}$

c) $\frac{7}{10} + \frac{3}{4} + \frac{5}{8}$

4.4 **10.** Subtract.

a) $\frac{1}{2} - \frac{1}{3}$ **b)** $\frac{7}{10} - \frac{2}{5}$

c) $\frac{3}{4} - \frac{1}{8}$ **d)** $\frac{5}{6} - \frac{1}{3}$

11. Find 2 fractions with a difference of $\frac{1}{4}$. Find as many pairs of fractions as you can. Remember to use fractions with different denominators.

4.5 **12.** Ali drank $\frac{3}{4}$ cup of water. Brad drank $\frac{2}{3}$ cup of water.

a) Who drank more water?

b) How much more water did he drink?

LESSON

13. Subtract.

a) $\frac{9}{10} - \frac{2}{5}$ b) $\frac{7}{3} - \frac{5}{6}$

c) $\frac{8}{5} - \frac{1}{4}$ d) $\frac{9}{4} - \frac{2}{3}$

14. Subtract.

a) $3\frac{3}{4} - 2\frac{1}{8}$ b) $4\frac{4}{5} - 2\frac{2}{3}$

c) $9\frac{1}{2} - 3\frac{1}{3}$ d) $6\frac{3}{4} - 6\frac{1}{5}$

4.6 **15.** Multiply. Draw a picture to show each answer.

a) $\frac{1}{4} \times 7$ b) $8 \times \frac{3}{8}$

c) $6 \times \frac{7}{10}$ d) $\frac{4}{5} \times 5$

16. Sasha had 16 tomatoes in his vegetable garden. He gave: Samira $\frac{1}{8}$ of the tomatoes; Sielen $\frac{1}{8}$ of the tomatoes; and Amina $\frac{1}{4}$ of the tomatoes.

a) What fraction of the tomatoes did Sasha have left?

b) How many tomatoes did Sasha have left?

17. Orit spends $\frac{1}{4}$ of her day at school, $\frac{1}{12}$ of her day playing soccer, and $\frac{1}{3}$ of her day sleeping.
How many hours are left in Orit's day?

4.7 **18.** A rectangular park has dimensions 2.8 km by 1.9 km. What is the area of the park?

4.8 **19.** Nuri has 10.8 L of water. He pours 1.5 L into each of several plastic bottles.

a) How many bottles can Nuri fill?

b) How much water is left over?

20. Delia works at the library after school. She earns \$7.50/h. She usually works 15 h a week.

a) What does Delia earn in a week? Use estimation to check your answer.

b) One week Delia only works $\frac{1}{2}$ the hours she usually works. What are her earnings that week?

4.9 **21.** Mr. Statler took his class to the local library 4 times last year. One student had a pedometer. She measured the trip as 1.7 km each way.
How far did the students walk back and forth to the library during the year?

22. Evaluate.

a) $5.3 + 5.1 \div 3$

b) $12.6 \times (1.5 + 2.5)$

c) $68.9 - 32.7 \times 2$

23. Evaluate.

a) $5.9 + 3.7 \times 2.8 - 1.5 \div 0.5$

b) $3.4 \times 1.9 \div 1.7 + 7.2 \div 1.2$

Practice Test

1. Add or subtract.

a) $\frac{5}{4} + \frac{3}{8}$ b) $\frac{3}{2} - \frac{3}{5}$ c) $\frac{11}{12} - \frac{2}{3}$ d) $\frac{4}{9} + \frac{7}{6}$

2. a) Find three pairs of fractions that have a sum of $\frac{3}{5}$.

b) Find three pairs of fractions that have a difference of $\frac{1}{5}$.

3. Add or subtract.

a) $6\frac{3}{8} - 2\frac{1}{5}$ b) $4\frac{1}{4} + 2\frac{2}{3}$

4. Lana does yard work. The table shows the approximate time for each job.

Job	Time
Mow small lawn	$\frac{1}{2}$ h
Mow large lawn	$\frac{3}{4}$ h
Mow lawn/tidy yard	$1\frac{1}{2}$ h
Plant annuals	$2\frac{1}{2}$ h

For one Saturday, Lana has these jobs:
– mow 3 small lawns
– mow 1 large lawn
– mow lawn/tidy yard in 2 places
– plant annuals in 1 place

Lana needs travel time between jobs, and a break for lunch. Do you think she will be able to do all the jobs? Justify your answer.

5. a) Evaluate.

i) $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ ii) $7 \times \frac{2}{3}$ iii) $\frac{7}{8} \times 8$

b) Convert each answer in part a to a decimal.

c) Order the decimals from least to greatest.

6. A 6-kg bag of fertilizer covers a rectangular area of 2.5 m by 5.0 m.

a) How many bags of fertilizer are needed for a rectangular lawn measuring 7.5 m by 10.2 m?

b) A 6-kg bag of fertilizer costs $15.50. How much does it cost to fertilize the lawn?

7. Evaluate. Explain each step.

$12.4 \times (2.9 + 4.6) + 23.7 \div 2.4$

Unit Problem Publishing a Book

The students at Garden Avenue School are preparing a special book for the school's 100th anniversary. They finance the book by selling advertising space to sponsors.

The students sold the following space:

Full page	$\frac{1}{2}$ page	$\frac{1}{3}$ page	$\frac{1}{4}$ page	$\frac{1}{6}$ page	$\frac{1}{8}$ page
1	1	1	3	4	5

All the advertisements are to fit at the back of the book.

Sam asks: "How many pages do we need for the advertisements?"
Mara asks: "Will the advertisements fill the pages?"
Kathleen asks: "Is there more than one way to arrange these advertisements?"
Can you think of other questions students might ask?

1. Find the total advertising space needed.

2. Sketch the pages to find how the advertisements can be placed. Use grid paper if it helps.

3. Compare your group's sketch with those of other groups. When you made your sketch, what decisions did you make about the shape of each advertisement? Did other groups make the same decisions? Explain.

4. What are the fewest pages needed to display the advertisements? Will there be room for any other advertisements? Explain.

5. What else might students need to consider as they prepare the layout for the book?

Here are the fees for the advertisements.

Size	Full page	$\frac{1}{2}$ page	$\frac{1}{3}$ page	$\frac{1}{4}$ page	$\frac{1}{6}$ page	$\frac{1}{8}$ page
Cost ($)	500	360	250	200	150	100

6. How much money will students get from the advertisers?

7. Students will do the layout on computer.
The cost to copy and bind 500 books is $4750. Use the income from question 6.
   a) What do the students have to charge per book so they do not incur a loss? Justify your answer.
   b) The students cannot print fewer than 500 books.
   What if they can sell only 350 books?
   What do the students have to charge per book so they do not incur a loss? Justify your answer.
   c) What if the students decide to charge $5 per book?
   How many books do the students need to sell so they do not incur a loss? Justify your answer.

Check List

Your work should show:

- ✓ all calculations in detail
- ✓ diagrams of the layout for the advertisements
- ✓ a clear explanation of how you prepared the layout
- ✓ a clear explanation of how students do not incur losses

Reflect on the Unit

Choose one operation with fractions and a different operation with decimals.
Write an example that illustrates how to carry out each operation.

Cumulative Review

UNIT

1

1. Find the number that is:
- a factor of 1000
- not a factor of 100
- a multiple of 5
- has fewer than 3 digits

Explain how you found the number.

2. A rectangular pen is built with 60 m of fencing.
The length of each side of the pen is a whole number of metres.
a) Which dimensions are possible? Make an organized list of your answers.
b) What are the dimensions of the pen with the greatest area? Explain.

3. The number 49 is a perfect square. It can be written as 7^2.
Some numbers that are not perfect squares can be written as the sum of a perfect square and a prime number.
For example: $21 = 4^2 + 5$
a) Find all the numbers between 40 and 50 that can be written this way.
b) Which numbers in part a can be written more than one way? Show each new way.

4. Write the next 3 terms in each pattern. Describe each pattern.
a) 49, 36, 25, 16, . . .
b) 3, 7, 15, 31, . . .
c) 6, 36, 216, 1296, . . .

2

5. Write each ratio in simplest form.

a) saxophones to trumpets
b) drums to flutes
c) saxophones to total number of instruments

6. The ratio of the number of hits to the times at bat for three baseball players is:
Irina, 3:5; Juana, 2:3; Carla, 3:4
a) Who has the best record?
b) Who had the least success?
Explain your answers.

7. The ratio of boys to girls in the sailing club is 4:5.
a) There are 15 girls in the club. How many boys are in the club?
b) What if 3 boys leave the club? What is the new ratio of girls to boys?

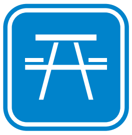
UNIT

3

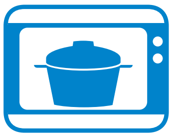
8. Identify the view (front, back, side, or top) and the object on each sign. Then draw a different view of that object.

a) Post office

b) Picnic area

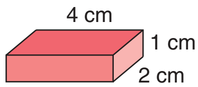


c) Microwave oven

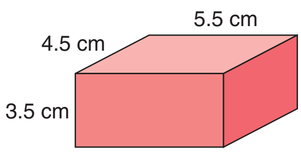


9. Find the surface area of each rectangular prism.

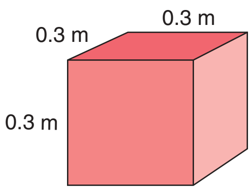
a)



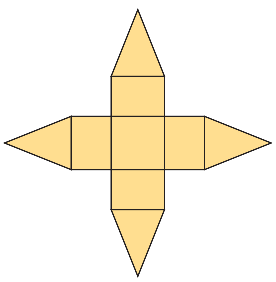
b)



c)



10. Use a large copy of this net.



a) Fold the net to make an object.

b) Sketch the front, back, side, and top view of the object.

4

11. Use fraction strips, number lines, or another model to add or subtract.

a) $\frac{3}{8} + \frac{3}{2}$ b) $\frac{2}{3} + \frac{1}{4}$

c) $\frac{7}{5} - \frac{1}{2}$ d) $\frac{11}{6} - \frac{4}{3}$

12. Skylar and Riley work together on a project.
Skylar does $\frac{3}{10}$ of the work.
Riley does $\frac{2}{5}$ of the work.

a) Who does more work? How much more does this person do?

b) What fraction of the project has yet to be done?

13. Multiply. Draw a picture to show each answer.

a) $4 \times \frac{5}{8}$ b) $\frac{3}{5} \times 5$

c) $3 \times \frac{3}{10}$ d) $5 \times \frac{5}{4}$

14. Multiply.
Record your work on grid paper.

a) 3.5×2.1 b) 0.8×1.2

15. The area of a rectangle is 7.35 m^2.
The width is 2.1 m.
What is the length?

16. Evaluate.

a) $1.25 + 3.6 \times 2.4 \div 1.2$

b) $10.5 \div 1.5 - 2.8 \times 0.3$

c) $10.1 - 4.5 \div 0.5 + 7.5$

d) $(10.1 - 4.5) \div (0.5 + 7.5)$

Why are the answers to parts c and d different?

UNIT 5 Data Management

David and his friend surveyed their classmates. They wanted to find out which outdoor winter activities the students participated in. They recorded the results in a tally chart.

Outdoor Winter Activities	
Activity	**Tally**
Skating	### ### ### ### ////
Skiing	### ### ###
Snowboarding	### ### //
Tobogganing	### ### ### ### ### ###

- What survey question might the students have asked to get the data in the chart?
- How could you graph the data?
- Which graph is most suitable for the data?

What You'll Learn

- Collect and organize data on tally charts and stem-and-leaf plots.
- Display data on frequency tables, bar graphs, pictographs, line graphs, and circle graphs.
- Describe and evaluate data presented on charts, tables, and graphs, and solve related problems.
- Use databases and spreadsheets.
- Calculate and use mean, median, and mode.
- Identify and describe trends in graphs.
- Use technology to draw graphs.

Why It's Important

- You see data and their interpretations in newspapers, magazines, and on TV.
  You need to understand how to interpret these data.

Key Words

- mean
- median
- mode
- primary data
- secondary data
- biased
- database
- fields
- Statistics Canada (Stats Can)
- frequency table
- stem-and-leaf plot
- cluster
- trend
- line graph
- spreadsheet

UNIT 5

Skills You'll Need

Calculating Mean, Median, and Mode

For any set of numbers:

- The **mean** is the sum of all the numbers divided by the number of numbers.
- The **median** is the middle number when the numbers are arranged in order. When there is an even number of numbers, the median is the mean of the two middle numbers. Half the numbers are above the median and half are below.
- The **mode** is the number that occurs most often. There may be more than one mode. There may be no mode.

Example

Calculate the mean, median, and mode of these numbers:
7, 13, 10, 12, 14, 8, 9, 7, 11, 6

Solution

There are 10 numbers in the set.

- For the mean, add the numbers, then divide by 10.

$$\text{Mean: } \frac{7 + 13 + 10 + 12 + 14 + 8 + 9 + 7 + 11 + 6}{10} = \frac{97}{10}$$
$$= 9.7$$

- For the median, arrange the numbers in order, beginning with the least number. The median is the middle number. Since there are 10 numbers, the median is the mean of the two middle numbers. For 10 numbers in order, the first middle number is $\frac{10}{2} = 5$, or the 5th number. The next middle number is the 6th number.

  6, 7, 7, 8, 9, 10, 11, 12, 13, 14

  Median: $\frac{9 + 10}{2} = \frac{19}{2}$, or 9.5

- The mode is the number that occurs most often. The mode is 7.

✓ Check

1. Calculate the mean, median, and mode for the numbers in each set.

a) 3, 9, 5, 8, 2, 0, 9, 5

b) 25, 24, 55, 30, 44, 21, 17, 19, 21

c) 14, 18, 16, 12, 11, 16

d) 76, 81, 50, 64, 67, 69, 72, 94, 81, 76

5.1 Collecting Data

Focus Analyse data collection and possible bias.

Electronic games are popular among Grade 7 students.
Which electronic game do you think Grade 7 students in your class like to play?
How could you find out?

Explore

Work in a group.
Which electronic game is most popular in your class?
Conduct a survey to find out.
What survey question will you ask? Record your results.

Reflect & Share

Compare your results with those of another group.
How did the survey question affect the results?

Connect

Mia wanted to find out the favourite singer of her classmates.
She conducted a survey. She asked this question:

"Who is your favourite singer: Bryan Adams ____, Susan Aglukark ____, Celine Dion ____, Sam Roberts ____, Shania Twain ____, or Other ____?"

Mia recorded the results in a tally chart.

Singer	Number of Students
Bryan Adams	###
Susan Aglukark	###
Celine Dion	### //
Sam Roberts	///
Shania Twain	//
Other	### ### /

Mia concluded that Celine Dion was the most popular singer of those named.
However, if all the votes in the "Other" category were for the same person, then that person would be the most popular.

The word "data" is plural. So, we say "The data are …" A single piece of information is called "datum."

Since Mia collected the data herself, they are called **primary data**.
Data that are found from the library or using the Internet are called **secondary data**.

It is important that a survey is conducted and data are collected in a fair way. Sometimes, the way a question is asked or written might persuade a person to answer a certain way. This type of question is **biased**. A survey question must be unbiased. That is, the question must not lead a person toward a particular answer.

Example

How is each survey question biased?
Rewrite the question so it is unbiased.

a) "Many students are bored at the end of August.
Should the school year be longer?
Yes ________ No ________"

b) "Some people get sick after visiting patients in a hospital.
Do you think patients in a hospital should have visitors?
Yes ________ No ________ No Opinion ________"

Solution

a) The question includes a statement about how some students feel. This may encourage more people to answer "Yes."
An unbiased survey question is:
"The school year is 10 months long. Should it be longer?
Yes _________ No ________"

b) The question includes a statement that may encourage people to answer "No."
An unbiased survey question is:
"Should patients in a hospital have visitors?
Yes ________ No _________ No Opinion ________"

Practice

1. Are primary data or secondary data collected in each situation? How do you know?

a) Elly used an encyclopedia to find the area of each continent.

b) Jason read the thermometer to find the outside temperature.

c) Jane looked in the newspaper to see which NHL team won the game.

2. Would you use primary or secondary data in each case? Explain.

a) To find the favourite car model in Canada

b) To find the favourite juice of Grade 7 students in your school

c) To find the favourite song of Grade 7 students in Ontario

d) To find the most popular type of transport used by students in your school

3. Biased data are unreliable. Yet, sometimes people use biased data. Why do people use biased data?

4. Think of a survey you could conduct in your school.

a) Write a biased survey question.

b) Write an unbiased survey question.

5. Comment on each survey question.
If it is biased, write an unbiased question.

a) "Sugar is bad for your teeth. Should children eat candy?
Yes ________ No ________ No Opinion ________"

b) "Children prefer snowboarding to skiing.
Which do you think is more fun?
Snowboarding ________ Skiing ________"

6. **Assessment Focus** Suppose a person intends to open a shoe store in the local mall.
The person is unsure of the style or make of shoes to stock.
What questions should the person ask to make the best decision?
Explain what role a survey might play in the decision.

7. **a)** Predict the favourite hobby or pastime of your classmates.

b) Write a survey question you could ask to find out.
Explain how you know your question is unbiased.

c) Conduct the survey. Tally the results.

d) How did your prediction compare with your results?

Number Strategies

How are these two patterns related?

- 0.75, 1.5, 3, 6, . . .
- 96, 192, 384, 768, . . .

Reflect

Why is it important that a survey question is unbiased?

Using Databases to Find Data

Focus Search databases for information.

A **database** is an organized collection of data.
Examples of databases include a telephone book, a dictionary, an encyclopedia, or a library catalogue.

A database is organized into **fields**. Each field contains specific information. In a library database, a book may be stored with information in each of these fields: title, author, publisher, subject, ISBN (international standard book number), or Dewey decimal number.

Statistics Canada (Stats Can) is the federal government department that collects, analyses, and stores data about Canada and Canadians. Two of its Internet databases are **CANSIM II** (**Can**adian **S**ocio-economic **I**nformation **M**anagement System) and **E-STAT**.

Stats Can charges a fee for some of its data on CANSIM II.
E-STAT is a free database for teachers and students. Your teacher will give you the website addresses for these databases.

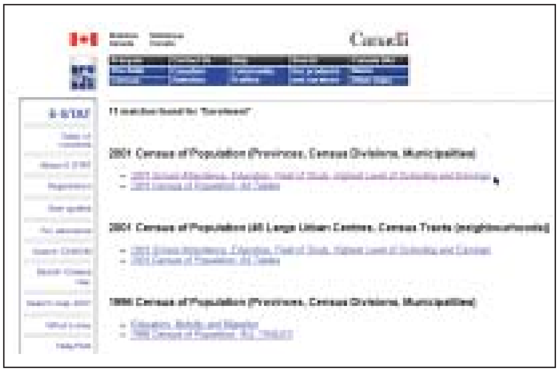
To use E-STAT to find data on school attendance for 15- to 24-year-olds, follow these steps:

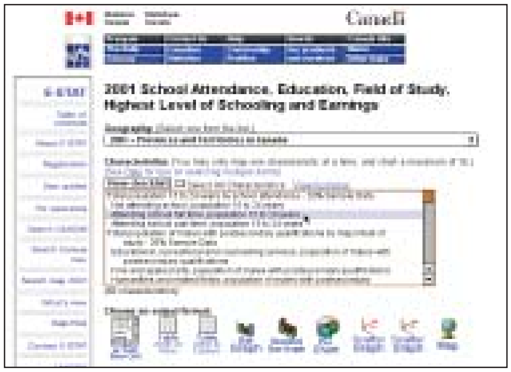
1. Open the E-STAT Website.
You may be asked for your username and password. Ask your teacher for these.

2. Click the “Data” tab. Click “Education,” as shown above left.

3. Under Census databases, click “Enrolment,” as shown below left.

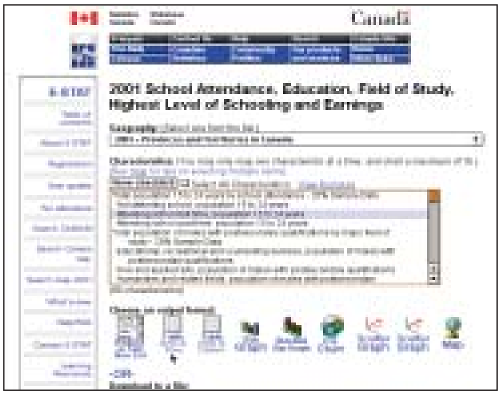
4. Under 2001 Census of Population (Provinces, Census Divisions, Municipalities), click "2001 School Attendance, Education, Field of Study, Highest Level of Schooling and Earnings," as shown below left.

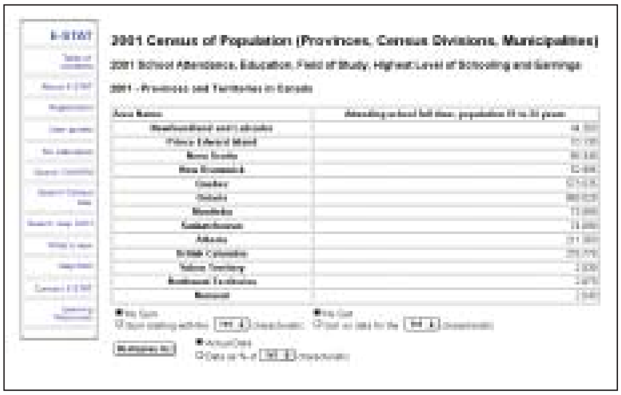




5. Click "Attending school, full time, population 15 to 24 years," as shown above right.

6. Choose an output format. For data in a table arranged in rows, click "Table Areas as Rows," as shown below left. The data are displayed as shown below right.





Check

1. Use E-STAT or another database.
   Research one of the topics below. Print your data.
   Be sure to state your source.
   - The number of Canadians who had a particular type of occupation at the time of the 1911 Census
   - The top 10 movies of the year
   - The top 10 music CDs of the year
   - A topic of your choice

   Write 3 things you know from looking at the data.

5.2 Recording Data

Focus Record data in tables, bar graphs, and pictographs.

Explore

Work on your own.
Use a novel you are currently reading.
Open it to any page.

➢ Count the number of letters in each of the first 50 words.
Record your data in a tally chart.

Number of Letters	Tally of Number of Words
1	
2	
3	
4	

➢ Graph your data. Justify your choice of graph.
➢ Which is the most common length of word?
➢ Who might be interested in these data?

Reflect & Share

Compare your most common length of word with that of a classmate. Should the lengths be the same? Explain.

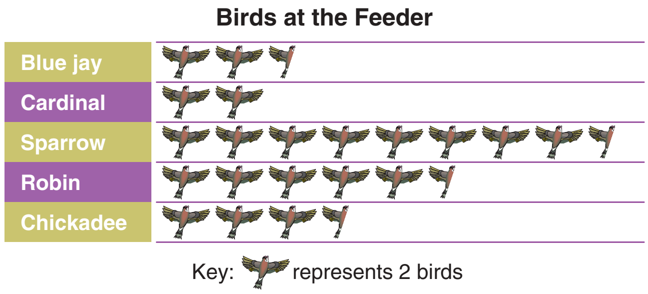
Connect

Data can be organized in a tally chart or **frequency table**.
A frequency table is a tally chart with an extra column.

Andrew recorded the different birds that visited his feeder one morning.

Birds at the Feeder		
Bird	Tally	Frequency
Blue jay	𝍸	5
Cardinal	////	4
Sparrow	𝍸 𝍸 𝍸 //	17
Robin	𝍸 𝍸 /	11
Chickadee	𝍸 //	7

To fill in the *Frequency* column, Andrew counted the tallies for each bird.
He graphed the data using a pictograph.
Andrew chose a key of 1 symbol represents 2 birds.



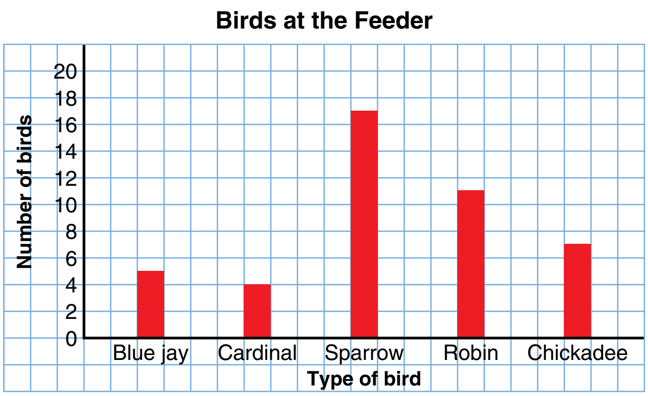
Example

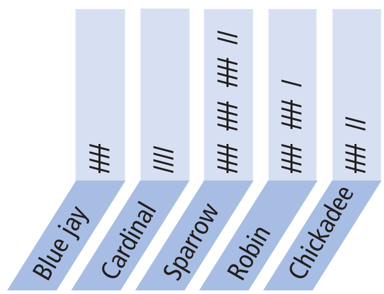
a) Construct a bar graph of Andrew's data.

b) Compare the graph and tally chart.
How are they alike?

Solution

a) Construct a bar graph, below left.
One square represents 2 birds.





b) The longest bar on the bar graph represents the same bird as the longest row on the tally chart.
When we rotate the *Tally* column $\frac{1}{4}$ turn counterclockwise, above right, its shape matches that of the bar graph, above left.

Practice

Favourite Canadian Hockey Team		
Hockey Team	**Tally**	**Frequency**
Calgary Flames	### ///	
Edmonton Oilers	### /	
Montreal Canadiens	###	
Ottawa Senators	###	
Toronto Maple Leafs	### ###	
Vancouver Canucks	### /	

Rolling a Number Cube		
Number	**Tally**	**Frequency**
1		
2		
3		

1. The table shows the favourite Canadian hockey teams of some Grade 7 students at Hamilton Junior High School.

a) Write the frequency of each team.

b) How many students were surveyed?

c) Which team is the most popular among the sample? The least popular? Give some possible reasons for these results.

2. Work with a partner. You will need a number cube labelled 1 to 6.

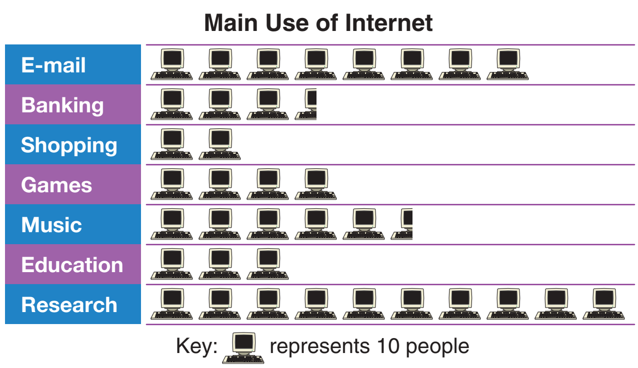
a) Roll the number cube 50 times. Make a table. Record the frequency of each number.

b) Predict the results if you were to roll the number cube 50 more times. Justify your answer.

c) Compare your results with those of another pair of students. If they are different, explain why.

d) Graph your data. Justify your choice of graph.

3. This pictograph displays the results of a survey.



a) What do you think the survey question was? Explain.

b) How many people were surveyed?

c) How would the pictograph change if represented 1 person? 50 people?

d) Draw a bar graph to display these data.

e) Is a bar graph better than a pictograph to display the data? Explain.

f) Write a question you could answer using the pictograph or bar graph. Answer the question.

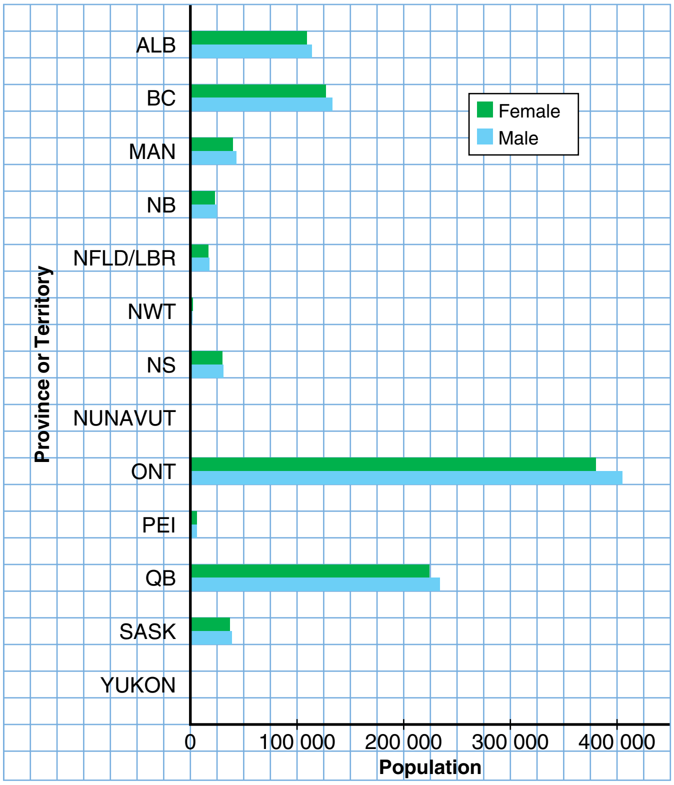
Number Strategies

Add.

- $\frac{1}{2} + \frac{2}{3}$
- $\frac{3}{4} + \frac{4}{5}$
- $\frac{5}{6} + \frac{6}{7}$

4. This is a double-bar graph.

Populations of 10- to 14-year-olds, 2001



a) How is this bar graph different from the others in this lesson?

b) How does the female population compare with the male population?

c) Which province(s) have approximately the same number of 10- to 14-year-old females as males?

d) Here are the data for the 3 territories.

	Nunavut	Northwest Territories	Yukon
Females	1550	1725	1090
Males	1685	1785	1225

Why are there no bars on the graph for Nunavut and Yukon?

e) Could you use a pictograph to display the data in the double-bar graph? Explain.

5. Use an atlas or an almanac.

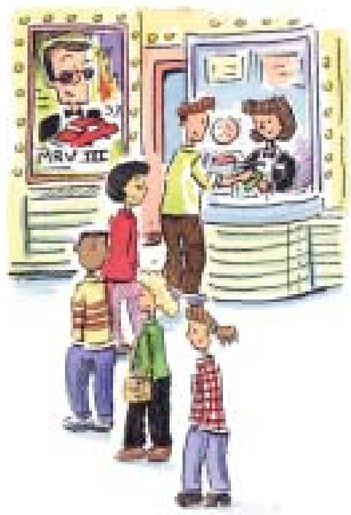
a) Choose 5 cities in Canada.

b) Choose a recent year.
Find the population of each city in that year.

c) Record the data in a table.

d) Graph the data. Justify your choice of graph.

e) What do you know from looking at the data?



6. Assessment Focus

a) Suppose you want to find out how your classmates spend their money. Write a question you could use to survey your classmates.

b) Conduct the survey. Record the data in a frequency table.

c) Graph the data. Justify your choice of graph.

d) What do most of your classmates spend their money on?

e) Use the graph to write 3 other things you know about how your classmates spend their money.

Spelling Mistakes

4 7 9 2 1 0 6 5 8 10 7
3 2 12 8 11 3 4 8 4 5
8 15 9 6 2 1 0 17 19
16 3 11 5 10 7 3 7 1 9
10 3 5 8 6 13 8 2 4 6
18 19 5 20 0 1 8 10 3

7. Two classes of Grade 7 students took a spelling test.
The number of words each student misspelled is recorded at the left.

Number of Mistakes	Tally	Frequency
0 – 2		
3 – 5		

The interval is 3 because the numbers of mistakes are counted in groups of 3.

a) Copy the frequency table. Continue the pattern of the intervals in the 1st column to 18–20.
Complete the 2nd and 3rd columns in the frequency table.

b) Graph the data.

Reflect

When is a pictograph more appropriate than a bar graph?
When is a bar graph more appropriate than a pictograph?
Give examples.

5.3 Stem-and-Leaf Plots

Focus Display data on a stem-and-leaf plot.

The data, and **stem-and-leaf plot**,
show science test scores for a Grade 7 class.
How are the two displays related?
What can you see in the stem-and-leaf plot that you cannot see in the chart?

Science Test Scores

64	75	89	99	58	53	69
66	72	81	88	67	75	86
52	97	77	75	57	61	85
94	72	81	65	78	81	93

Science Test Scores

Stem	Leaf
5	2 3 7 8
6	1 4 5 6 7 9
7	2 2 5 5 5 7 8
8	1 1 1 5 6 8 9
9	3 4 7 9

This is a stem-and-leaf plot.

Explore

Work with a partner.
You will need a metre stick or a tape measure.
Measure each other's height in centimetres.
Write it on the board.
Use the data for the whole class.
Draw a stem-and-leaf plot to show the heights, in centimetres, of your classmates.

➢ Are any students the same height?
➢ Which is the shortest height? The tallest?
➢ What is the median height?
➢ What is the mode height?
➢ What else do you know from looking at the stem-and-leaf plot?

Reflect & Share

Compare your stem-and-leaf plot with that of another pair of classmates. Do your plots match?
If you answer no, how are they different?

Connect

Here are the numbers of books students in one Grade 7 class read in one year.
Each number is the number of books one student read.

Books Read in One Year							
23	19	43	12	16	18	31	17
15	23	27	18	13	16	24	10
16	15	29	25	33	42	20	24
17	38	36	11	26	31	32	19

We can make a stem-and-leaf plot from these data.

- The least number of books is 10. The greatest number is 43. Students read between 10 and 43 books.
  Write the tens digits 1 to 4 in a column.
  These are the **stems**.

Books Read in One Year

Stem	Leaf
1	0 1 2 3 5 5 6 6 6 7 7 8 8 9 9
2	0 3 3 4 4 5 6 7 9
3	1 1 2 3 6 8
4	2 3

In a stem-and-leaf plot, the leaves are always numbers from 0 to 9.

- Start with the least number, 10.
  Write 0 as the **leaf** next to the stem, 1.
  Continue with the next greatest number, 11.
  Write 1 as the next leaf.
  Continue in this way.
  Record all the leaves in order from the least number to the greatest number.

- Write a title for the plot.

Example

Here are the heights, in centimetres, of the members of St. Mark's junior football team.

a) What is the range of the heights of the football players?
b) Draw a stem-and-leaf plot for the data.
c) What is the median height?
d) What is the mode height?

Heights of Football Players in Centimetres								
147	153	162	171	159	168	145	183	164
187	153	164	148	164	167	187	179	
148	190	188	173	155	177	176	189	192

Solution

a) The least height is 145 cm.
The greatest height is 192 cm.
The range is 192 cm – 145 cm = 47 cm.

b) Each number has 3 digits, so the stem will be the hundreds and tens digits.
The leaf will be the ones digit.
The stems are from 14 to 19.
Start with the least number, 145.
Write the leaf, 5, next to the stem, 14.
Continue to write the leaves, from the least number to the greatest number.

Heights of Football Players in Centimetres

Stem	Leaf
14	5 7 8 8
15	3 3 5 9
16	2 4 4 4 7 8
17	1 3 6 7 9
18	3 7 7 8 9
19	0 2

c) There are 26 numbers.
This is an even number.
The median is the mean of the two middle numbers when the numbers are arranged in order.
For 26 numbers in order, the first middle number is $\frac{26}{2} = 13$,
or the 13th number.
The next middle number is the 14th number.
Count the leaves, beginning at 5.
The 13th and 14th numbers are 167 and 168.

Heights of Football Players in Centimetres

Stem	Leaf
14	5 7 8 8
15	3 3 5 9
16	2 4 4 4 7 8
17	1 3 6 7 9
18	3 7 7 8 9
19	0 2

The median is: $\frac{167 + 168}{2} = 167.5$
The median height is 167.5 cm.

d) The mode is the number that occurs most often.
Leaf 4 occurs three times next to stem 16.
The mode is 164 cm.

The *Example* shows that, when the data are displayed in a stem-and-leaf plot, the range, the median, and the mode can be found from the plot.
When you organize data in a stem-and-leaf plot, the original data are visible. A frequency table only shows how many numbers are in each group, and not what the numbers are.

Practice

Hours Worked by Part-Time Staff at a Video Store in One Month

Stem	Leaf
9	1 6 9
10	2 5 6 8
11	0 3 4 4 4 7
12	0 1 2 2 5 6 6 8 9
13	2 3 5 6 7 7 7 7 8 9 9

1. a) What does this stem-and-leaf plot show?

b) How many part-time employees work at the video store?

c) What is the least number of hours worked? The greatest number of hours worked?

d) What is the range of hours worked?

e) What is the median number of hours worked?

f) What is the mode number of hours worked?

2. The masses of parcels, in kilograms, are given.

a) Display the data in a stem-and-leaf plot.

b) Find the greatest mass. The least mass.

c) What is the range of masses?

d) What is the median mass?

e) What is the mode mass?

Masses in Kilograms

28	32	40	36	31
40	48	35	38	34
43	35	41	36	52
37	47	29	42	35
33	39	32	48	37
38	30	35	44	54

3. Which type of data cannot be shown in a stem-and-leaf plot? Explain.

4. Work with a partner. Use a metre stick to measure each other's stride, to the nearest centimetre. Record the measures on the board. Use the measures for the whole class.

a) Make a stem-and-leaf plot.

b) What did you find out about the strides of your classmates? Write down as much as you can.

Calculator Skills

Find two numbers with a difference of 2.75 and a product of 60.125.

5. A food manufacturer claims:
"We guarantee an average of 50 g of peanuts per bag."
In 6 months, Devon found the masses of peanuts, in grams, in 24 bags. Look at the data.

a) Is the food manufacturer's claim true?

b) How could you use a stem-and-leaf plot to justify your answer?

Masses of Peanuts in Grams

50.1, 49.9, 49.0, 48.9,
50.8, 50.3, 51.3, 49.8,
48.8, 49.0, 50.3, 51.2,
49.4, 49.0, 48.1, 49.6,
49.8, 51.3, 50.5, 50.7,
48.7, 51.0, 49.3, 52.5

6. **Assessment Focus**
   a) Collect data on the points scored by a basketball team in its last 15 games. The points could be from games played by your school team or a professional team.
   b) Make a stem-and-leaf plot.
   c) Write 3 things you know from looking at the plot.

Reflect

Explain why it is easier to read data in a stem-and-leaf plot than data in a table.

The World of Work: Meteorologist

A meteorologist is often a specialist. She uses data collection tools suitable for the weather condition being studied. The data collected are pooled nationally and internationally, and studied by a variety of meteorologists and others in related fields. The information and analysis produce severe weather alerts, which can save thousands of lives.

We may not experience surviving a tornado or a hurricane. But there are other weather conditions that are just as destructive.
On January 6, 1998, a severe ice-storm hit Quebec, eastern Ontario, and northeastern U.S. Many people called it the largest ecological disaster in the history of Quebec. On February 6, 1998, Quebec was restored to some normalcy.

When was the last time you heard a "severe weather warning"?
Was the prediction correct?
What kinds of data management tools might have been used to predict this event?

Mid-Unit Review

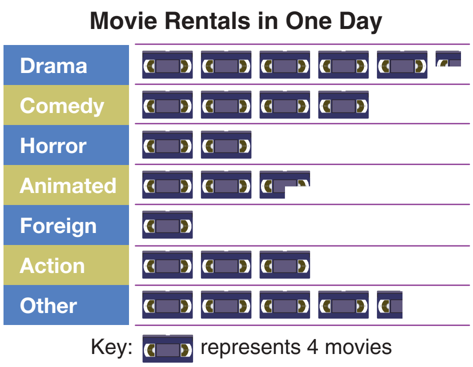
LESSON

5.1 **1.** A survey was conducted to decide if a new hockey arena should be built. The question was, "We have one hockey arena that is always in use. Do we need a new hockey arena?
Yes _____ No _____"

a) Is the question biased? Explain.

b) If your answer to part a is yes, rewrite the question so it is not biased.

5.2 **2.** The pictograph shows the types of movies rented from a video store in one day.



a) Record the data in a frequency table.

b) Construct a bar graph.

c) Write a question you could answer from the bar graph or pictograph.
Answer the question.

d) What else do you know from the graphs?

3. Use an atlas or other database.

a) Choose 5 countries other than Canada.

b) Find the area of each country and Canada.

c) Record the data in a table.

d) Graph the data.

e) How does each country's area compare with Canada's area?

f) What else do you know from the table or graph?

5.3 **4.** Here are the history test scores for students in Ms. Epstein's class.

History Test Scores

56	87	98	34	66	75	50
69	70	83	99	55	83	56
62	90	47	92	75	85	68
98	78	62	51	59	75	81
58	79	80	94	92	63	71

a) Draw a stem-and-leaf plot to represent the data.

b) What is the range of the scores?

c) The pass mark is 50.
How many students did not pass the test?

d) Students who scored below 60 had to rewrite the test.
How many students rewrote the test?

e) Find the median score.

f) Find the mode score.