Chapter 3: Number Relationships

Lesson 1 - Identifying Factors
Learning Goals: Identify factors to solve problems.

Question:
A school in Cochrane, Alberta, has 24 solar panels on the roof to produce electricity.

a) Sketch the possible arrays that can be made with 24 solar panels.

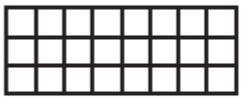
b) How can you use the arrays you sketched in part a) to identify all the factors of 24?

Answer:

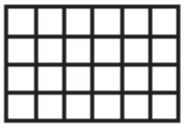
a)

1-by-24

2-by-12



3-by-8



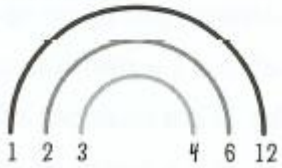
4-by-6

b) For example, each number of rows and columns in an array represents a factor of 24. So the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

At-Home Help

A **factor** is any one of the numbers you multiply.
Here are two ways to identify factors of a number such as 12:

- **Sketch arrays.**
  For example,
  1 row of 12
  2 rows of 6
  3 rows of 4
- **Use a factor rainbow.**
  For example,



Lesson 2 - Identifying Multiples
Learning Goals: Identify multiples to solve problems.

Question:
Pauline expects between 70 and 80 people at the Chinese Mid-Autumn Moon Festival party.
She wants to buy picnic plates and picnic glasses.

a) Plates come in packages of 8. How many plates are in 1 to 10 packages?
b) How many packages of plates does Pauline need to buy? Explain your thinking.
c) Glasses come in packages of 12. How many glasses are in 1 to 10 packages?
d) How many packages of glasses does Pauline need to buy? Explain your thinking.

Answer:

a) 8, 16, 24, 32, 40, 48, 56, 64, 72, 80 plates
b) For example, 10 packages; Pauline needs to buy plates for 80 people, and 80 plates are in 10 packages.
c) 12, 24, 36, 48, 60, 72, 84, 96, 108, 120
d) For example, 7 packages; Pauline needs to buy at least 80 glasses, and 6 packages have 72 glasses, which is too little, but 7 packages have 84 glasses, which is enough.

At-Home Help

A **prime number** is a whole number that has only two different factors: 1 and itself. For example, 7 is a prime number.
A **composite number** is a whole number greater than 1 that has more than two different factors. For example, 6 is a composite number.

Lesson 3 - Prime and Composite Numbers
Learning Goals: Identify prime and composite numbers.

Question:

Suppose you want to arrange the candles on a birthday cake in an array to show your age each year from now until age 18.

a) For what ages can the candles be arranged in only one array? Show your work.

b) How does knowing how to identify prime and composite numbers help you answer part a)?

Answer:

a) For example, every number of candles that is prime can be arranged in only one row or in one column. I will be 12 next month. So when I am 13 or 17, I can arrange the number of candles on a birthday cake in only one array. For all other ages up to 18, I can arrange the number of candles in more than one array.

b) For example, I know prime numbers have only two factors and one of the factors has to be 1. So you can represent the numbers in only one row or one column. Composite numbers have more than two factors so you can arrange them in more than one array. So I just had to identify the prime numbers from 12 to 18 to answer part a).

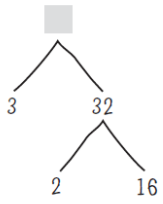
At-Home Help

A **prime number** is a whole number that has only two different factors: 1 and itself. For example, 7 is a prime number.

A **composite number** is a whole number greater than 1 that has more than two different factors. For example, 6 is a composite number.

Lesson 4 - Identifying Factors by Dividing
Learning Goals: Identify factors by dividing composite numbers by primes.

Question:



Manon used a factor tree to help her identify some factors of a number.

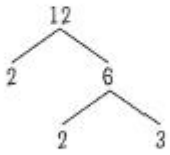
a) What number is on the top of her factor tree? How do you know?
b) How can you tell that 2 and 3 are the only prime numbers that are factors of the number?
c) How can you use Manon's factor tree to identify another factor of her number?

Answer:

a) 96; for example, she divided a number by 3 and got 32. So the number must be $3 \times 32 = 96$.
b) For example, once you get to 16, the only prime number you can divide by is 2.
c) For example, if I divide 16 by 2, I get another factor, 8.

At-Home Help

A factor tree breaks down a composite number by dividing it by prime numbers. For example, here is a factor tree for 12:



This factor tree shows that 2, 6, and 3 are three of the factors of 12.

Lesson 6 - Solving Problems Using an Organized List,
Learning Goals: Use an organized list to solve problems that involve number relationships.

Question:

Natalie and Gwen spin the spinner twice to form a two-digit number. Natalie scores a point if the number is an even multiple of 7. Gwen scores a point if the number is an odd multiple of 9. Who has more ways to score a point? Explain your reasoning.

Answer:

Natalie. For example, use the problem-solving process.

Understand: Since the spinner contains the numbers 1 through 9 and each girl spins the spinner twice, it is possible to create any two-digit number between 11 and 99 that doesn't have 0 as the ones digit. I need to determine even multiples of 7 and odd multiples of 9.

Make a Plan: I will list all the two-digit numbers between 11 and 99 that are multiples of 7 and 9 and don't have 0 as the ones digit. Then, I will circle the even multiples of 7 and odd multiples of 9.

Carry Out the Plan: This is the list of two-digit numbers between 11 and 99 that are multiples of 7 or multiples of 9 and don't have 0 as the ones digit.

Multiples of 7:	14	21	28	35	42	49	56	63	77	84	91	98
Multiples of 9:	18	27	36	45	54	63	72	81	99			

The circled numbers are the even multiples of 7 and the odd multiples of 9.

Multiples of 7:	(14)	21	(28)	35	(42)	49	(56)	63	77	(84)	91	(98)
Multiples of 9:	18	(27)	36	(45)	54	(63)	72	(81)	(99)			

There are six even multiples of 7 between 11 and 99 that don't have 0 as the ones digit, so Natalie has seven ways to score 1 point. There are five odd multiples of 9 between 11 and 99, so Gwen has five ways to score 1 point. Natalie has more ways to score 1 point.

Look Back: I checked all the circled numbers to see if they match the conditions.

$14 = 7 \times 2$ $28 = 7 \times 4$ $42 = 7 \times 6$
$56 = 7 \times 8$ $84 = 7 \times 12$ $98 = 7 \times 14$
$27 = 9 \times 3$ $45 = 9 \times 5$ $63 = 9 \times 7$
$81 = 9 \times 9$ $99 = 9 \times 11$

My solutions are reasonable.

At-Home Help

An organized list can help you solve a problem. For example:

Problem: Which numbers between 10 and 20 are multiples of both 2 and 3?
Solution: List the numbers between 10 and 20 that are multiples of 2. Then circle the numbers that are also multiples of 3.
10, (12), 14, 16, (18), 20
The answer is 12 and 18.

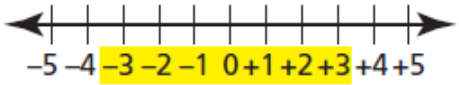
Lesson 7 - Representing Integers
Learning Goals: Use integers to describe situations.

Question:
What integers are between each pair of integers?
Use a number line.

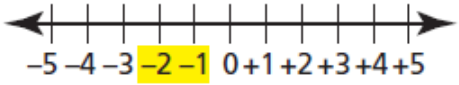
a) −4 and +4
b) −3 and 0
c) −2 and −5
d) 0 and −1

Answer:

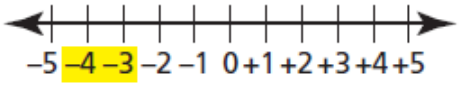
a) The number line shows the integers between –4 and +4. The integers −3, −2, −1, 0, +1, +2, and +3 are between −4 and +4.



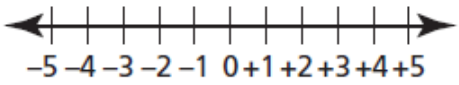
b) The number line shows the integers between –3 and 0. The integers between −3 and 0 are −2 and −1.



c) The number line shows the integers between –2 and –5. The integers between −2 and −5 are −3 and −4.



d) The number line shows that there are no integers between 0 and –1.



At-Home Help

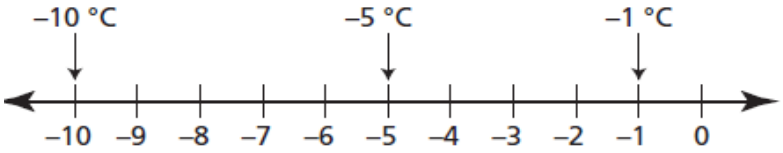
Integers are the counting numbers (+1, +2, +3, ...), zero (0), and the opposites of the counting numbers (−1, −2, −3, ...).

Opposite integers are integers that are the same distance from 0 but on opposite sides of a number line. For example, +4 and −4 are opposite integers.

Lesson 8 - Comparing and Ordering Integers
Learning Goals: Use a number line to compare and order integers.

Question:
How can you tell whether a temperature is colder or warmer than −5 °C? Use a number line to explain.

Answer:
For example, −5 °C is below the freezing point of water so it is to the left of zero on a number line. Temperatures like −10 °C are to the left of −5 °C and are colder than −5 °C. Temperatures like −1 °C are to the right of −5 °C and are warmer than −5 °C. Positive temperatures are to the right of zero, which is to the right of −5 °C, so any positive temperature is warmer than −5 °C.



Lesson 9 - Order of Operations

Learning Goals: Apply the rules for order of operations with whole numbers.

Question:

Each expression has four 4s and is equal to 1.

$4 + 4 \div 4 - 4$ $\quad (4 + 4) \div (4 + 4)$ $\quad 44 \div 44$

a) Show that each expression equals 1.

b) Make new expressions that equal each whole number from 2 to 5.

- You must use four 4s in each expression.
- You may combine digits to form a two-digit number such as 44.
- You may use any operation plus brackets.

Answer:

a) $4 + 4 \div 4 - 4 = 4 + 1 - 4$
$= 5 - 4$
$= 1$

$(4 + 4) \div (4 + 4) = 8 \div (4 + 4)$
$= 8 \div 8$
$= 1$

$44 \div 44 = 1$

b) For example,

$2 = 4 \div 4 + 4 \div 4$

$3 = (4 + 4 + 4) \div 4$

$4 = (4 - 4) \div 4 + 4$

$5 = (4 \times 4 + 4) \div 4$

At-Home Help

Here are the rules for **order of operations**:

- Do the operations in brackets first.
- Next, divide and multiply from left to right.
- Finally, add and subtract from left to right.

For example, calculate
$2 + 5 \times (3 + 1)$.
$2 + 5 \times (4) = 2 + 20 = 22$